Regla de la cadena

(20)

$$\frac{\partial f_1}{\partial x} = 2x + y \qquad \frac{\partial f_1}{\partial y} = x \qquad \frac{\partial f_2}{\partial x} = 0 \qquad \frac{\partial f_2}{\partial y} = 2y$$

$$\Rightarrow Df = \begin{pmatrix} 2x+y & x \\ 0 & 2y \end{pmatrix}$$

$$\frac{\partial g}{\partial u} = (1, 2, 0) \qquad \frac{\partial g}{\partial v} = (1, 0, 2v)$$

$$Dg = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 2v \end{pmatrix} \qquad Dg_{f(x,y)} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 2y^2+4 \end{pmatrix}$$

$$Dg_{f(x,y)} \cdot Df = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 2y^2+4 \end{pmatrix} \begin{pmatrix} 2x+y & x \\ 0 & 2y \end{pmatrix}$$

$$= \begin{pmatrix} 2x+y & x+2y \\ 4x+2y & 2y \\ 0 & 4y^4+8y \end{pmatrix} \Rightarrow D(g \circ f)_{f(1,1)} = \begin{pmatrix} 3 & 3 \\ 6 & 4 \\ 0 & 12 \end{pmatrix}$$

(18) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}$

$\frac{\partial f}{\partial u} = v\, e^{uv} \operatorname{sen}(u^2+v^2) + 2u\, e^{uv} \cos(u^2+v^2)$

$\frac{\partial f(x+y, xy)}{\partial u} = xy\, e^{(x+y)xy} \operatorname{sen}((x+y)^2+(xy)^2) + 2(x+y)\, e^{xy(x+y)} \cos((x+y)^2+(xy)^2)$

$\frac{\partial u}{\partial x} = 1$

$$\boxed{\frac{\partial f}{\partial x} = xy\, e^{(x+y)xy} \operatorname{sen}((x+y)^2+(xy)^2) + 2(x+y)\, e^{xy(x+y)} \cos((x+y)^2+(xy)^2)}$$

$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial v} \frac{\partial v}{\partial y}$

$\frac{\partial f}{\partial v} = u\, e^{uv} \operatorname{sen}(u^2+v^2) + 2v\, e^{uv} \cos(u^2+v^2)$

$\frac{\partial f(x+y, xy)}{\partial v}$