Philosophiae Naturalis Principia Mathematica

Newton, Isaac
1687

DEFINITIONS.

DEFINITION I

***The quantity of matter is a measure of the same arising jointly from the density and magnitude* [*volume*].**

AIR with the density doubled, in a volume also doubled, shall be quadrupled; in triple the volume, six times as great. You understand the same about snow and powders with the condensing from melting or by compression. And the account of all bodies is the same which are condensed in different ways by whatever causes. Here meanwhile I have no account of a medium, if which there were, freely pervading the interstitia of the parts [of the body]. But I understand this quantity everywhere in what follows under the name of the body or of the mass. That becomes known through the weight of any body: for the proportion to the weight is to be found through experiments with the most accurate of pendulums set up, as will be shown later.

DEFINITION II.

The quantity of motion is a measure of the same arising from the velocity and quantity of matter jointly.

The whole motion is the sum of the motions within the single parts ; and therefore in a body twice as great, with equal velocity, it is doubled, & with the velocity doubled four times as great.

DEFINITION III.

The innate force of matter is the resisting force, by which each individual body, however great it is in itself, persists in its state either of rest or of moving uniformly straight forwards.

This innate [or *vis insita*] force is proportional always to its body, and nor does it differ at all from the inertia of the mass, unless in the required manner of being considered. From the inertia of matter it arises, that each body may be disturbed with difficulty either from its state of rest or from its state of motion. From which the *vis insita* will be possible also to be called by a most significant name the *vis inertiae* [force of inertia]. Truly the body exercises this force only in the change made of its state by some other force impressed on itself ; and the exercise is under that difference with respect to resistance and impetus: Resistance: in as much as the body is resisting a change in its state by the force acting; Impetus: in as much as the same body, with the force of resistance requiring to concede to the obstacle, tries to change the state of this obstacle. One commonly attributes resistance to states of rest and impetus to states of movement : but motion and rest, as they are considered commonly, are distinguished only in turn from each other; nor are [bodies] truly at rest which may be regarded commonly as being in a state of rest.

[The use of the word *directum* , direct or straight forward rather than straight line, as is given in texts on mechanics removes a circular argument from the definition, as the body can only so move in the absence of forces, and cannot be part of the definition as well as a consequence; and there are of course no lines drawn in space, although we could in principle detect deviations of motion along a given direction. Clearly, this was Newton's original meaning, where he uses the word *directum,* and his thoughts on the predominance of Mechanics over Geometry are set out in the Preface to the first edition.]

DEFINITION IV.

The impressed force is the action exercised on the body, to changing the state either of rest or of motion uniform in direction.

This force is in position only during the action, nor remains in the body after the action. For the body may persevere in any new state by the force of inertia only. Moreover the impressed force is of diverse origins, as from a blow, from pressure, or from the centripetal force.

DEFINITION V.

It is the centripetal force, by which bodies are drawn, impelled, or tend in some manner from all sides towards some point, as towards a centre.

Gravity is [a force] of this kind, by which bodies tend towards the centre of the earth ; the magnetic force, by which iron seeks a loadstone; and that force, whatever it may be, by which the planets are drawn perpetually from rectilinear motion, and are forced to revolve along curved lines. A stone rotating in a sling is trying to depart from the turning hand; and in its attempt has stretched the sling, and with that the more the faster it revolves, and it flies off as soon as it is released. I call the force contrary to that endeavour the centripetal force, by which the sling continually pulls the stone back to the hand and keeps it in orbit, as it is directed to the hand or the centre of the orbit. And the account is the same of all bodies, which are driven in a circle. All these are trying to recede from the centre of the orbit; and unless some other force shall be present trying the opposite to this, by which they may be confined and retained in the orbits, and each thus I call centripetal, they will depart with a uniform motion in straight lines. A projectile, if it were abandoned by the force of gravity, would not be deflected towards the earth, but would go in a straight line to the heavens; and that with a uniform motion, but only if the air resistance may be removed. By its gravity it is drawn from the rectilinear course and always is deflected to the earth, and that more or less for its gravity and with the velocity of the motion. So that the smaller were the gravity for a quantity of matter or the greater the velocity with which it was projected, by that the less will it deviate from a rectilinear course and the further it will go on. If a leaden sphere is projected from the peak of some mountain with a given velocity along a horizontal line by the force of gunpowder, it may go on in a curved line for a distance of two miles, before it falls to earth : since here with the velocity doubled it may go on twice as far as it were, and with ten times the velocity ten times as far as it were: but only if the resistance of the air is removed. And by increasing the velocity it may be possible to increase the distance to any desired distance in which it is projected, and the curvature of the line that it may describe be lessened , thus so that it may fall only according to a distance of ten or thirty or ninety degrees ; or also so that it may encircle the whole earth or finally depart into the heavens, and from the departing speed to go on indefinitely. And by the same account, by which the projectile may be turned by the force of gravity in obit and may be able to encircle the whole earth, also the moon is able, either by the force of gravity, but only it shall be of gravity, or some other force , by which it may be acted on, always to be drawn back from a rectilinear course towards the earth, and to be turning in its orbit : and without such a force the moon would not be able to be retained in its orbit. This force, if it should be just a little less, would not be enough to turn the moon from a rectilinear course : if just a little greater, would turn the moon more and it would be led from its orbit towards the earth. Certainly it is required, that it shall be of a just magnitude : and it is required of mathematicians to find the force, by which a body will be able to be retained carefully in some given orbit ; and in turn to find the curved path, in which a body departing from some given place with a given velocity may be deflected by a given force. Moreover the magnitude of this centripetal force is of three kinds, absolute, accelerative, and motive.

[Newton uses some of his later dynamical ideas to refine the centripetal force acting on a body under the influence of a large mass into three parts: the *absolute* force, which depends primarily on the magnitudes of the large mass and small mass, e.g. if the centripetal force were produced on a body near the sun, or near the earth, all else being equal; the *accelerative* force is simply the acceleration due to gravity on a small mass at some location: the force of gravity on a unit mass (i.e. g) ; by *motive* force Newton means the rate of change $\frac{d(mv)}{dt}$ of the quantity of motion mv or momentum, which in turn he calls simply *motion*.]

DEFINITION VI.

The absolute magnitude of a centripetal force is a measure of the same, greater or less, for the effectiveness of the cause of propagating that from the centre into the orbital regions.

So that the magnetic force for the size of the loadstone either extends more in one loadstone of greater strength, or lesser in another.

DEFINITION VII.

The accelerative magnitude of the centripetal force is the measure of this proportional to velocity, that it generates in a given time.

As the strength of the same loadstone is greater in a smaller distance, smaller in greater : or the force of gravity is greater in valleys, less at the tops of high mountains, and small still (as it will become apparent afterwards) at greater distances from the globe of the earth ; but at equal distances it is the same on all sides, because therefore all falling bodies (heavy or light, large or small) with the air resistance removed, accelerate equally.

DEFINITION VIII.

The motive magnitude of the centripetal force is the measure of this, proportional to the motion, which it generates in a given time.

As the weight is greater in a greater body, less in a smaller ; and in the same body greater near the earth, less in the heavens. This magnitude is the centripetency or the propensity of the whole body to move towards the centre, and (as thus I have said) the weight; and it becomes known always by that force equal and opposite to itself, by which the descent of the body can be impeded.

And the magnitudes of these forces for the sake of brevity can be called the motive, accelerative, and absolute forces, and for the sake of being distinct refer to bodies attracted towards the centre, to the locations of these [moving] bodies, and to the centre of the forces: there is no doubt that the motive force for a body, as the attempt of the whole towards the centre [of the attracting body] is composed from the attempts of all the parts ; and the accelerative force at the position of the body, as a certain effectiveness, spread out in the orbit from the centre through the individual locations to the bodies towards moving the bodies which are in these places ; but the absolute force towards the centre, as being provided by some cause, without which the motive forces may not be propagated through the regions in the revolution; or for that cause there shall be some central body (such is the loadstone at the centre of the magnetic force, or the earth from the centre of the force of gravity) or some other [cause] which may not be apparent. The concept here is only mathematical : For I do not consider the causes and physical seats of the forces.

Therefore the accelerative force is to the motive force as the velocity is to the motion. For the quantity of the motion arises from the velocity and also from the quantity of matter; the motive force arises from the accelerative force taken jointly with the same quantity of matter. For the sum of the actions of the accelerative force on the individual particles of the body is the whole motive force [*i.e.* the weight]. From which next to the surface of the earth, where the accelerative gravity or the gravitating force is the same in all bodies, the motive gravity or weight is as the body : but if it may ascend to regions were the accelerative weight shall be less, the weight equally may be diminished, and it will be always as the body and accelerative gravity jointly. Thus in regions were the accelerative gravity is twice as small, the weight of the body small by two or three times will be four or six times as small. Again I name attraction and impulses, in the same sense, accelerative and motive forces. But for these words attraction, impulse, or of any propensity towards the centre, I use indifferently and interchangeably among themselves; these forces are required only to be considered from the mathematical point of view and not physically. From which the reader may be warned, lest by words of this kind he may think me to define somewhere either a kind or manner of action or a physical account, or to attribute truly real forces to the centres (which are mathematical points); if perhaps I have said to draw from the centre or to be forces of the centres.

Scholium.

Up to this stage it has been considered to explain a few notable words, and in the following in what sense they shall be required to be understood. Time, space, position and motion, are on the whole the most notable. Yet it is required to note that ordinary people may not conceive these quantities otherwise than from the relation they bear to perception. And thence certain prejudices may arise, with which removed it will be agreed to distinguish between the absolute and the relative, the true and the apparent, the mathematical and the common usage.

l. Absolute time, true and mathematical, flows equably in itself and by its nature without a relation to anything external, and by another name is called duration. Relative, apparent, and common time is some sensible external measure of duration you please (whether with accurate or with unequal intervals) which commonly is used in place of true time; as in the hour, day, month, year.

II. Absolute space, by its own nature without relation to anything external, always remains similar and immovable: relative [space] is some mobile measure or dimension of this [absolute] space, which is defined by its position to bodies according to our senses, and by ordinary people is taken for an unmoving space: as in the dimension of a space either underground, in the air, or in the heavens, defined by its situation relative to the earth. Absolute and relative spaces are likewise in kind and magnitude; but they do not always endure in the same position. For if the earth may move, for example, the space of our air, because relative to and with respect to the earth it always remains the same, now there will be one part of absolute space into which the air moves, now another part of this; and thus always it will be moving absolutely.

llI. The position [or place] is a part of space which a body occupies, and for that reason it is either an absolute or relative space. A part of space, I say, not the situation of [places within] the body, nor of the surrounding surface. For there are always equal

positions[within] equal solid shapes; but not so surfaces as most are unequal on account of dissimilarities of the figures;
[for a surface is liable to change, due to air resistance, etc.]
Truly positions may not have a magnitude on speaking properly, nor are they [to be considered] as places rather than as affectations of places [*i.e*. the position is not a physical property of the body, but rather an indication of where the body is situated at some time in space]. The whole motion is the same as the sum of the motions of the parts, that is, the translation of all from its place is the same as the sum of the translations of the parts from their places ; and thus the place of the whole is the same as the sum of all the parts of the places and therefore both inside and with the whole body.

IV. An absolute motion is the translation of a body from one absolute place into another absolute place, a relative [motion] from a relative [place] into a relative [place]. Thus in a ship which is carried along in full sail, the relative position of the body is that region of the ship in which the body moves about, or the part of the whole cavity of the ship [hold] which the body fills up, and which thus is moving together with the ship : and relative quiet is the state of being of the body in that same ship or in the part of the hold. But the persistence of the body is true rest in the same part of space in which the ship is not moving, in which the ship itself together with the hold and all the contents may be moving. From which if the earth truly is at rest, the body which relatively at rest in the ship, truly will be moving and absolutely with that velocity by which the ship is moving on the earth. If the earth also is moving; there is the true and absolute motion of the body, partially from the motion of the ship truly in an unmoving space, partially from the motion of the ship relative to the earth: and if the body is moving relatively in the ship, the true motion of this arises, partially from the true motion of the earth in motionless space, partially from the relative motion both of the ship on the earth as well as of the body in the ship ; and from these relative motions the motion of the body relative to the earth arises. So that if that part of the earth, where the ship is moving, truly is moving to the east with a speed of 10010 parts; and by the wind in the sails the ship is carried to the west with a velocity of ten parts ; moreover a sailor may be walking on the ship towards the east with a velocity of one part : truly the sailor will be moving and absolutely in the immobile space with a velocity of 10001 to the east, and relative to the earth towards the east with a speed of nine parts.

Absolute time is distinguished from relative time in astronomy by the common equation of time. For the natural days are unequal, which commonly may be taken as equal for the measure of time. Astronomers correct this inequality, so that they measure the motion of the heavens from the truer time. It is possible, that there shall be no uniform motion, by which the time may be measured accurately. All motions are able to be accelerated and retarded, but the flow of absolute time is unable to change. The duration or the perseverance of the existence of things is the same, either the movement shall be fast or slow or none at all: hence this is distinguished by merit from the sensibilities of their measurement, and from the same [the passage of time] is deduced through an astronomical equation. But a need prevails for phenomena in the determination of this equation, at some stage through an experiment with pendulum clocks, then also by the eclipses of a satellite of Jupiter.

As the order of the parts of time is unchangeable, thus too the parts of space. These could be moved from their own places (as thus I may say), and they will be moved away from each other [*i.e.* out of sequence]. For the times and the spaces are themselves of this [kind] and as if the places of all things: in time according to an order of successions, and in space according to an order of positions, to be put in place everywhere. Concerning the essence of these things, it is that they shall regarded as places : and it is absurd to move the first places. These therefore are absolute places ; and only the translations from these places are absolute motions.

In truth since these parts of space are unable to be seen, and to be distinguished from each other by our senses; we use in turn perceptible measures of these. For we define all places from the positions and distances of things from some body, which we regard as fixed : and then also we may consider all motion with respect to the aforementioned place, as far as we may conceive bodies to be transferred from the same. Thus in exchange of absolute places and motions we make use of relative ones ; not to be an inconvenience in human affairs : but required to be abstracted from the senses in [natural] philosophical matters. And indeed it can happen, that actually no body may be at rest, to which the position and motion may be referred to.

But rest and motion, both absolute and relative, are distinguished in turn from each other by their properties, causes and effects. The property of rest is, that bodies truly at rest are at rest among themselves. And thus since it shall be possible, that some body in the regions of fixed [stars], or far beyond, may remain absolutely at rest ; moreover it is impossible to know in turn from the situation of bodies in our regions, whether or not any of these given at a remote position may serve [to determine true rest in the absolute space for local bodies]; true rest cannot be defined from the situation of these bodies between themselves.

A property of motion is, that the parts which maintain given positions to the whole, share in the motion of the whole. For all the rotating parts are trying to recede from the axis of the motion, and the impetus of the forwards motion arises from the impetus of the individual parts taken together. Therefore with the motion for circulating bodies [*e.g.* planets], they do move in circles [*i.e.* orbits] in which they are relatively at rest. And therefore true and absolute motion cannot be defined by a translation from the vicinity of such bodies, which [otherwise] may be regarded as being in a state of rest. For external bodies [introduced by way of example] must not only seem as being in a state of rest, but also truly to be at rest. Otherwise everything included also participates in the true orbiting motion, besides a translation from the vicinity of the orbiting body ; and with that translation taken away they are not truly at rest, but they may be seen only at rest in this manner. For the orbiting bodies are to the included, as the total exterior part to the interior part, or as a shell to the kernel. But with the shell moving also the kernel is moving, or a part of the whole, without a translation from the vicinity of the shell.

The relation to the preceding property is this, because in the place moved a single location is moved : and thus a body, which is moved with the place moved, also shares the motion of its place. Therefore all [relative] motions, which are made from moved places, are only parts of both the whole and absolute motions, and every whole motion is composed from the motion of the body from its first place, and from the motion of this place from its own place in turn, and thus henceforth ; until at last it may arrive at a stationary place, as in the example of the sailor mentioned above. From which whole and absolute motions can be defined only from unmoved places: and therefore above I have referred to these as immovable places, and relative places to be moveable places. But they are not immovable places, unless all the given positions may serve in turn from infinity to infinity ; and so always they remain immovable, and I call the space which they constitute immovable.

The causes, by which true and relative motions can be distinguished from each other in turn, are the forces impressed on bodies according to the motion required to be generated. True motion neither can be generated nor changed, other than by forces impressed on the motion of the body itself: but relative motion can be generated and changed without forces being impressed on this body. For it suffices that they be impressed on other bodies alone to which the motion shall be relative, so that with these yielding, that relation may be changed from which it consisted, of rest or relative motion. On the contrary true motion always is changed from the forces impressed on a moving body ; but the relative motion from these forces is not changed by necessity. For if the same forces thus may be impressed on other bodies also, for which a relation is made, thus so that the relative situation will be conserved on which the relative motion is founded. Therefore all relative motion can be changed where the true may be conserved, and to be conserved where the true may be changed ; and therefore true motion in relations of this kind are considered the least.

The effects, by which absolute and relative motions are to be distinguished from each other, are the forces of receding from the axis of circular motion. For none of these forces in circular motion are in mere relative motion, but are in a true [circular] motion greater from true absolute motion for a quantity of motion. If a vessel may hang from a long thread, and always is turned in a circle, while the thread becomes very stiff, then it may be filled with water, and together with the water remains at rest; then by another force it is set in motion suddenly in the opposite sense and with the thread loosening itself, it may persevere a long time in this motion; the surface of the water from the beginning was flat, just before the motion of the vessel: But after the vessel, with the force impressed a little on the water, has the effect that this too begins to rotate sensibly; itself to recede a little from the middle, and to ascend the sides of the vessel, adopting a concave figure, (as I have itself tested) and by moving faster from the motion it will rise always more and more, while the revolutions by being required to be completed in the same times with the vessel, it may come to relative rest with the same vessel. Here the ascent indicates an attempt to recede from the axis of the motion, and by such an attempt it becomes known, and the true and absolute circular motion of the water is measured, and this generally is contrary to the relative motion. In the beginning, when the motion of the water was a maximum relative to the vessel, that motion did not incite any attempt to recede from the axis: the water did not seek circumference by requiring to ascend the sides of the vessel, but remained flat, and therefore the true circular motion had not yet begun. Truly later, when the relative motion had decreased, the ascent of this to the sides of the vessel indicated an attempt of receding from the axis; and this trial showed this true circular motion always increasing, and finally made a maximum when the water remained at rest relative to the vessel. Whereby this trial does not depend on the translation of the water with respect to orbiting bodies, and therefore true circular motion cannot be defined by such translations. Truly the circular motion of each revolving body is unique, corresponding to a singular and adequate effort to be performed : but relative motions are for innumerable and varied external relations ; and corresponding to a relation, generally they are lacking in true effects, unless in as much as they share in that true and single motion. And by those who wish, within a system of these [rotational motions], our heavens [*i.e.* local space] to revolve in a circle below the heavens of the fixed stars [*i.e.* distant interstellar space], and the planets to defer with it ; the individual parts of the heavens, and the planets which truly are moving, which indeed within their nearby heavens [*i.e.* the local part of space relative to themselves] are relatively at rest, truly are moving. For they change their positions in turn (as otherwise the system truly passes into rest) and together with the deferred heavens they participate in the motion of these, and so that the parts of the revolving total are trying to recede from the axes of these.

Relative quantities are not therefore these quantities themselves, the names of which they bear, but those perceptible measures (true or mistaken) of them which are used by ordinary people in place of the measured quantities. [Thus, a length is related to a standard length, etc.] But if the significances of words are required to be defined from the use; these measures perceptible [to the senses] are to be particularly understood by these names : Time, Space, Location and Motion ; and the discourse will be contrary to custom and purely mathematical, if measured quantities here are understood. [Here Newton is expressing the fact that he uses such quantities in our sense as abstract variables, rather than as mere units for measuring the amounts of physical quantities, as one might use in arithmetic.] Hence they carry the strength of holy scriptures, which may be interpreted there by these names regarding measured quantities. Nor do they corrupt mathematics or [natural] philosophy any less, who combine true quantities with the relations of these and with common measures.

Indeed it is most difficult to know the true motion of bodies and actually to discriminate from apparent motion ; therefore because the parts of that immobile space, in which bodies truly are moving, do not meet the senses. Yet the cause is not yet quite desperate. For arguments are able to be chosen, partially from apparent motions which are the differences of true motions, partially from forces which are the causes and effects of true motions. So that if two globes, to be connected in turn at a given distance from the intervening thread, may be revolving about the common centre of gravity; the exertion of the globes to recede from the axis of the motion might become known from the tension in the thread, and thence the quantity of the circular motion can be computed. Then if any forces acting equally likewise may be impressed mutually on the faces of the globes to increase or diminish the circular motion, the increase or decrease in the circular motion may become known from the increase or decrease in the tension of the thread ; and thence finally the faces of the globes on which the impressed forces must be impressed, so that the motion may be increased maximally; that is, the faces to the rear, or which are following in the circular motion. But with the faces which are following known, and with the opposite faces which precede, the determination of the motion may be known. In this manner both the quantity and the determination of the motion of this circle may be found in a vacuum however great, where nothing may stand out externally and perceptibly by which the globes may be able to be brought together [in comparison]. If now bodies may be put in place in that space with a long distance maintained between themselves, such as the fixed stars are in regions of the heavens : indeed it may not be possible to know from the relative translation of the globes among the bodies, whether from these or those a motion may be required to be given. But if attention is turned to the string, and the tension of that itself is taken to be as the required motion of the globes ; it is possible to conclude that the motion is that of the globes, and [the distant] bodies to be at rest ; & then finally from the translation of the globes among the bodies, the determination of this motion can be deduced. But the true motion from these causes, are to be deduced from the effects and from the apparent differences, or on the contrary from the motions or forces, either true or apparent, the causes and effects of these to be found, will be taught in greater detail in the following. For towards this end I have composed the following treatise.

AXIOMS,

OR

THE LAWS OF MOTION.

LAW I.

Every body perseveres either in its state of resting or of moving uniformly in a direction, unless that is compelled to change its state by impressed forces.

PRojectiles persevere in their motion, unless in as much as they may be retarded by the resistance of the air, and they are impelled downwards by the force of gravity. A child's spinning top, the parts of which by requiring to stick together always, withdraw themselves from circular motion, does not stop rotating, unless perhaps it may be slowed down by the air. But the greater bodies of the planets and comets preserve both their progressive and circular motions for a long time made in spaces with less resistance .

LAW II.

The change of motion is proportional to the [*magnitude of the*] *impressed motive force, and to be made along the right line by which that force is impressed.*

If a force may generate some motion ; twice the force will double it, three times triples, if it were impressed either once at the same time, or successively and gradually. And this motion (because it is determined always in the same direction generated by the same force) if the body were moving before, either is added to the motion of that in the same direction, or in the contrary direction is taken away, or the oblique is added to the oblique, and where from that each successive determination is composed.

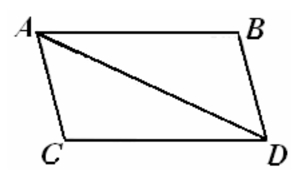
LAW III.

To an action there is always an equal and contrary reaction : *or the actions of two bodies between themselves are always mutually equal and directed in opposite directions.*

Anything pressing or pulling another, by that is pressed or pulled just as much. If anyone presses a stone with a finger, the finger of this person is pressed by the stone. If a horse pulls a stone tied to a rope, and also the horse (as thus I may say) is drawn back equally by the stone: for the rope stretched the same on both sided requiring itself to be loosened will draw upon the horse towards the stone, and the stone towards the horse; and yet it may impede the progress of the one as much as it advances the progress of the other. If some body striking another body will have changed the latter's motion in some manner by the former's force, the same change too will be undergone in turn on its own motion in the contrary direction by the force of the other (on account of the mutual pressing together). By these actions equal changes are made, not of the velocities but of the motions ; obviously in bodies not impeded otherwise. For changes of the velocity, are made likewise in the contrary parts, because the motions are changed equally, they are inversely proportional with the bodies [*i.e.* with their masses]. This law is obtained with attractions also, as will be approved in a nearby scholium.

COROLLARY I.

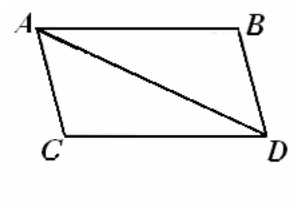
A body with forces added together describes the diagonal of the parallelogram in the same time, in which the separate sides are described.



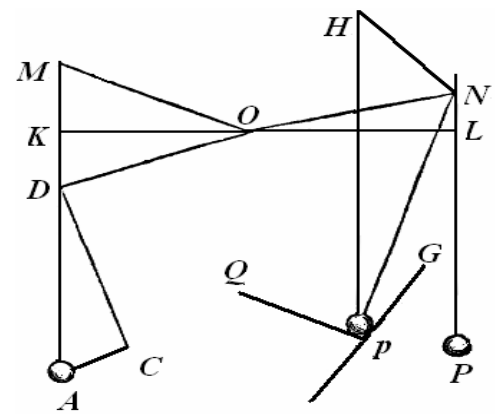
If the body in a given time, by the force alone *M* impressed at the place *A*, may be carried with a uniform motion from *A* to *B;* and by the single force *N* impressed at the same place, may be carried from A to *C*: the parallelogram *ABDC* may be completed, and by each force that body may be carried in the same time on the diagonal from *A* to *D.* For because the force *N* acts along the line *AC* parallel to *BD* itself, this force by law II will not at all change the velocity required to approach that line *BD* generated by the other force. Therefore the body approaches the line *BD* in the same time, whether the force *N* may be impressed or not ; and thus at the end of the time it may be found somewhere on that line *BD.* By the same argument at the end of the same time the body will be found somewhere on the line *CD,* and on that account it is necessary to be found at the concurrence *D* of each of the lines. Moreover it will go in rectilinear motion from *A* to *D* by law I.

COROLLARY II.

And hence the composition of the force directed along AD is *apparent from any oblique forces* AB *and* BD*, and in turn the resolution of any force directed along* AD *into any oblique forces* AB *and* BD. *Which composition and resolution indeed is confirmed abundantly from mechanics.*



So that if from the centre of some wheel *O* the unequal radii *OM*, *ON* emerge , the weights *A* and *P* may be sustained by the threads *MA*, *NP*, and the forces of the weights are required towards moving the wheel: Through the centre *O* a right line *KOL* is drawn, meeting the threads perpendicularly at *K* and *L*, and with the centre *O*, and *OL* the greater of the intervals *OK*, *OL*, a circle is described meeting the thread *MA* in *D:* and *AC* shall be parallel to the right line made *OD* , and the perpendicular *DC*. Because it is of no importance, whether the points of the threads *K, L, D* shall be fastened or not to the plane of the wheel; the weights will prevail the same, and if they may be suspended from the points *K* and *L* or *D* and *L*. But the total force of the weight *A* is set out by the line *AD*, and this is resolved into the forces *A*C, *CD*, of which A*C* by pulling [*drawing* in the original text] the radius *OD* directly from the centre provides no force to the required wheel movement ; but the other force *DC*, by pulling on the radius *DO* perpendicularly, accomplishes the same, as if it pulls the radius *OL* the equal of *OD* itself; and that is, the same weight *P*, but only if that weight shall be to the weight *A* as the force *DC* to the force *DA*, that is (on account of the similar triangles *ADC*, *DOK*,) as *OK* to *OD* or *OL*. Therefore the weights *A* and *P*, which are inversely as the radii *OK* and *OL* placed in line, will exert the same influence, and thus remain in equilibrium : which is the most noticeable property of scales, levers, and of a wheel and axle. If either weight shall be greater than in this ratio, the force of this will be so much greater requiring the wheel to rotate.



But if a weight *p*, equal to the weight *P*, is suspended partly by the thread *Np*, and partly by resting on the oblique plane *pG* : [the forces] *pH* and *NH* are acting, the former perpendicular to the horizontal, the latter perpendicular to the plane *p*G; and if the force of the weight *p* acting downwards, is set out by the line *pH*, this can be resolved into the forces *pN, HN*. If some plane *pQ* shall be perpendicular to *pN*, cutting the other plane *pG* in a line parallel to the horizontal [here we have to think in 3 dimensions]; and the weight *p* lies only on the planes *pQ*, *p*G ; that it may press on these planes by these forces *pN*, *HN*, without doubt the plane *pQ* perpendicularly by the force *pN*, and the plane *pG* by the force *HN*. And thus if the plane *pQ* may be removed, so that the weight may stretch the thread ; because now the thread in turn by being required to sustain the weight, performs the function of the plane removed, may be extended by that same force *pN*, which before acted on the plane. From which, the tension of this oblique thread will be to the tension of the thread of the other perpendicular *PN*, as *pN* to *pH.* And thus if the weight *p* shall be to the weight *A* in a ratio, which is composed from the reciprocal ratio of the minimum distances of their threads *pN, AM* from the centre of the wheel, and in the direct ratio *pH* to *pN;* the same weights likewise will prevail for the wheel being moved, and thus mutually will sustain each other, as any test can prove.

But the weight *p*, pressing on these two oblique planes, can be compared to a wedge making a split between the internal surfaces: and thence the forces of the wedge and of the hammer become known: seeing that since the force by which the weight *p* presses hard on the plane *pQ* to the force, by which the same either by its weight or impelled by the blow of the hammer acting along the line *pH* in the plane, shall be as *pN* to *pH;* and to the force, by which it presses hard on the other plane *pG*, as *pN* to *NH.* But also the force of a screw can be deduced by a similar division of the forces; obviously which wedge is pushed against by a lever. Therefore the uses of this corollary appear to be the widest, and by extending widely the truth of this prevails; since from now with what has been said, all mechanical devices may depend on explanations shown in different ways by authors. And from these indeed we may derive easily the forces of machines, which from wheels, revolving cylinders, pulleys, levers, stretched cords and weights directly or obliquely ascending, and with the rest from the powers of mechanics are accustomed to be assembled, and as also the forces of tendons requiring the bones of animals to be moved.

[The diagram has been altered a little, as in the original the oblique angle appears to be 90^0. Note that Newton pays a little attention here to statics and simple machines.]

COROLLARY III.

The quantity of motion which is deduced by taking the sum of the motions of the contributing factors in the same direction and the difference of the contributing factors in the opposite direction , may not change from the action of bodies among themselves.

And indeed the action and the contrary reaction to that are equal by law III, and thus by law II bring about equal changes in the motions towards the contrary directions. Therefore if the motions are made in the same direction; whatever is added to the motion of the departing body, is taken from the motion of the following body thus, so that the sum may remain the same as before. If the bodies may get in each other's way, an equal among will be taken from the motion of each, and thus the difference of the contributing factors of the motions in the opposite directions will remain the same.

So that if a spherical body *A* shall be three times greater than a spherical body *B*, and it may have two parts of velocity ; and *B* may follow on the same right line with a velocity of ten parts, and thus the motion of *A* shall be to the motion of *B* itself, as 6 to 10 : the motion from these may be put to be of 6 parts and of 10 parts, and the sum will be of 16 parts. Therefore in the meeting of the bodies, if the body *A* may gain a motion of 3, 4, or 5 parts, the body *B* will lose just as many parts, and thus the body *A* will go on after the reflection with 9, 10 or 11 parts, and *B* with 7, 6, or 5 parts, with the same sum of the parts present as before. If the body *A* may gain 9, 10, 11, or 12, and thus may move past the meeting with 15, 16, 17, or 18 parts ; the body *B*, by losing as many parts as *A* may gain, may be progressing with one part, with 9 parts lost, or it may be at rest with the 10 parts of its motion missing, or with one part it may be moving backwards with one part more (as thus I may say) missing from its motion, or it may be moving backwards with two parts on account of subtracting 12 parts of the motion forwards. And thus the sum of the motions in the same direction $15+1$ or $16+0$, and the differences in the opposite directions $17-1$ and $18-2$ will be always 16 parts, as before the meeting and the reflection. But with the motions known with which bodies go on after the reflection, the velocities of which may be found, on putting that to be to the velocity before the reflection, as the motion after is to the motion before. So that in the final case, where the motion of the body *A* were of six parts before reflection and of eight parts afterwards, and the velocity of two parts before the reflection; the velocity of six parts after the reflection may be found, on being required to say, that the motion of 6 parts before the reflection to the 18 parts afterwards, thus of 2 parts of velocity before the reflection to 6 parts of velocity afterwards.

But if bodies are incident between themselves mutually obliquely either non spherical or with differing rectilinear motions, and the motions of these may be required after reflection ; the position of the plane in which the meeting bodies are touching at the point of concurrence is required to be known: then the motion of each body (by Corol.II) is required to be separated into two parts, one perpendicular to this plane, the other parallel to the same : but the parallel motions, therefore because the bodies act in turn between themselves along a line perpendicular to this plane, are required to have retained the same motion before and after the reflection, and the changes in the perpendicular motions thus required are to be attributed equally in opposite directions, so that both the sum of the acting together and the difference of the contrary may remain the same as before. From reflections of this kind also circular motions are accustomed to arise about their own centres. But I will not consider these cases in the following, and it would be exceedingly long to consider showing all this here.

COROLLARY IV.

The common centre of gravity of two or more bodies, from the actions of the bodies between themselves, does not change its state either of motion or of rest ; and therefore the common centre of gravity of all bodies in the mutual actions between themselves (with external actions and impediments excluded) either is at rest or is moving uniformly in direction.

For if two points may be progressing with a uniform motion in right lines, and the distances of these is divided in a give ratio, the dividing point either is at rest or it is progressing uniformly in the right line. This is shown later in a corollary to lemma XXIII of this work, if the motion of the points is made in the same plane ; and by the same account can be demonstrated, if these motions are not made in the same plane. Hence if some bodies are moving uniformly in right lines, the common centre of gravity of any two either is at rest or progresses uniformly in a right line; because the line joining the centres of these bodies therefore is required to be progressing uniformly in right lines, it is divided by this common centre in a given ratio. And similarly the common centre of these two and of any third either is at rest or progressing uniformly in a right line ; because the distances between the common centre of the two bodies and of the third body from that therefore is divided in a given ratio . In the same manner also the common centre of these three and of some fourth body either is at rest or is moving uniformly in a straight line; because from that the distance between the common centre of the three and the centre of the fourth body therefore is divided in a given ratio, and thus *ad infinitum*. Therefore in a system of bodies, which in the interactions among themselves in turn and in general are free from all extrinsic forces, and therefore singly are moving uniformly in individual right lines, the common centre of gravity of all either is at rest or is moving in a direction uniformly.

Again in a system of two bodies acting on each other in turn, since the distances of the centres of each from the common centre of gravity shall be reciprocally as the bodies [*i.e.* the masses of the bodies]; the relative motions of the bodies from the same shall be equally between themselves either approaching to that centre or receding from the same. Hence that centre, from the equal changes of the motions made in opposite directions, and thus from the actions of these bodies between themselves, neither will move forwards nor be retarded nor suffers a change in its state as far as motion or rest is concerned. But in a system of several bodies, because the common centre of gravity of any two bodies acting mutually between themselves on account of that action allows no change at all in its state ; and of the remaining , the action from which does not hinder these, the common centre of gravity thence suffers nothing; but the distance of these two centres of gravity is divided by the common centre of all the bodies into parts to which the total [masses] of the bodies are reciprocally proportional ; and thus with the state of their moving or resting maintained from these two centres, the common centre of all also maintains its own state: because it is evident that common centre of all on account of the actions of two bodies between themselves at no time changes its own state as far as motion and rest are concerned. But in such a system all the actions of the bodies between each other, are either between two bodies, or between the actions of two composite bodies ; and therefore at no time do they adopt a change of everything from the common centre in the state of this of motion or rest. Whereby since that centre where the bodies do not act among themselves in turn, either is at rest, or is progressing uniformly in some right motion, will go on the same, without the opposition of bodies, without actions between themselves, either to be at rest or always to be progressing uniformly in a direction ; unless it may be disturbed from that state by some external forces. Therefore it is the same law of a system of many bodies, which of a solitary body, as far as persevering in a state of motion or of rest. Indeed the progressive motion either of a solitary body or of a system of bodies must always be considered from the motion of the centre of gravity.

COROLLARY V.

The motions of bodies between themselves included in a given space are the same, whether that space be at rest, or the same may be moving in a direction without circular motion.

For the differences of the motions given in the same direction , and the sums to be given in the opposite directions, are the same initially in each case (by hypothesis), and with these sums or differences the collisions and impulses arise with which the bodies strike each other. Therefore by law II the effects of the collisions are equal in each case ; and therefore the motion between themselves in one case will remain equal to the motions between themselves in another case. The same may be proven clearly by an experiment. All motions between bodies occur in the same way on a ship, whither that is at rest or is moving uniformly in a direction.

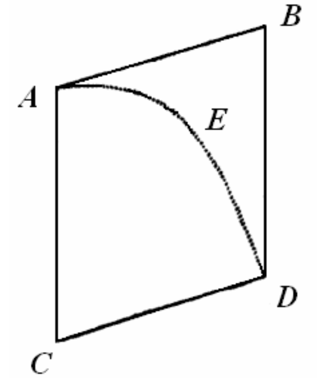
COROLLARY VI.

If bodies may be moving in some manner among themselves, and from the forces with equal accelerations may be impressed to move along parallel lines ; all will go on to move in the same manner between themselves, and if by these forces they are not to be disturbed.

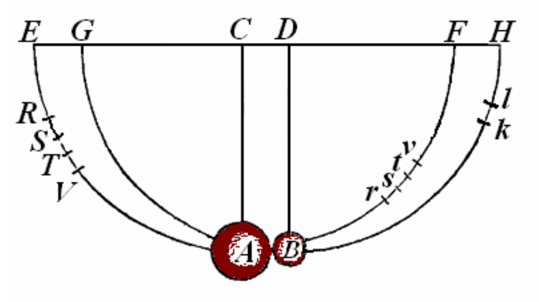
For these forces equally (for the quantities of the bodies required to be moved) and by acting along parallel lines, all the bodies will be moved equally (as far as velocity is concerned) by law II, and thus at no time will the positions and motions of these between each other be changed.

Scholium.

Thus far I have treated the fundamentals with the usual mathematics and confirmed many times by experiment. By the first two laws and from the first two corollaries Galileo found the fall of weights to be in the duplicate ration of the time, and the motion of projectiles to be in a parabola; by agreeing with experiment, unless as far as the those motions are retarded a little by the resistance of the air. With a body falling uniform gravity, by acting equally in equal small intervals of time, will impress equal forces on that body, and generates equal velocities: and in the total time the total force impressed and the total velocity it generates is proportional to the time. And the distances described in the proportional times, are as the velocities and times taken together ; that is in the duplicate ratio of the times. And with the body projected up uniform gravity impresses forces and velocities taken proportional to the times ; and the times required to rise the greatest heights are as the velocities required to be taken away, and these heights are as the velocities and the times taken together or, in the duplicate ratio of the velocities. [*Duplicate ratio* means of course, *as the square*.] And the motion arising of a body projected along some right line is composed from the motion arising from gravity. So that if the body *A* by its motion of projection only in a given time can describe the right line *A B* and from its motion only of requiring to fall, can describe the height *AC* in the same time*:* the parallelogram *ABDC* may be completed, and that body will be found at the end of the time at the place *D* from the composed motion ; and the curved line *AED*, which that body



describes, will be a parabola which the right line *AB* touches at *A*, and the ordinate *BD* of which is as *AB* squared. The demonstrations of the times of oscillations of pendulums will depend on the same laws and corollaries, from the daily experience with clocks. From the very same, and with the third law Sir *Christopher Wren, Dr. John Wallis* and *Christian Huygens,* easily the principle outstanding geometers of the age, have discovered separately the rules of hard bodies colliding and rebounding, and almost at the same time they communicated the same with the *Royal Society*, among themselves (as regards these laws) everything is in agreement: and indeed first *Wallis*, then *Wren* and *Huygens* produced the discovery. But the truth has been established by *Wren* in person before the *Royal Society* through an experiment with pendulums : which also the most illustrious *Mariotte* soon deemed worthy to explain in a whole book. Truly, so that this experiment may agree to the precision with theories, and account is required to be had, both of the resistance of the air, as well as of the elasticity of the colliding bodies.

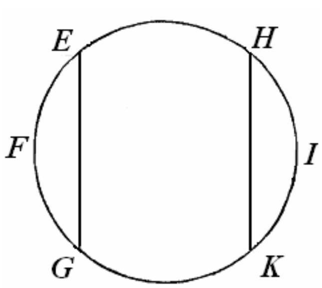


The spherical bodies *A*, *B* may hang from parallel threads and with *A*C, *BD* equal from the centres *C*, *D.* From these centres and intervals the bisected semicircles *EAF*, *GBH* are described, with radii *CA*, *DB.* The body *A* may be drawn to some point *R* of the arc *EAF,* and thence may be released (with the body *B* taken aside), and after one oscillation it may return to the point *V. RV* is the retardation from the air resistance. *ST* is made the fourth part of *RV* placed in the middle, thus evidently so that *R S* and *TV* are equal, and *R S* to *ST* shall be as 3 to 2. And thus *ST* will show the retardation in falling from *S* to *A* approximately. [A rule gained from experience perhaps for light damping.]

Body *B* may be restored to its place *B*. Body *A* may fall from the point *S*, and the velocity of that at the place of reflection *A* will be without so great a sensible error, and as if it had fallen from the location *T* in a vacuum. Therefore this velocity is set out by the chord of the arc *TA.* For the velocity of the pendulum at the lowest point is as the chord of the arc, that it has described on falling, the proposition is well known from geometry. After the reflection the body *A* may arrive at the location *s*, and the body *B* at the location *k.* The body *B* may be taken away and the position *v* may be found *;* from which if the body *A* may be sent off and after one oscillation may return to the location *r*, let *st* be the fourth part of that *rv* placed in the middle, thus so that it may be considered that *rs* and *tv* are equal ; and the velocity may be set out by the chord of the arc *tA*, that the body *A* had approximately after the reflection at the place *A*. For *t* will be that true and correct place, to which the body *A*, with the resistance of the air removed, ought to be able to rise. The location *k* to which the body *B* has risen is required to be corrected by a similar method, and requiring to find the location *l*, to which that body ought to ascend in a vacuum. With this done it is possible to test everything, in the same way as if we were placed in a vacuum. Yet the body *A* will be required to adopt (as thus I may say) the chord *TA* of the arc, which shows the velocity of this, so that the motion may be had approximately at the place *A* before the reflection ; then the chord *tA* of the arc, so that the motion of this may be had approximately at the place *A* after the reflection. And thus the body *B* will be required to adopt the chord of the arc *Bl,* so that the approximate motion of this may be had after the reflection. And by a similar method, where the bodies are sent off at the same time from two different locations, the motions of each are required to be found both before, as well as after the reflection ; and then finally the motions among themselves are to be brought together and the effect of the reflection deduced. In this manner with the matter requiring to be tested with ten feet pendulums, and that with bodies both unequal as well as equal, and by arranging so that bodies may concur from the greatest intervals, such as 8, 12, or 16 feet ; I have found [the text has *reperi* or *you find* in the singular command mode, which has been taken as a misprint, rather than *repperi* which has been adopted for translation] always within an error of 3 inches in the measurement, where the bodies themselves were meeting each other directly, equal changes were obtained in the contrary parts of the motions for the bodies, and thus the actions and reactions always to be equal. So that if the body *A* were incident on body *B* at rest with 9 parts of motion, and with 7 parts removed it went on with 2 parts after the reflection ; body *B* was rebounding with these 7 parts. If the bodies were going against each other, *A* with 12 parts and *B* with 6, and *A* was returning with 2 parts ; *B* was returning with 8, each with the removal of 14 parts. From the motion of *A* 12 parts are removed and nothing remains: 2 other parts are taken away , and there is made a motion of 2 parts in the opposite direction: and thus from the motion of the body *B* of 6 parts by requiring 14 parts to be taken away, 8 parts are made in the opposite direction. But if the bodies were going in the same direction, *A* faster with 14 parts, and *B* slower with 5 parts, and after the reflection *A* was going on with 5 parts ; *B* was going on with 14 parts, with the translation made of 9 parts from *A* to *B.* And thus for the rest. From a running together and collision of bodies at no time does the quantity of motion change, which is deduced from the sum of the motions acting in the same directions or from the differences in contrary directions. For the error of an inch or two I may attribute to the difficulty required in performing the individual measurements accurately enough. It was with difficulty, not only that the pendulums thus be dropped at the same time, so that the bodies could strike each other at the lowest position *A B;* but also the locations *s, k* to be noted, to which the bodies were ascending after the collision. But the unequal density of the parts of the pendulous bodies, and the construction from other causes of irregularity, were leading to errors.

Again lest anyone may object to the rule requiring approval which this experiment has found, to presuppose the bodies either to be completely hard, or perhaps perfectly elastic, none are to be found of this kind in natural compositions ; because I add now experiments described which succeed equally with soft or hard bodies, without doubt by no means depending on the condition of hardness. For if that rule is required to be extended to bodies which are not perfectly hard, the reflection is to be diminished only in a certain proportion to the magnitude of the elastic force. In the theory of *Wren* and *Huygens* absolutely hard bodies return with the speed of the encounter. Most surely that will be proven with perfectly elastic bodies. In imperfectly elastic bodies the return speed is required to be diminished likewise with the elastic force ; therefore because that elastic force, (unless where the parts of bodies are struck by their coming together, or extended a little as if they suffer under a hammer,) and certainly shall be required to be determined (as much as I know) and may be made so that bodies may return with a relative velocity in turn, which shall be in a given ratio to the relative velocity of approach. This I have tested thus with balls of wool closely piled together and strongly constricted. First by dropping pendulums and by measuring the reflection, I have found the magnitude of the elastic force; then with this force I have determined the reflections in other cases of concurrence, and they have answered the trials. Always the wool have returned with a relative velocity, which is to be to the relative velocity of concurrence as around 5 to 9. Balls of steel return with almost the same velocity ; others from cork with a little less : but with glass the proportion was around 15 to 16. And with this agreed upon, the third law has agreed with theory as far as impacts and reflections are concerned, which clearly agree with experiment.

I briefly show the matter for attractions thus. With any two bodies *A* and *B* mutually attracting each other, consider some obstacle placed between each, by which the meeting of these may be impeded. If either body *A* is drawn more towards the other body *B*, than that other *B* towards the first *A*, the obstacle will be urged more by the pressing of body *A* than by the pressing of body *B;* and hence will not stay in equilibrium. The pressing will prevail stronger, and it will act so that the system of the two bodies and the obstacle may move in the direction towards *B*, and in motions in free spaces always by accelerating, may depart to infinity. Which is absurd and contrary to the first law. For by the first law the system must persevere in it state of rest or of uniform motion in a direction, and hence the bodies will press equally on the obstacle, and on that account are drawn equally in turn. I have tested this with a loadstone and iron. If these placed in their own vessels touching separately they may float next to each other in still water ; neither propels the other, but from the equality of the attraction they sustain mutual attempts between themselves, and finally they remain in an established equilibrium.



Thus also is the gravity between the earth and the mutual parts of this. The earth *FI* is cut by some place *EG* into two parts *EGF* and *EGI:* and the mutual weights of these shall be equal mutually between themselves. For if by another plane *HK* which shall be parallel to the first part *EG*, the greater part *EGI* shall be cut into the two parts *EGKH* and *HKI*, of which *HKI* shall be equal to the first part cut *EFG*: it is evident that the middle part *EGKH* by its own weight will not be inclined to either of the extreme parts, but between each in equilibrium, thus so that I may say, it may be suspended and it is at rest. But the extreme part *HK*I by its own weight presses on the middle part, and will urge that into the other extreme part *EGF;* and thus the force by which the sum of the parts *HKI & EGKH* , *EGI* tends towards the third part *EGF,* is equal to the weight of the third part *HKI,* that is to the weight of the third part *EGF.* And therefore the weights of the two parts *EGI*, *EGF* are mutually in equilibrium, as I had wished to show. And unless these weights shall be equal, the whole earth floating on the free aether may go towards the greater weight, and from that required flight would go off to infinity.

Just as bodies in coming together and reflecting may exert the same influence [on each other], the velocities of which are reciprocally as their innate forces: thus the agents exert the same influence in the movements of mechanical devices, and by contrary exertions mutually sustain each other, the velocities of which, following the determination of the forces considered, are reciprocally as the forces. Thus the weights exert the same influence towards moving the arms of scales, which with the scales oscillating are reciprocally as the velocities of these up and down: that is, the weights, if they ascend up and down rightly [*i.e.* vertically], exert the same influences, which are reciprocally as the distances of the points from which they are suspended from the axis of the scales; if, by an oblique plane or from other obstacles to the motion, the ascents or descents are oblique, they exert the same influence, which are reciprocally as the ascent and descent, just as made along the perpendicular: and that on account of the determination of the weight acting downwards.

Similarly with a pulley or a pulley system, the force of the hand directly drawing on the rope which shall be to the weight , ascending either directly or obliquely, as the perpendicular speed of ascent to the speed of the hand pulling directly on the rope will sustain the weight. In clocks and similar devices, which have been constructed from little wheels joined together, the forces required contrary to promoting and retarding the motions of the wheels if they are reciprocally as the speeds of the wheels on which they are impressed, will mutually sustain each other. The force of a screw required to press upon a body is to the force of hand turning the handle, as the rotational speed of the handle in that part where it is pressed on by the hand, to the speed of progress of the screw towards the body pressed. The forces by which a wedge urges the two parts of wood to be split are to the force of the hammer to the wedge, as the progress of the wedge following a determined force impressed by the hammer on itself, to the speed by which the parts of the wood, following lines perpendicular to the faces of the wedge. And an account of all machines is the same.

The effectiveness and use of these consists in this only, that by diminishing the speed we augment the force, and vice versa: From which the general problem is solved in all kinds of suitable mechanical devices : *a given weight is to be moved by a given force,* or some given resistance is to be overcome by a given force. For if machines may be formed thus, so that the speeds of the driving force and of the resistance shall be reciprocally as the forces ; the driving force will sustain the resistance : and it may overcome the same with a greater difference of the speeds. Certainly if the disparity of the speeds shall be so great, so that all resistance may also overcome, which is accustomed to arise both from the slipping and friction of nearby bodies between each other, as well as from the cohesion and in turn of the separation and continued elevation of bodies ; with all that resistance overcome, the excess force will produce an acceleration motion proportional to itself, partially within the parts of the machine, and partially within the resisting body. It is not the intention of this work to treat everything mechanical. I have wished only to show, both how wide and sure the third law of motion shall be. For if the action of the driving force may be estimated from the speed and this force taken jointly ; and similarly the reaction of the resistance may be estimated conjointly from the velocities of the individual parts of this, and from the friction of these, from the cohesion, and from the weight, and from the acceleration arising ; the action and the reaction will always be equal to each other in turn, in every use of instruments. And as far as the action is propagated by the instrument and finally may be impressed on any resisting body, the final determination of this will always be contrary to the determined reaction.