Emulating OP_RAND in Bitcoin

Rarimo Protocol

Oleksandr Kurbatov ok@distributedlab.com

Abstract

This paper proposes a method of emulation of OP_RAND opcode on Bitcoin through a trustless interactive game between transaction counterparties. The game result is probabilistic and doesn't allow any party to cheat and increase their chance to win on any protocol step. The protocol can be organized in a way unrecognizable to any external party and doesn't require some specific scripts or Bitcoin protocol updates. We will show how the protocol works on the simple **Thimbles Game** and provide some initial thoughts about approaches and applications that can use the mentioned approach.

1 Introduction

Bitcoin [1] is a decentralized payment system that focuses on censorship resistance and cryptographic control of funds. Its payments have limited programmability, allowing the realization of the simplest spending conditions: signatures, hash, and time locks. Bitcoin script doesn't directly allow putting randomness and constructing the spending flow based on that. So, realizing the flow "Alice and Bob put for 5 BTC each, and Bob takes everything if the coin comes up tails" wasn't possible upon the following assumptions:

- 1. The transaction can't derive or take randomness from somewhere at the moment of confirmation
- 2. Bitcoin Script can't inspect the block, past or future transactions
- 3. Each party can receive the same stack's state after each opcode processing
- 4. We can't control the ECDSA or Schnorr signature determinism
- 5. Bitcoin doesn't support OP_RAND opcode =)

All the limitations mentioned led to the situation where we couldn't find trustless solutions that allow scrambling randomness and use it for a trustless protocol operating with bitcoins. This paper proposes a way to organize it via a 2-party interactive protocol and shows how these properties can be applied in the example of a thimbles game that takes bids in β .

1.1 Related work

We list the set of protocols we inspired to construct the approach described in the paper. First of all, it's HTLC contracts and Lightning Network introduced by [2]. These technologies allow you to lock bitcoins on the multisig address and then send payments off-chain, changing only a reconciliation transaction. We use these properties to lock coins initially, allowing us to unlock them after the locktime if the game doesn't start.

Then, we reuse the properties provided by Atomic Swaps technology, which allows us to swap assets between different chains using secret knowledge and its commitment as a locking condition (totally trustless). The same technology can be modified using adaptor signatures introduced by [3] and taprootized untraceable version proposed by [4].

2 Preliminaries

 \mathbb{G} a cyclic group of prime order p written additively, $G \in \mathbb{G}$ is the group generator. $a \in \mathbb{F}_p$ is a scalar value and $A \in \mathbb{G}$ is a group element. $\mathsf{hash}_p(m) \to h \in \mathbb{F}_p$ is the cryptographic hash function that takes as an input an arbitrary message m and returns the field element h. $\mathsf{hash}_{160}(P) \to \mathsf{addr} \in \mathcal{A}$ is the function of hashing the public key with sha-256 and ripemd160 functions and receiving a valid bitcoin address as an output.

We define the relation for the proof π as $\mathcal{R} = \{(w; x) \in \mathcal{W} \times \mathcal{X} : \phi_1(w, x), \phi_2(w, x), \dots, \phi_m(w, x)\}$, where w is a witness data, x is a public data and $\phi_1(w, x), \phi_2(w, x), \dots, \phi_m(w, x)$ the set of relations must be proven simultaneously.

We define a bitcoin transaction as $\mathsf{TX}\{(\mathsf{id},\mathsf{i},\mathsf{proof})^{(n)};(\mathsf{aB},\mathsf{cond})^{(m)}\}$ with n inputs and m outputs, where id is the hash of the previous transaction, i - output's index, proof - the list of data which is needed to transaction spending, a - the number of coins in the output, cond - scriptPubKey conditions. For example, the P2PKH method requires $\mathsf{proof} \leftarrow \langle \mathsf{PK}, \sigma \rangle$ and $\mathsf{cond} \leftarrow \langle \mathsf{OP_DUP}, \mathsf{OP_HASH160}, \mathsf{addr}, \mathsf{OP_EQUALVERIFY}, \mathsf{OP_CHECKSIG} \rangle$. We are going to simplify the condition notation above to addr when referring to the P2PKH approach.

3 EC Point covenant

First of all, let's see how we can implement the transaction with two counterparties and the following conditions: "It's possible to spend the second transaction output only in the case the first is spent". Traditionally, it could be organized using a hash lock contract, but 1 – it's recognizable; 2 – it won't help us to implement the final game.

Algorithm 1 Creating the conditional output that can be spent in the case of spending another output Condition: Alice and Bob deposit 1 BTC each. Bob must be able to spend his 1 BTC only in the case Alice spends her 1 BTC. Bob's public key P_b is known in advance.

Flow:

1. Alice generates:

$$sk_a \leftarrow \mathbb{F}_p$$

$$P_a = sk_aG$$

$$\mathsf{addr}_a = \mathsf{hash}_{160}(P_a)$$

$$C = \mathsf{hash}_p(P_a) \cdot G$$

and creates a proof π_c for the relation:

$$\mathcal{R}_c = \{P_a; \mathsf{addr}_a, C, G : \mathsf{hash}_{160}(P_a) \to \mathsf{addr}_a \ \land \mathsf{hash}_p(P_a) \cdot G \to C\}$$

2. Bob verifies the proof π_c , takes C and calculates:

$$\mathsf{addr}_b = \mathsf{hash}_{160}(P_b + C)$$

3. Bob creates a transaction and sends it to Alice:

$$\mathsf{TX}_1\{(\mathsf{prev}_\mathsf{A},\mathsf{i}_\mathsf{A},-),(\mathsf{prev}_\mathsf{B},\mathsf{i}_\mathsf{B},\sigma_\mathsf{B}(\mathsf{TX}_1));(1\rlap{\/B},\mathsf{addr}_\mathsf{a}),(1\rlap{\/B},\mathsf{addr}_\mathsf{b})\}$$

4. Alice co-signs the transaction and propagates it to the network:

$$\mathsf{TX}_1\{(\mathsf{prev}_\mathsf{A},\mathsf{i}_\mathsf{A},\sigma_\mathsf{a}(\mathsf{TX}_1)),(\mathsf{prev}_\mathsf{B},\mathsf{i}_\mathsf{B},\sigma_\mathsf{B}(\mathsf{TX}_1));(1\rlap{\/B},\mathsf{addr}_\mathsf{a}),(1\rlap{\/B},\mathsf{addr}_\mathsf{b})\}$$

If Alice wants to spend her output, she needs to create a transaction and reveal a public key P_a and the signature value.

$$\mathsf{TX}_2\{(\mathsf{TX}_1,1,\langle \mathsf{P_a},\sigma_{\mathsf{P_a}}(\mathsf{TX}_2)\rangle);(1\rlap{\/B},\mathsf{addr}_{\mathsf{a}'})\}$$

After the transaction is published, Bob can extract P_a and recover the $\mathsf{hash}_p(P_a)$ value. Then the secret key for the second output is calculated as $sk = \mathsf{hash}_p(P_a) + sk_b$ (only Bob controls sk_b), and Bob can construct the signature related to $P_b + C$ public key and corresponding address.

$$\mathsf{TX}_3\{(\mathsf{TX}_1, 2, \langle \mathsf{P}_\mathsf{b} + \mathsf{C}, \sigma_{\mathsf{P}_\mathsf{b} + \mathsf{C}}(\mathsf{TX}_3) \rangle); (1 \ \ \mathsf{B}, \mathsf{addr}_{\mathsf{b}'})\}$$

So, we have built the first part needed for emulating the randomness and our thimbles game. We need to note that in the previous example, if Alice doesn't spend her output and doesn't publish P_a anywhere, Bob can't recover the key and spend his output as well. If we need to provide an ability to spend these outputs after some time (if the game hasn't started), we can do it similarly to the payment channel construction — lock coins to the multisignature and create a tx that can spend them after a defined period.

4 OP_RAND emulation protocol

We propose to emulate the OP_RAND opcode with an interactive protocol between parties involved in the transaction. Introducing the Challenger \mathcal{C} and Accepter \mathcal{A} roles we can define the OP_RAND emulation protocol as follows:

- 1. \mathcal{C} and \mathcal{A} have their cryptographic keypairs $\langle sk_{\mathcal{C}}, P_{\mathcal{C}} \rangle$ and $\langle sk_{\mathcal{A}}, P_{\mathcal{A}} \rangle$. Only $P_{\mathcal{C}}$ value is public
- 2. C generates the set of random values a_1, a_2, \ldots, a_n and creates a first rank commitments for them as $A_i = a_i G, i \in [1, n]$
- 3. C selects one commitment A_x , assembles it with own public key as $R_C = P_C + A_x$ and publishes only the hash value of the result $\mathsf{hash}(R_C)$
- 4. C creates second rank commitments as $h_i = \mathsf{hash}(A_i), i \in [1, n]$ and third rank commitments as $H_i = h_i G, i \in [1, n]$
- 5. C creates a proof π_a that all third rank commitments were derived correctly, and one of the first rank commitments is used for assembling with P_C
- 6. C proposes the set of third rank commitments to the A and provides π_a
- 7. \mathcal{A} verifies the proof π_a and selects one of the third-rank commitments H_y to assemble it with $P_{\mathcal{A}}$. The result $R_{\mathcal{A}} = P_{\mathcal{A}} + H_y$ is hashed hash $(R_{\mathcal{A}})$ and published
- 8. \mathcal{A} creates a proof π_r that one of the third rank commitments was used for assembling with $P_{\mathcal{A}}$ and sends it to \mathcal{C} . Additionally the proof covers the knowledge of the discrete log of $P_{\mathcal{A}}$
- 9. C verifies the proof π_r and if it's valid publishes the R_C
- 10. \mathcal{A} calculates $A_x = R_{\mathcal{C}} P_{\mathcal{C}}$
- 11. If $hash(A_x) \cdot G = H_y$, \mathcal{A} won. Otherwise lost

5 An example of Thimbles Game

Finally, we can show how the interactive protocol we introduced allows the organization of a trustless thimbles game between two counterparties. So, having Alice and Bob, the game could be described as follows:

- 1. Alice and Bob lock their coins.
- 2. Alice generates two values and selects one of them (don't reveal the selected value to Bob). In other words, Alice chooses a thimble with a ball under it.
- 3. Bob selects the thimble: takes value from proposed by Alice (and also doesn't reveal it).

- 4. Alice reveals the value she selected initially.
- 5. If Bob selected the same value he can take all deposited coins. If not, Alice can spend coins after locktime.

Algorithm 2 Thimbles game

Condition: Alice must deposit 6 BTC and Bob 5 BTC, and then Bob can take all coins only in the case he guessed the value selected by Alice.

Flow:

1. Alice generates:

$$sk_a \leftarrow \mathbb{F}_p$$
 $P_a = sk_aG$ $a_1, a_2 \leftarrow \mathbb{F}_p$ $A_1 = a_1G, A_2 = a_2G$ $h_1 = \mathsf{hash}_p(A_1), h_2 = \mathsf{hash}_p(A_2)$ $H_1 = h_1G, H_2 = h_2G$ $\mathsf{addr}_a = \mathsf{hash}_{160}(P_a + A_1)$

and creates a proof π_a for the relation:

$$\mathcal{R}_a = \{a_1, a_2; H_1, H_2, P_a, G, \mathsf{addr}_a : \\ a_1G \to A_1 \ \land a_2G \to A_2 \ \land \ \mathsf{hash}_p(A_1) \to h_1 \ \land \ \mathsf{hash}_p(A_2) \to h_2 \ \land \\ h_1G \to H_1 \ \land \ h_2G \to H_2 \ \land \\ (\mathsf{hash}_{160}(P_a + \textcolor{red}{A_1}) \to \mathsf{addr}_a \ \lor \ \mathsf{hash}_{160}(P_a + A_2) \to \mathsf{addr}_a)\}$$

2. Alice creates a transaction

$$\mathsf{TX}_1\{(\mathsf{prev}_\mathsf{A},\mathsf{i}_\mathsf{A},-),(\mathsf{prev}_\mathsf{B},\mathsf{i}_\mathsf{B},-);(1\rlap{\/B},\mathsf{addr}_\mathsf{a}),(10\rlap{\/B},-)\}$$

3. Bob generates $sk_b, P_b = sk_bG$, verifies the proof π_a , takes H_1, H_2 and selects only one value of them $(H_1 \text{ for example})$. Then Bob generates an address as:

$$\mathsf{addr}_b = \mathsf{hash}_{160}(P_b + H_1)$$

calculates the signature proving the knowledge of sk_b :

$$\sigma \leftarrow \mathsf{sigGen}(sk_b, \mathsf{addr}_b)$$

and generates a proof π_r for the relation:

$$\mathcal{R}_r = \{P_b, \sigma; \mathsf{addr}_b, H_1, H_2: \\ \mathsf{sigVer}(\sigma, P_b, \mathsf{addr}_b) \to \mathsf{true} \ \land (\mathsf{hash}_{160}(P_b + H_1) \to \mathsf{addr}_b \ \lor \ \mathsf{hash}_{160}(P_b + H_2) \to \mathsf{addr}_b) \}$$

4. Bob fills the transaction with the following data:

$$\mathsf{TX}_1\{(\mathsf{prev}_\mathsf{A},\mathsf{i}_\mathsf{A},-),(\mathsf{prev}_\mathsf{B},\mathsf{i}_\mathsf{B},\sigma_\mathsf{B}(\mathsf{TX}_1));(1\mbox{\ensuremath{\baselighta}}, \mathsf{addr}_\mathsf{a}),(10\mbox{\ensuremath{\baselighta}},\mathsf{addr}_\mathsf{b}\lor(\mathsf{P}_\mathsf{a}+\mathsf{LT}))\}$$

5. Alice verifies the proof π_r and complete the transaction with:

$$\mathsf{TX}_1\{(\mathsf{prev}_\mathsf{A},\mathsf{i}_\mathsf{A},\sigma_\mathsf{A}(\mathsf{TX}_1)),(\mathsf{prev}_\mathsf{B},\mathsf{i}_\mathsf{B},\sigma_\mathsf{B}(\mathsf{TX}_1));(1\mbox{$\rlap/$B},\mathsf{addr}_\mathsf{a}),(10\mbox{$\rlap/$B},\mathsf{addr}_\mathsf{b}\vee(\mathsf{P}_\mathsf{a}+\mathsf{LT}))\}$$

6. Alice propagates the transaction to the network.

So Bob doesn't know which value was selected by Alice, Alice doesn't know what Bob selected. And we can go to the final stage when Alice takes her coins. For that Alice spends the first output, so publishes $P_a + A_1$ value. Bob knows P_a so he can easily receive A_1 value and corresponding $h_1 = \mathsf{hash}_p(A_1)$.

If the secret key $h_1 + sk_b$ satisfies the address addr_b , Bob can take 10 β , locked on the second output. If not — Alice can spend them after the timelock.

6 Future work

Although it is quite difficult to find cases that require randomness in Bitcoin, we found that the parts described in the paper can be used to constrain the potential states and their sequence. In other words, we can emulate a minimalistic virtual machine within transaction UTXOs.

Imagine Alice has some hidden state s she doesn't want to reveal in advance. There is a set of operations $f_1(), f_2()$ (we reduce the number of possible operations to 2 for simplicity), which can be applied to the state. So, we have a limited set of states we can receive after the particular function is executed.

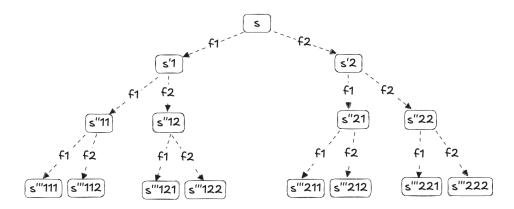


Figure 1: Alternative execution traces

Then Alice does the following:

- 1. Creates a commitment $P_0 = P_a + sG$
- 2. Creates the set of commitments for possible transitions:

$$P_1 = P_a + \mathsf{hash}(P_0)G + f_1(s)G$$

$$P_2 = P_a + \mathsf{hash}(P_0)G + f_2(s)G$$

3. Commitment for the following layer:

$$\begin{split} P_{11} &= P_a + \mathsf{hash}(P_1)G + f_1(f_1(s))G \\ P_{12} &= P_a + \mathsf{hash}(P_1)G + f_1(f_2(s))G \\ P_{21} &= P_a + \mathsf{hash}(P_2)G + f_2(f_1(s))G \\ P_{22} &= P_a + \mathsf{hash}(P_1)G + f_2(f_2(s))G \end{split}$$

4. And finally, for the last layer:

$$\begin{split} P_{111} &= P_a + \mathsf{hash}(P_{11})G + f_1(f_1(f_1(s)))G \\ & \dots \\ P_{222} &= P_a + \mathsf{hash}(P_{22})G + f_2(f_2(f_2(s)))G \end{split}$$

- 5. Then Alice generates the addr for each commitment and the proof that all commitments and addresses were generated correctly.
- 6. Then Alice creates the transaction:

$$\begin{split} \mathsf{TX}\{(\mathsf{prev}_\mathsf{A},\mathsf{i}_\mathsf{A},\sigma_\mathsf{a}(\mathsf{TX}));&(1\rlap{B},\mathsf{addr}_0+\mathsf{hashlock}(\mathsf{s})),\\ &(1\rlap{B},\mathsf{addr}_1\ \mathsf{OR}\ \mathsf{addr}_2)\\ &(1\rlap{B},\mathsf{addr}_{11}\ \mathsf{OR}\ \mathsf{addr}_{12}\ \mathsf{OR}\ \mathsf{addr}_{21}\ \mathsf{OR}\ \mathsf{addr}_{22})\\ &\dots\} \end{split}$$

7. Note that all addresses can be put as alternative spending paths in the Taproot. After the transaction is submitted, Alice can spend outputs one by one, proving the correctness of all states regardless of the exact trace that was executed.

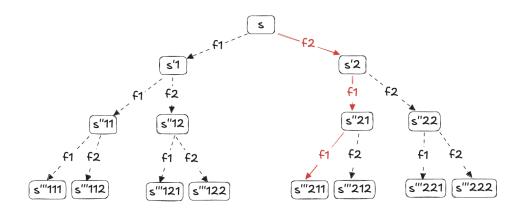


Figure 2: Actual state transitions

We see potential in the mentioned approach, but still, a huge number of improvements and potential risks MUST be considered.

References

- [1] Satoshi Nakamoto. Bitcoin: A Peer-to-Peer Electronic Cash System, 2009. URL: http://www.bitcoin.org/bitcoin.pdf
- [2] Joseph Poon and Thaddeus Dryja. The Bitcoin Lightning Network: Scalable Off-Chain Instant Payments, 2016. URL: https://lightning.network/lightning-network-paper.pdf
- [3] Andrew Poelstra. Scriptless scripts. Presentation Slides, 2017
- [4] Oleksandr Kurbatov et al. Multichain Taprootized Atomic Swaps: Introducing Untraceability through Zero-Knowledge Proofs, 2024. URL: https://arxiv.org/pdf/2402.16735