③ $A \in \mathbb{M}^{n \times n}$ A def. pos simétrica

veré $f(x) = x^T A x \Rightarrow \nabla f = 2Ax$

demo

$$f(x) = \langle x^T, Ax \rangle = \langle x^T, \begin{bmatrix} \sum_{j=1}^{n} A_{1j} x_j \\ \vdots \\ \sum_{j=1}^{n} A_{nj} x_j \end{bmatrix} \rangle$$

$$= \sum_{k=1}^{n} x_k \sum_{j=1}^{n} A_{kj} x_j$$

$$= \sum_{k=1}^{n} \sum_{j=1}^{n} x_k A_{kj} x_j$$

$$\frac{\partial f}{\partial x_i}(x) = \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{d}{dx_i} x_k A_{kj} x_j$$

$$= \sum_{j=1}^{n} \frac{d}{dx_i} x_i A_{ij} x_j + \sum_{\substack{k=1 \\ k \neq i}}^{n} \sum_{j=1}^{n} \frac{d}{dx_i} x_k A_{kj} x_j$$

$$= 2 x_i A_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} A_{ij} x_j + \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{d}{dx_i} \underset{\substack{\parallel \\ x_k A_{ki}}}{x_k A_{ki} x_i}$$

$$= 2x_i A_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} A_{ij} x_j + \sum_{\substack{k=1 \\ k \neq i}}^{n} A_{ki} x_k$$

// son números reales

$$\Downarrow$$

$$A^T = A$$

$$\Downarrow$$

$$= 2x_i A_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} A_{ij} x_j + \sum_{\substack{k=1 \\ k \neq i}}^{n} A_{ik} x_k$$

$$= \underbrace{2x_i A_{ii}}_{= 2A_{ii}x_i} + 2\sum_{\substack{j=1 \\ j \neq i}}^{n} A_{ij} x_j$$

$$= 2\sum_{j=1}^{n} A_{ij} x_j = 2(Ax)_i$$

$$\Rightarrow \frac{\partial f}{\partial x_i}(x) = 2(Ax)_i$$

$$f'(x) = \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}\right) = \begin{bmatrix} (Ax)_1 \\ \vdots \\ (Ax)_n \end{bmatrix}^T$$

$$\Rightarrow f'(x) = 2(Ax)^T$$

$$(\nabla f = f'^T)$$

$$\Rightarrow \nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix} = \begin{pmatrix} 2(Ax)_1 \\ \vdots \\ (2Ax)_n \end{pmatrix}$$

$$= 2Ax$$

notar que

$$f'(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_n}{x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} = \partial$$

en particular si $f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$$f'(x) = \left(\frac{\partial f_1}{\partial x_1} \cdots \frac{\partial f_1}{\partial x_n} \right)$$

$$= (\nabla f)^T$$

•) $\|x - A^{-1}b\|_A^2 = (x - A^{-1}b)^T A (x - A^{-1}b)$

$$(f_1'(x)) = \left[\frac{1}{2}\Big(\underbrace{\sqrt{(x-x^*)^T A (x-x^*)}}_{f_1(x)}\Big)^2\right]'$$

$(x - x^*)' = Id$ $\quad \left(\frac{1}{2}(x - x^*)^T A (x - x^*)\right)'$

$$= (A(x-x^*))^T \cdot Id$$

$$\Rightarrow \nabla f_1 = (f_1')^T = Id\, A(x - x^*)$$
$$= Ax - Ax^*$$
$$= Ax - b$$

$$\left(f_2'(x)\right) = \left(\frac{1}{2}(Ax - b)^T A^{-1}(Ax - b)\right)'$$

$$= \left(A^{-1}(Ax - b)\right)^T A \qquad (Ax - b)' = A$$

$$\Rightarrow \nabla f_2(x) = A^T\left(A^{-1}(Ax - b)\right)$$
$$= \underline{\underline{A^T x - A^T A^{-1} b}} = A^T(x - x^*)$$
$$= Ax - b$$

$$f_3'(x) = (Ax)^T - (x^T b)'$$

$$(x^T b) = x_1 b_1 + x_2 b_2 + \dots + x_n b_n$$

$$\frac{\partial (x^T b)}{\partial x_1} = b_1 \qquad \frac{\partial (x^T b)}{\partial x_i} = b_i$$

$$\Rightarrow (x^T b)' = b^T$$

$$f_3'(x) = (Ax)^T - b^T \qquad \Rightarrow \nabla f_3 = Ax - b$$

transpuesta

(4)

$$d_i^k = \begin{cases} \dfrac{r_i^k}{|r_i^k|} & r_i^k \neq 0 \\ 0 & cc \end{cases}$$

P110

$$r^{k^T} d^k = \sum r_i^k d_i^k = \sum_{\substack{i=1 \\ r_i \neq 0}}^{n} \frac{|r_i^k|^2}{|r_i|} = \sum_{\substack{i=1 \\ r_i \neq 0}}^{n} |r_i^k| = \|r^k\|_1$$

$$1 - \frac{\|r^k\|_1^2}{\underbrace{((d^k)^T A d^k)((r^k)^T A^{-1} r^k)}} \leq 1 - \frac{\|r^k\|_2^2 \; n}{((d^k)^T A d^k)((r^k)^T A^{-1} r^k)}$$

$$\langle d^k, A d^k \rangle \langle r^k, A^{-1} r^k \rangle$$

$$\leq \|d^k\|_2 \|A d^k\|_2 \|r^k\|_2 \|A^{-1} r^k\|_2 \leq \|d^k\|_2 \|A\| \|d^k\|_2 \|r^k\|_2 \|A^{-1}\|_2 \|r^k\|_2$$

C-S

$$\|d^k\|^2 = \sum_{\substack{i=0 \\ r_i \neq 0}}^{n} (d_i^k)^2 = \sum_{\substack{i=0 \\ r_i \neq 0}}^{n} 1 \leq n$$

$$\leq 1 - \frac{n \; 1}{\|d^k\|_2^2 \|A\| \|A^{-1}\|}$$

$$= 1 - \frac{\cancel{n}}{n^{\cancel{2}} k_2(A)}$$

(6) Derivar $f \Rightarrow f'(x) = \underline{(Ax)^T - b^T}$

$\underline{f(x^k + de^i)} \Rightarrow \underline{f'(x^k + de^i)\, e^i}$

$$f'(x^k + de^i) \cdot e^i = \left((A(x^k + de^i))^T - b^T \right) e^i$$

$$= \left((Ax^k)^T + (Ade^i)^T - b^T \right) e^i$$

$$= (Ax^k)^T e^i + d\, e^{iT} A^T \underline{e^i} - b^T e^i$$

$$= x^{kT} A^T e^i + d\, a_{ii} - b_i$$

$$= x^{kT} c_i(A^T) + d\, a_{ii} - b_i$$

$$0 = \sum_{j=1}^{n} x_j^k a_{ij} + d\, a_{ii} - b_i$$

esto igual a 0 es sol de

$$\min_{d \in \mathbb{R}} f(x^k + de^i)$$

igualo a 0 $\Rightarrow$ $\boxed{d_i^k = \frac{b_i}{a_{ii}} - \sum_{j=1}^{n} \frac{x_j^k a_{ji}}{a_{ii}}}$

este d lo meto en

$$\boxed{x^{k+1} = x^k + d^k}$$

y veo que es igual a el
x^{k+1} generado por jacobi

$$x_i^{k+1} = \underline{x_i^k} + b_i - \sum_{j=1}^{n} x_j^k a_{ji}$$

$=$

(10) $\cdot)\ g_1(x) = \frac{1}{2}(x - x^*)^T A^T A (x - x^*)$

$$\nabla f(x) = A^T A (x - x^*) = A^T (Ax - b)$$

$\cdot)\ g_2(x) = \frac{1}{2}\|A(x - x^*)\|_2^2 = \frac{1}{2}\langle A(x-x^*), A(x-x^*)\rangle$

$$= \frac{1}{2}(x - x^*)^T A^T A (x - x^*)$$

$$= g_1(x)$$

$a_{11}\ a_{21} \dots a_{n1}$

$\Rightarrow \nabla g_2 = \nabla g_1$

$\cdot)\ g_3(x) = \frac{1}{2}\langle Ax, Ax\rangle = \langle x, A^T b\rangle$

$$\underset{f(x)}{\underset{\shortparallel}{\langle x, A^T b\rangle}} = \left\langle x, \begin{bmatrix} b_1 a_{11} + \dots + b_n a_{n1} \\ b_1 a_{12} \\ \\ \\ b_1 a_{1n} + \quad + b_n a_{nn} \end{bmatrix} \right.$$

$$= \sum_{j=1}^{n} x_j \sum_{i=1}^{n} b_i a_{ij}$$

$$\frac{\partial f}{\partial x_k} = \sum_{i=1}^{n} b_i a_{ik} = (A^T b)_k$$

$$f'(x) = ((A^T b)_1, \dots, (A^T b)_n) = \begin{bmatrix} (A^T b)_1 \\ \vdots \\ (A^T b)_n \end{bmatrix}^T$$

$$= (A^T b)^T = \underline{b^T A}$$

$$\Rightarrow (g_3'(x)) = (A^T A x)^T + b^T A$$

$$A x^* = b$$
$$\Rightarrow A^T A x^* = A^T b$$

$$\nabla g_3(x) = A^T A x + A^T b$$
$$= A^T A x + A^T A x^*$$
$$= A^T A (x - x^*)$$