Práctica 2

Consider the simplified planar model of the control system for self balancing of a motorcycle represented in Figure 1, in which the motorcycle is represented by a bar that rotates around point $\mathbf{b}$. Points $\mathbf{c}_1$ and $\mathbf{c}_2$ are the positions of the centers of mass of the motorcycle and the reaction wheel, respectively. The control system is composed of an actuated reaction wheel, which rotates around point $\mathbf{c}_2$. A motor controls the rotation of the reaction wheel applying a torque τ, which is the only input of the control system. Angles q_1, q_2 and their time derivatives are the state variables of the system.

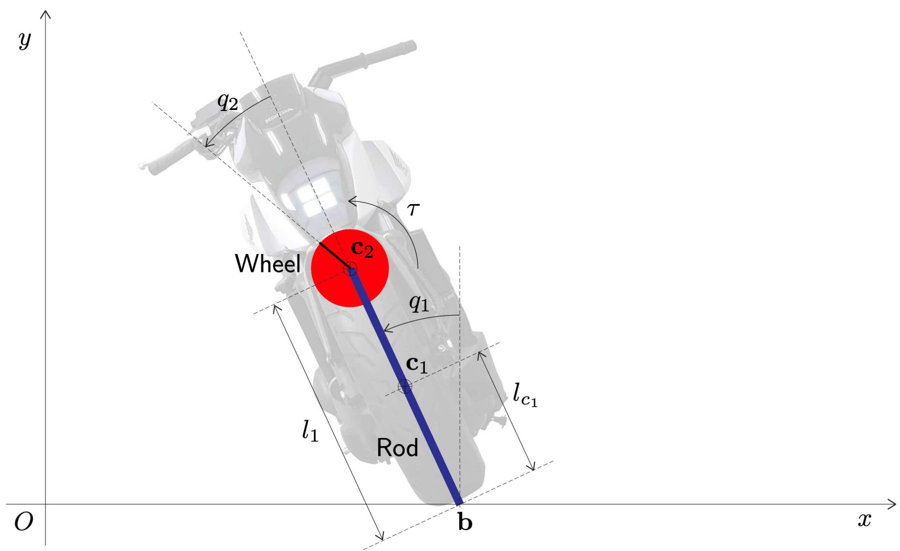

Figure 1: Sketch of the control system for self balancing of a motorcycle

The dynamic model of this system is

$$\begin{aligned}(m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 + I_2)\ddot{q}_1 + I_2\ddot{q}_2 - (m_1 l_{c_1} + m_2 l_1) g \sin(q_1) &= 0,\\ I_2\ddot{q}_1 + I_2\ddot{q}_2 &= \tau,\end{aligned}$$

with the following parameters

- mass of the motorcycle $m_1 = 200$ [kg],
- mass of the reaction wheel $m_2 = 50$ [kg],
- barycentric moment of inertia of the motorcycle $I_1 = 25$ [kg m^2],
- barycentric moment of inertia of the reaction wheel $I_2 = 5$ [kg m^2],
- length of the rod $l_1 = 1$ [m],

- $l_{c_1} = \frac{l_1}{2} = 0.5$ [m],
- gravity acceleration: $g = 9.81$ [m/s^2].

1) Intentionally omitted.

2) Calculate the state space representation of the system, assuming that $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$, where angles are expressed in [rad] and angular velocities in [rad/s]. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `statespace.m`)

3) Calculate all the operating points of the system and explain the obtained result. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `equilibrium.m`)

4) Find the operating point that corresponds to $\overline{x}_1 = 0, \overline{u} = 0$. Linearize the system around this operating point. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `linearization.m`)

5) Is the linearized system controllable? (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `controllability.m`)

6) Is the linearized system observable by means of the output $y = x_1$? Is the linearized system controllable by means of the output $y = x_2$? (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `observability.m`)

7) Using the pole placement method and an observer, design an output feedback controller to control the inclination of the motorcycle. We want to steer the motorcycle from the state $\mathbf{x} = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T = (0.26179, 0, 0, 0)^T$ to the state $\mathbf{x} = (0, 0, 0, 0)^T$. Give the eigenvalues that have been assigned to the controlled system and illustrate the behaviour of the controller by plotting the relevant state and control variables and by a graphical animation. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en la carpeta `controller`)

Originality and completeness of the written answers will be the aspects that will be taken into account in the grading of the exam, and therefore, the `Matlab` code alone will not be considered.

Solution

1) Intentionally omitted.

2) Calculate the state space representation of the system, assuming that $\mathbf{x} = (x_1, x_2, x_3, x_4)^T = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$, where angles are expressed in [rad] and angular velocities in [rad/s]. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `statespace.m`)

   Answer here

   File `statespace.m`

   ```
   Matlab code here
   ```

3) Calculate all the operating points of the system and explain the obtained result. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `equilibrium.m`)

   Answer here

   File `equilibrium.m`

   ```
   Matlab code here
   ```

4) Find the operating point that corresponds to $\overline{x}_1 = 0, \overline{u} = 0$. Linearize the system around this operating point. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `linearization.m`)

   Answer here

   File `linearization.m`

   ```
   Matlab code here
   ```

5) Is the linearized system controllable? (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `controllability.m`)

   Answer here

   File `controllability.m`

   ```
   Matlab code here
   ```

6) Is the linearized system observable by means of the output $y = x_1$? Is the linearized system controllable by means of the output $y = x_2$? (Contesta en el informe y sube el código `Matlab` a Aula Virtual en el fichero `observability.m`)

Answer here

File `observability.m`

```
Matlab code here
```

7) Using the pole placement method and an observer, design an output feedback controller to control the inclination of the motorcycle. We want to steer the motorcycle from the state $\mathbf{x} = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T = (0.26179, 0, 0, 0)^T$ to the state $\mathbf{x} = (0, 0, 0, 0)^T$. Give the eigenvalues that have been assigned to the controlled system and illustrate the behaviour of the controller by plotting the relevant state and control variables and by a graphical animation. (Contesta en el informe y sube el código `Matlab` a Aula Virtual en la carpeta `controller`)

Answer here