Coursework template CO343

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1 Problem 1

1.
$$|x_i| \leq ||(x_1, \ldots, x_n)||$$

Proof.
$$|x_1| = \sqrt{x_1^2} \leq \sqrt{x_1^2 + \cdots + x_n^2} = ||(x_1, \ldots, x_n)||$$
$\square$

2.
$$||(x_1, \ldots, x_n)|| \leq \sqrt{n} \max\{|x_i|, i = 1, \ldots, n\}$$

Proof.
$$\sqrt{x_1^2 + \cdots + x_n^2} \leq \sqrt{n(\max_{i=1,\ldots,n} |x_i|)^n} = \sqrt{n}\sqrt{(\max_{i=1,\ldots,n} |x_i|)^n} = \sqrt{n} \max_{i=1,\ldots,n} |x_i|$$
$\square$

3.
$$||(x_1, \ldots, x_n)|| \leq |x_1| + \cdots + |x_n|$$

Proof. Usando Cauchy Schwarz
$$||(x_1, \ldots, x_n)|| = \sqrt{x_1^2 + \cdots + x_n^2} \leq |x_1| + \cdots + |x_n|$$
$$\iff |x_1^2 + \cdots x_n^2| \leq (|x_1| + \cdots |x_n|)^2$$
$$\iff x_1^2 + \cdots x_n^2 \leq |x_1|^2 + \cdots |x_n|^2 + \sum_{i \neq j}^{n} |x_i||x_j|$$
$$\iff x_1^2 + \cdots x_n^2 \leq x_1^2 + \cdots x_n^2 + \sum_{i \neq j}^{n} |x_i||x_j|$$
$$\iff 0 \leq \sum_{i \neq j}^{n} |x_i||x_j|$$
$\square$