Midterm 1

PSTAT 5A, Summer B, 2018

Name:

Perm #:

Section Time & TA Name (draw a circle):	D.Bernstein TW 2 pm	D.Bernstein TW 3 pm	K. Wang TW 5 pm

Instructions:

- You have 80 minutes to complete the exam.
- Read each question carefully and answer all questions.
- **Round numbers to 3 decimal places.**
- You must show your work clearly: NO WORK=NO CREDIT.
- Anyone found copying another students' work will be given an F for the course.
- You are **NOT ALLOWED** to consult any notes or textbook during this exam.
- You are **NOT ALLOWED** to consult any cellphones, smartphones, computers or electronic device of any form during this exam.
- All cellphones, smartphones and computers must be turned off.
- You may use a calculator. You cannot not use a phone as a calculator.

Questions	Points
1 (40 pts)	
2 (30 pts)	
3 (30 pts)	
TOTAL	

Good Luck!!!

Question 1

a) You flip one coin, roll one dice and draw one card from a deck. All at the same time. Unfortunately, the deck of cards is very old and you can only recognize whether a card is a face or not. And the dice is even worse; only the 6 is still visible. **What is the sample space?**

Sample space of the coin: $\{H, T\}$
Sample space of the deck of cards: $\{\text{Face,Not a Face}\}$
Sample space of the deck of cards: $\{\text{Six,Not a Six}\}$

Hence, the sample space of the random experiment is:

$$\Omega = \{H, T\}\times\{\text{Face,Not a Face}\}\times\{\text{Six,Not a Six}\}$$

That is:

$$\Omega = \{(H, F, 6), (H, F, not\ 6), (H, not\ F, 6), (H, not\ F, not\ 6),$$
$$(T, F, 6), (T, F, not\ 6), (T, not\ F, 6), (T, not\ F, not\ 6)\}$$

b) Your friend is running a binomial experiment for his master's thesis project and needs your help. After 50 trials, he believes the mean is 30. **What is the standard deviation?**

$n = 50$
$\mu = 30$
$\mu = np$ then: $p = \frac{\mu}{n} = \frac{30}{50} = 0.6$

Hence:

$$\sigma = \sqrt{np(1-p)} = \sqrt{50\times0.6\times(1-0.6)} = 3.464$$

c) You draw a card from a deck of cards. And you have the following random variable:

$$Y = \begin{cases} 1 & \text{if the card is an ace or an odd number} \\ 2 & \text{if the card is an even number} \\ 3 & \text{if the card is a face (that is jack, queen or king)} \end{cases}$$

What is the mean of Y?

y	1	2	3
pdf $P(Y = y)$	$\frac{20}{52}$	$\frac{20}{52}$	$\frac{12}{52}$

$$\mu = 1\times\frac{20}{52} + 2\times\frac{20}{52} + 3\times\frac{12}{52} = \frac{96}{52} = 1.846$$

d) Same setup as c). **What is the standard deviation of Y?**

$$\sum_{\text{for all } x} x^2\, P(X = x) = 1^2\times\frac{20}{52} + 2^2\times\frac{20}{52} + 3^2\times\frac{12}{52} = \frac{208}{52} = 4$$

$$\sigma = \sqrt{\left(\sum_{\text{for all } x} x^2\, P(X = x)\right) - \mu^2} = \sqrt{4 - 1.846^2} = 0.770$$

Question 2

a) You draw a card from a deck of cards. What's the probability that the card is an even number given that is a number?

B=card is an even number
A= card is a number

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{20/52}{36/52} = \frac{5}{9} \approx 0.555$$

b) What is the probability that you draw an odd number given that card is black?

B=card is an odd number
A= card is black

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{8/52}{26/52} = \frac{8}{26} = \frac{4}{13} \approx 0.308$$

c) Now you draw two cards. One at a time with replacement. What's the probability of obtaining two black cards?

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$$

Question 3

Jennifer draw cards from a deck, one at a time with replacement. She smiles only if she gets a number. She runs this experiment 3 times.

a. Is this a binomial experiment? Check the conditions.

In this case the binomial experiment conditions hold

- The number of trials is fixed, that is $n = 3$
- The trials are all independent
- The outcome of the trial is a success whenever Jennifer smiles (that is whenever she gets a number) and a failure otherwise.
- The probability of success is constant, that is $p = \frac{36}{52}$

b. Find the cdf of the number of smiles

$Y = \#$ of times Jennifer smiles

y	0	1	2	3
pdf $P(Y = y)$	0.029	0.197	0.442	0.332
cdf $P(Y \leq y)$	0.029	0.226	0.668	1

c. Find the mean number of smiles

$$\mu = np = 3 \times \frac{36}{52} = 2.077$$

d. If the trials were done without replacement instead. Will this be a binomial experiment? Explain why.

No because:

- the probability of success would not be constant
- the trials will not be independent anymore