Binomial $\sim (n,p)$ $EX = np$ $Var = np(1-p)$

$$P(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0,1,\dots,n \\ 0 & cc \end{cases}$$

GEOM
$$\begin{cases} \mu = \frac{1-p}{p} & (1-p)^x p = P(X=x) \quad x \in [0,1,\dots) \quad Var = \frac{1-p}{p^2} \\ \mu = \frac{1}{p} & (1-p)^{x-1} p = P(X=x) \quad x \in [1,2,\dots n) \end{cases}$$

HIPERGEOM $\sim (n,K,N)$ $\mu = \frac{nK}{N}$

$$f_X(x) = P(X=x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

Multinomial $(n, p_1, p_2, \dots, p_r)$

$$P(Y_1 = y_1, Y_2 = y_2, \dots Y_r = y_r) = \begin{cases} \frac{n!}{y_1! y_2! \cdots y_r!} p_1^{y_1} \cdots p_r^{y_r} & \text{si } y_j \in \{0,1,\dots n\} \ \forall j = 1,2\dots r, \\ & y_1 + y_2 + \dots + y_r = n \\ 0 & \text{en caso contrario} \end{cases}$$

UNIFORME $\sim U([a,b])$

$\mu = \frac{1}{2}(a+b) \qquad var = \frac{1}{12}(b-a)^2$

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & cc \end{cases}$$

$$F_X(x) = P(X \leq x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a,b] \\ 1 & x > b \end{cases}$$

BINEG $\sim BN(r,p)$

$\mu = r\frac{(1-p)}{p} \qquad var \quad r\frac{(1-p)}{p^2}$

$$P(X=x) = \binom{x+r-1}{x}(1-p)^x p^r$$

POISSON $\sim P(\lambda)$

$\mu = \lambda \qquad var = \lambda$

$$P(X=x) = \frac{\lambda^x e^{-\lambda x}}{x!}$$

EXPONENCIAL

Sea X v.a, $X \sim \mathcal{E}(\lambda)$ $(\lambda > 0)$. $\mu = \frac{1}{\lambda}$ $Var = \frac{1}{\lambda^2}$

Entonces

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & c.c. \end{cases}$$

Proposición

Sea $X \sim \mathcal{E}(\lambda)$. Entonces $P\left(X > a+b / X > a\right) = P(X > b)$ $\forall a, b \geq 0$

Proposición (Caracterización de la Distribución exponencial)

Sea X v.a continua tal que $\forall\, a, b \geq 0$,

$$P(X > a+b) = P(X > a) . P(X > b)$$

Entonces: $P(X > 0) = 0$ ó $X \sim \mathcal{E}(\lambda)$ para algún $\lambda > 0$

NORMAL ESTANDAR $N(0,1)$

$f : \mathbb{R} \longrightarrow (0, +\infty)$, dada por:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \forall x \in \mathbb{R}$$

NORMAL GRAL $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

GAMMA

$$\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{u}\, du$$

.) $\Gamma(\alpha+1) = \alpha\,\Gamma(\alpha)$

.) $\Gamma(n+1) = n!$ $\qquad \Gamma(1) = 1$

Sea X v.a absolutamente continua con función de densidad dada por:

$$f(x) = \begin{cases} \dfrac{\lambda^\alpha}{\Gamma(\alpha)}\, x^{\alpha-1} e^{-\lambda x} & x > 0 \\ 0 & \text{c.c.} \end{cases}$$

donde $\lambda > 0$, $\alpha > 0$. $\quad EX = \dfrac{\alpha}{\lambda} \quad Var = \dfrac{\alpha}{\lambda^2}$

Entonces decimos que X tiene distribución Gamma con parámetros α y λ. Notación $X \sim \Gamma(\alpha, \lambda)$

$$\boxed{X \sim \Gamma(1,\lambda) \iff X \sim \mathcal{E}(\lambda)}$$

Sea X v.a tal que $X \sim N(0,\sigma^2)$; entonces $X^2 \sim \Gamma\left(\frac{1}{2}, \frac{1}{2\sigma^2}\right)$

CAUCHY

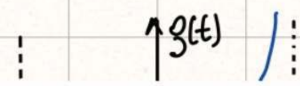
Sea X v.a. Se dice que X tiene distribución de Cauchy si su función de densidad es de la forma:

$$f_X(x) = \frac{1}{\pi(1+x^2)} \quad \forall x \in \mathbb{R}$$

Proposición

Sea X v.a tal que $X \sim \mathcal{U}(0,1)$. Sea $Y = g(X) = tg\left(\pi X - \frac{\pi}{2}\right)$

Entonces $Y \sim$ de Cauchy



Cauchy es simétrica

$\boxed{\text{NORMAL BIVARIADA}}$

Caso Particular: $A = id$ y $\mu = (0,0)$

Distribución Normal Bivariada Estándar

$$f_{X,Y}(x,y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$$

Sumas

$X_i \sim Bi(n_i, p)$

$$\sum_{i=0}^{h} X_i \sim B\left(\sum n_i, p\right)$$

$X_i \sim P(\lambda_i)$

$$\sum_{n=0}^{h} X_i \sim P\left(\sum_{i=0}^{h} \lambda_i\right)$$

$X_i \sim BN(n_i, p)$

$$\sum_{n=0}^{h} X_i \sim BN\left(\sum_{i=0}^{h} n_i, p\right)$$

Proposición

Sean X e Y v.a independientes definidas sobre $(\Omega, \mathcal{A}, P)$ tal que $X \sim \Gamma(\alpha_1, \lambda)$ e $Y \sim \Gamma(\alpha_2, \lambda)$. Entonces $(X+Y) \sim \Gamma(\alpha_1+\alpha_2, \lambda)$

Proposición

Sean X e Y v.a independientes definidas sobre $(\Omega, \mathcal{A}, P)$ tal que $X \sim N(\mu_1, \sigma_1^2)$ e $Y \sim N(\mu_2, \sigma_2^2)$. Entonces $(X+Y) \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

Definición

Sea X v.a. definida sobre $(\Omega, \mathcal{A}, P)$. Entonces X se dice simétrica si X y $-X$ tienen exactamente la misma distribución. Esto es:

$$P(X \leq t) = P(-X \leq t) \quad \forall\, t \in \mathbb{R}$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} (-1)^{n-1} S_n \qquad 1 \leq i_1 < i_2 < \dots < i_n \leq n$$

$$S_n = \sum P(A_{i_1} \cap P_{i_2} \dots \cap P_{i_n})$$

$$F_{X+Y}(z) = \int_{-\infty}^{\infty} \int_{0}^{z} f_{XY}(x, v-x)\, dv\, dx$$

$$= \int_{-\infty}^{\infty} \int_{0}^{z} f_{XY}(u-y, y)\, du\, dy$$

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y)\, dy$$

$$= \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx$$

Discreta

$$f_{X+Y}(t) = \sum_{i=0}^{t} f_X(i) f_Y(t-i)$$

$$P(X+Y=t) = P(X=t)\, P(X=t-i)$$

Entonces definimos la función $f_{Y/X} : \mathbb{R}\times\mathbb{R} \longrightarrow [0,1]$ como:

$$f_{Y/X}(y/x) = \begin{cases} \dfrac{f(x,y)}{f_X(x)} & \text{si } f_X(x) > 0 \\ 0 & \text{si } f_X(x) = 0 \end{cases}$$

$Z = XY$

$$F_Z(z) = \int_{-\infty}^{+\infty} \underbrace{\int_{-\infty}^{z} \frac{f_{XY}(x, \frac{v}{x})}{|x|} \, dv \, dx}_{= f_Z(z)}$$

Esperanzas

$U \sim U([a,b])$ $\qquad EU = \dfrac{b+a}{2}$

$X \sim \Gamma(\alpha, \lambda)$ $\qquad EX = \dfrac{\alpha}{\lambda}$

PROPIEDADES

·) X e Y indeptes $\Rightarrow E(XY) = EX \cdot EY$ $\quad \not\Leftarrow$

·) $E(aX+Y) = aEX + EY$

·) $P(X \geq Y) = 1 \Rightarrow EX \geq EY$

$$\boxed{\begin{array}{ll} P(X \geq t) \leq \dfrac{EX}{t} & \text{(Markov)} \\[2ex] P(|X-\mu| \geq t) \leq \dfrac{\sigma^2}{t^2} & \text{(Chevychev)} \end{array}}$$

LEY DEBIL $P(X_n \leq x) \simeq \Phi\left(\dfrac{x_n - EX}{\operatorname{var} X_n}\right)$

$$P\left(\left|\frac{X_n}{n} - \mu\right| \geq \varepsilon\right) \xrightarrow[n \to \infty]{} 0$$

·) $\operatorname{Cov}(X,Y) = EXY - EXEY = \operatorname{var}(X)$ si $Y = X$

$= 0$ si X e Y indeptes

·) $\operatorname{Var}(X) = EX^2 - (EX)^2$

·) $(EXY)^2 \leq E(X^2)E(Y^2)$

•) $|EXY| \leq \sqrt{EX^2}\sqrt{EY^2}$

•) $Var(X+Y) = Var X + Var Y + 2Cov(X,Y)$

•) $Var(X+c) = Var X$

•) $Var(cX) = c^2 Var(X)$

Lema 3:

si $X_1, X_2 \dots X_n$ son v.a tal que $E(X_i^2) < \infty$ $\forall i = 1, 2, \dots n$

entonces:

$$Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i) + \sum_{1 \leq i \neq j \leq n} Cov(X_i, X_j)$$

$$= \sum_{i=1}^{n} Var(X_i) + 2 \sum_{1 \leq i < j \leq n} Cov(X_i, X_j)$$