Consider the causal model

$$x_{k+1} = OR(x_k, y_k) \; XOR \; e_k$$
$$y_{k+1} = OR(x_k, y_k) \; XOR \; e'_k$$

where

(i) X_k, Y_k, e_k, and e'_k are binary variables, $k = 0, 1, \ldots, 5$,

(ii) e_k, e'_k are independent Bernoulli variables taking on the value 1 with probability p.

(iii) X_0, Y_0 are independent Bernoulli variables with $P(X_0 = 1) = P(Y_0 = 1) = 0.25$.

Use this model to generate n random samples, with each sample consisting of six pairs

$$\{X_k = x, Y_k = y_k\}, k = 0, 1, \ldots, 5.$$

1. Apply the IC algorithm to learn the structure of the generating model, and give the resulting structure for

$$n = 100, 1000.$$
$$p = 0, 0.05, 0.2$$

2. Repeat using the IC* algorithm.

3. For the case $n = 1000$, $p = 0.2$, explore and explain the reasons that the structures discovered by your algorithm differ from those of the generating model.

4. Repeat question 3 for the case $n = 100$, $p = 0$