$$T \ / \ [T]_C = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

$T \nearrow p_T = (X-c_1)^{d_1} \dots (X-c_k)^{d_k}$ $\Rightarrow$ puedo calcular jordan o forma racional si no se factoriza bien en $\mathbb{K}$

$\searrow m_T = (X-c_1)^{r_1} \dots (X-c_k)^{r_k}$

T-invar

$$W_i = Nu(T - c_i I)^{r_i} \Rightarrow V = W_1 \oplus \dots \oplus W_k$$

$B = \{W_1 \dots W_k\}$ base

$B = B_1 \cup \dots \cup B_k$

$$\Rightarrow [T]_B = \begin{pmatrix} \boxed{[T|_{W_1}]_{B_1}} & & 0 \\ & \ddots & \\ 0 & & \boxed{[T|_{W_k}]_{B_k}} \end{pmatrix}$$

$[T_{W_j}]_{B_j} \in \mathbb{K}^{d_j \times d_j}$

Además $T|_{W_i} = c_i I + N_i$ $\quad N_i = (T - c_i I)|_{W_i}$

en este caso

$$p_A(x) = (x-2)^5 (x+1)^1$$

$$m_A(x) = (x-2)^a (x+1) \qquad 1 \leq a \leq 5$$

$$W_1 = Nu\,(T-2I)^{r_1}$$

$$W_2 = Nu\,(T+I) \qquad (\dim W_2 = 1)$$

$$\boxed{V = W_1 \oplus W_2} \Rightarrow \dim W_1 = 5$$

$$Nu(T-2I) \subset Nu(T-2I)^2 \qquad \subset Nu(T-2I)^{r_1} = W_1$$

$$T-2I = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 \end{pmatrix}$$

Tiene 4 pivotes

$\dim Nu\,(T-2I) = 2 \rightarrow$ no alcanza para gr de $x-2$

vemos
$$(T-2I)^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & -3 & 9 \end{pmatrix}$$

Tiene 3 pivotes $\rightarrow \dim Nu(T-2I) = 3$

$$(T-2I)^3 = \begin{pmatrix} 0 & & & & & \\ 0 & & & & & \\ 0 & & & \bigcirc & & \\ 0 & & & & & \\ 1 & \dots & \dots & & & \\ -3 & -2 & -3 & -3 & 9 & -27 \end{pmatrix}$$

2 pivotes $\Rightarrow \dim (T-2I)^3 = 4$

pero $Nu(T-2I)^3 \subsetneq Nu(T-2I)^4$

$\Rightarrow \dim Nu(T-2I)^4 > 4$

pero es menor o igual a 5

$\therefore Nu(T-2I)^4 = 5 \Rightarrow r_1 = 4$

$\boxed{m_T = (X-2)^4 (X+1)}$

igual vamos

$$(T-2I)^4 = \begin{pmatrix} 0 & \dots & & & \dots & 0 \\ & & & & & \\ 10 & 6 & 9 & 9 & -27 & 81 \end{pmatrix}$$

$x_1 = -\frac{1}{10}(6x_2 + 9x_3 \dots + 81x_6)$

$$Nu(T-2I)^4 = \{\underset{v_1}{(-\tfrac{6}{10}, 1, 0, 0, 0, 0)}\ \underset{v_2}{(-\tfrac{9}{10}, 0, 1, 0, 0, 0)}$$

$$\underset{v_3}{(-\tfrac{9}{10}, 0, 0, 1, 0, 0)}\ \underset{v_4}{(\tfrac{27}{10}, 0, 0, 0, 1, 0)}$$

$$\underset{v_5}{(-\tfrac{81}{10}, 0, 0, 0, 0, 1)}\} = W_1 = B_1$$

$B_2 = W_2 = \{ (0,0,0,0,0,1) \}$
$\quad\quad\quad\quad\quad\quad\quad v_6$

a ojo se ve que v_1 es avec de aval -1
$\Rightarrow v_6 \in Nu(T+I)$

$$T \begin{pmatrix} -\frac{6}{10} \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{12}{10} \\ \frac{14}{10} \\ \frac{6}{10} \\ 2 \\ \frac{4}{10} \\ 0 \end{pmatrix} = \frac{14}{10} v_1 + \frac{6}{10} v_2 + 2 v_3 + \frac{4}{10} v_4 + 0 v_5$$

$$\left(\frac{14}{10}, \frac{6}{10}, 2, \frac{4}{10}, 0, 0 \right)_{W_1}$$

$$T(0,0,0,0,0,1) = (0,0,0,0,0,-1) = v_{6 W_2}$$

$$\Rightarrow [T]_{B = B_1 \cup B_2} = \left(\begin{array}{ccccc|c} \frac{14}{10} & -\frac{9}{10} & -\frac{7}{10} & \frac{27}{10} & -\frac{81}{10} & 0 \\ \frac{6}{10} & \frac{29}{10} & \frac{9}{10} & -\frac{27}{10} & \frac{81}{10} & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 \\ \frac{4}{10} & \frac{1}{10} & \frac{1}{10} & \frac{47}{10} & -\frac{81}{10} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -1 \end{array} \right)$$

haciendo $Tv_2, Tv_3, \dots Tv_5$ completamos

y esta matriz es $2I + N_1 = T_1$

$\Rightarrow T_1 - 2I = N_1 \quad (N_1 = N|_{W_1})$

$$\Rightarrow N_1 = (T - 2I)_{W_1}$$

$$\Rightarrow N = \begin{pmatrix} -\frac{6}{10} & -\frac{9}{10} & -\frac{9}{10} & \frac{27}{10} & \frac{9}{10} \\ \frac{6}{10} & \frac{9}{10} & \frac{9}{10} & -\frac{27}{10} & \frac{81}{10} \\ 2 & 0 & 0 & 0 & 0 \\ \frac{4}{10} & \frac{1}{10} & \frac{1}{10} & \frac{27}{10} & -\frac{81}{10} \\ 0 & 0 & 0 & 1 & -3 \end{pmatrix}$$

$P_{N_1} = X^5$

$m_{N_1} = X^r$ para $r = 4$

por descomp. cíclica

$m_N = p_1 = X^4$ $\quad p_2 \mid p_1$ $\quad$ y $\quad p_1 p_2 \dots p_j = P_N = X^5$

$$\Rightarrow p_2 = X$$

Sabemos también por descomp

$W_1 = Z(v_1, T|_{W_1}) \oplus Z(v_2, T|_{W_1})$

v_1 anulado por X^4 $\quad v_2$ por X

$B' = B_1' \cup B_2'$ con $B_i = Z(v_i, T|_{W_1})$

$$(N_1)_{B'} = \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

y B' tiene 5 vectores de $\mathbb{R}^6$

$$\Rightarrow \underset{\substack{\shortparallel \\ T|_{W_1} \\ \shortparallel \\ (N_1+2I)_{W_1}}}{A_1} \sim \left(\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ \hline 0 & 0 & 0 & 0 & 2 \end{array}\right) = (N_1+2I)_{B'} = (N_1)_{B'} + (2I)_{B'}$$

$$\Rightarrow \underset{\substack{\shortparallel \\ (T)_{B' \cup B_2}}}{A} \sim \left(\begin{array}{ccccc|c} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{2} & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \boxed{-1} \end{array}\right) = (T)_{B' \cup B_2}$$

Ahora calculemos dicha B'

buscando v_j / $Z(v_j, T|_W) = B_1'$

como $Nu(T-2I) \subsetneq Nu(T-2I)^2 \subsetneq Nu(T-2I)^3 \subseteq (Nu-2I)^4$

Si $v_j \in Nu(T-2I)$

$\Rightarrow v_j, (T-2I)v_j$ no genera B_1'
no le da dimension

lo mismo con $(T-2I)^2, {}^3$ N

necesitamos $v_j \in Nu(T-2I)^4$

tq $v_j \notin Nu(T-2I)^3$

(porque tampoco estaría en $Nu(T-2I)^{2,1}$)

entonces usamos v_3 de antes de W_1

$\Rightarrow$ pero si vimos $B = \{\overline{v_1 \dots v_5}\}$ los de antes

$[v_5]_B = (0,0,0,0,1)$

$\Rightarrow \{e_5, N_1 e_5, N_1^2 e_5, N^3 e_5\} = B_1'$

para B_2' queremos $v \in Nu(T-2I) = \{w_2, w_3\}$ que sabíamos tenía dim 2

ahora hay que ver alguno de esos dos que sea Li con B_1

supongo $w_1 \Rightarrow [w_1]_B = B_2'$

$B_1' \cup B_2' = B' = \{e_5, N_1 e_5, N^2 e_5, N^3 e_5, [w_1]_B\}$

$\Updownarrow$

$\tilde{B}_1 = \{v_5, (T-2I)v_5, (T-2I)^2 v_5, (T-2I)^3 v_5, w_1\}$

Luego $B = \tilde{B}_1 \cup B_2$ y ya tenemos la base que nos da form de jordan