① a) Givens

$$c = \frac{x_1}{\sqrt{x_1^2 + x_2^2}} \qquad s = -\frac{x_2}{\sqrt{x_1^2 + x_2^2}} \qquad G_{21} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \qquad \begin{matrix} x_1 = 2 \\ x_2 = 5 \end{matrix}$$

$$\Rightarrow \quad c = \frac{2}{\sqrt{29}} \qquad s = -\frac{5}{\sqrt{29}}$$

$$G_{21} A = \begin{pmatrix} \frac{2}{\sqrt{29}} & \frac{5}{\sqrt{29}} \\ -\frac{5}{\sqrt{29}} & \frac{2}{\sqrt{29}} \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$$

$$= \frac{1}{\sqrt{29}} \begin{pmatrix} 2 & 5 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$$

$$= \frac{1}{\sqrt{29}} \begin{pmatrix} 29 & 41 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} G_{21} A = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 29 & 41 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{29}} \begin{pmatrix} 29 & 41 \\ 0 & 1 \end{pmatrix}$$

G_{31}

(b)

$$G_{21} \begin{pmatrix} 12 \\ 9 \end{pmatrix} = \frac{1}{\sqrt{29}} \begin{bmatrix} 2 & 5 \\ -5 & 2 \end{bmatrix} \begin{pmatrix} 12 \\ 9 \end{pmatrix} = \frac{1}{\sqrt{29}} \begin{bmatrix} 69 \\ -42 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} G_{21} \begin{pmatrix} 12 \\ 9 \end{pmatrix} = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 69 \\ 42 \end{pmatrix} = \frac{1}{\sqrt{29}} \begin{bmatrix} 69 \\ 42 \end{bmatrix}$$

$$G_{31} G_{21} A x = G_{31} G_{21} b$$

$$\cancel{\frac{1}{\sqrt{29}}} \begin{pmatrix} 29 & 41 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \cancel{\frac{1}{\sqrt{29}}} \begin{bmatrix} 69 \\ 42 \end{bmatrix}$$

$$\boxed{x_2 = 42}$$

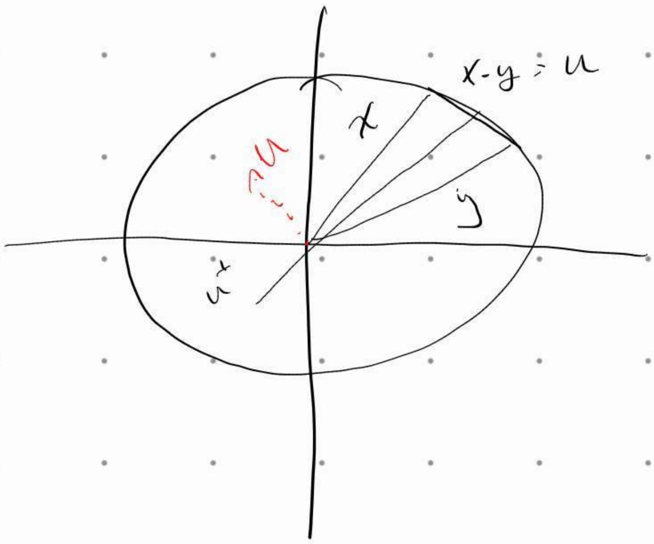
$$29 x_1 + 41 \cdot 42 = 69$$

$$29 x_1 = -1653$$

$$\boxed{x_1 = -57}$$

$$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} -57 \\ 42 \end{pmatrix} = \begin{bmatrix} 12 \\ 9 \end{bmatrix} \quad \square$$

2) Planteo



Ahora dado $u = x - y$

por teórico si

$$Q = I - \frac{2uu^T}{\|u\|_2}$$

cumple $Q\bar{x} = \bar{x} \quad \bar{x} \in u^{\perp}$

$$Qu = -u$$

$$\langle x+y, x-y \rangle$$

$$\|x\| - \|y\| = 0$$

$\Rightarrow \{x-y, x+y\}$ bog

$$x = \alpha(x+y) + \beta(x-y)$$

se llega a $\quad \alpha = \frac{1}{2} \quad \beta = \frac{1}{2}$

$$x = \frac{1}{2}(x+y) + \frac{1}{2}(x-y)$$

$$Qx = \frac{1}{2}Q(x+y) + \frac{1}{2}Q(x-y)$$

$$Qx = \frac{1}{2}Q(x+y) + \frac{1}{2}Qu$$

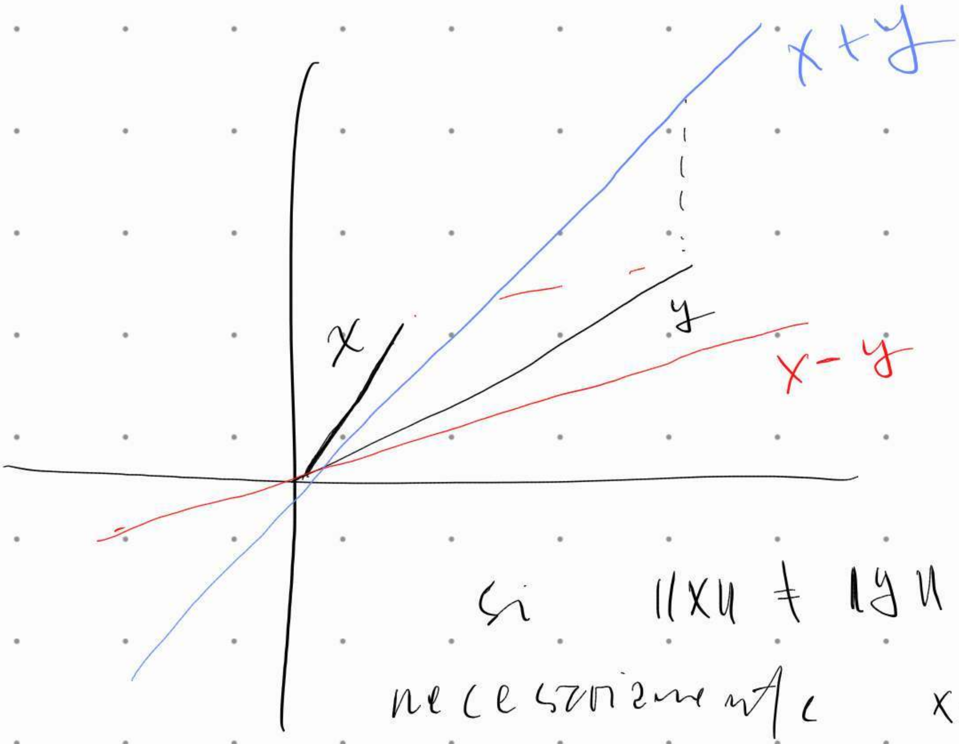
$x+y \in u^{\perp}$

$x-y = u$

$$= \frac{1}{2}(x+y) - \frac{1}{2}u$$

$$= \frac{1}{2}(x+y) - \frac{1}{2}(x-y)$$

$$\frac{1}{2}x + \frac{1}{2}y - \frac{1}{2}x + \frac{1}{2}y = y$$



si $\|x\| \neq \|y\|$ no necessariamente $x+y \perp x-y$

(5) $\|Ax - b\|_2^2 = \left\| \begin{bmatrix} Rx_1 + Wx_2 - c \\ Vx_2 - d \end{bmatrix} \right\|_2^2$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= \|Rx_1 + Wx_2 - c\|_2^2 + \|Vx_2 - d\|_2^2$

igualo a 0 pq quiero mín

Sabemos que existe (x_1, x_2)

tq $Rx_1 + Wx_2 - c = 0$

pq R rango completo

$Rx_1 + Wx_2 = c$

$\tilde{x}_1 + \tilde{x}_2 = c$

$\tilde{x}_1 = c - \tilde{x}_2$ y $\tilde{x}_1$ lo puedo armar como quiera pq R rango completo

resta ver $\|Vx_2 - d\|_2^2 = 0$

$\langle Vx_2 - d, Vx_2 - d \rangle = 0$

$\|Vx_2\|^2 + \|d\|^2 - Vx_2 d - d Vx_2 = 0$

⑥ $\Leftarrow)$ $\|Ax-b\|_2^2 = \|A(x-\bar{x})+A\bar{x}-b\|_2^2$

$$= \langle A(x-\bar{x})+A\bar{x}-b,\ A(x-\bar{x})+A\bar{x}-b\rangle$$

$$= \left((A(x-\bar{x}))^T+(A\bar{x}-b)^T\right)\cdot\left((A(x-\bar{x}))+(A\bar{x}-b)\right)$$

$$= \|A(x-\bar{x})\|^2 + (A\bar{x}-b)^T A(x-\bar{x})$$

$$(A(x-\bar{x}))^T(A\bar{x}-b)$$

escalar igual a su T

$$= \|A\bar{x}-b\|^2$$

$$= \|A(x-\bar{x})\|^2 + 2(A(x-\bar{x}))^T(Ax-b) + \|A\bar{x}-b\|^2$$

$$\|A(x-\bar{x})\|^2 + 2\cancel{(x-\bar{x})A^T(A\bar{x}-b)} + \|A\bar{x}-b\|^2$$

$\cdot)$ como $A^TA\bar{x} - A^Tb = 0$

$$A^T(A\bar{x}-b) = 0$$

$\geq \|A\bar{x}-b\|^2 \quad \forall x \in \mathbb{R}^n$

$\Rightarrow \bar{x}$ solución de minimizar

$\frac{1}{2}\|Ax-b\|_2^2$

$(\Rightarrow)$ Supongo $\bar{x}$ sol de

$$\min_{x \in \mathbb{R}^n} \frac{1}{2}\|Ax-b\|_2^2$$

$\Rightarrow \nabla f(\bar{x}) = 0$

$\nabla f(\bar{x}) = A^T(A\bar{x}-b)$ PR6 ej10

$\Leftrightarrow A^T(A\bar{x}-b) = 0$

$A^TA\bar{x} = A^Tb$

$Ax - yId \leq b$

$-Ax - yId \leq -b$