

**Midterm 2**  
**PSTAT 5A, Summer B, 2018**

Name:

Perm #:

Section Time & TA Name  
(draw a circle):

D.Bernstein  
TW 2 pm

D.Bernstein  
TW 3 pm

K. Wang  
TW 5 pm

**Instructions:**

- You have 80 minutes to complete the exam.
- Read each question carefully and answer all questions.
- **Round numbers to 3 decimal places.**
- You must show your work clearly: NO WORK=NO CREDIT.
- Anyone found copying another students' work will be given an F for the course.
- You are **NOT ALLOWED** to consult any notes or textbook during this exam.
- You are **NOT ALLOWED** to consult any cellphones, smartphones, computers or electronic device of any form during this exam.
- All cellphones, smartphones and computers must be turned off.
- You may use a calculator. You cannot not use a phone as a calculator.

Questions	Points
1 (40 pts)	
2 (30 pts)	
3 (30 pts)	
TOTAL	

**Good Luck!!!**





### Question 1

Let  $Z$  be a standard normal random variable. You sample this variable and you record its value each time. Based on the value of  $Z$ , you assign a value to  $W$  as follows:

$$W = \begin{cases} 2 & \text{if } Z \leq -0.51 \\ 4 & \text{if } Z \geq 0.51 \\ 7 & \text{otherwise} \end{cases}$$

- a) (10 points) What's the probability that  $W$  is even?
- b) (15 pts) You sample  $Z$  ten thousand times and you record all the values. What's the probability that  $W$  turns out odd between 4000 and 5000 (inclusive) times? Find an approximate value.

c) (10 pts) Find the approximate probability that  $W$  turns out even 6000 times.

d) (5 pts) Check any conditions used in b) or c) for your approximation to hold.

### Question 2

You graduate from UCSB and your first job is to manage the Lusail Iconic Stadium for the 2022 Qatar FIFA World Cup with capacity for 86,250 people. As a stadium manager, you believe that estimating the number of people attending a match is crucial for your decisions. To start working on this, you analyze data from this year's FIFA World Cup and obtain the following values:

# of matches	Average	St. Dev.	Min	Max
64	45363	13000	31359	78011

- a) (10 pts) At first you believe that attendance to any match is uniform. What's the probability that a match has an attendance between 70000 and 80000 people?

After a few days of examining the data you realize that it actually looks more like a Normal Distribution.

- b) (10 pts) The FIFA President emails you. He wants a 98% confidence interval for the mean match attendance. What would you report to him?

- c) (10 pts) A week later, the FIFA President tells the press that “the FIFA World Cup is a crowded event with a regular match having at least 50000 people attending”. Would you agree with him at a 1% significance level? The FIFA President thinks the real standard deviation is actually 12000.

### Question 3

Continuing with the stadium manager job and same data from Question 2:

# of matches	Average	St. Dev.	Min	Max
64	45363	13000	31359	78011

This time you don't believe the FIFA President anymore. Forget everything he said or claimed.

- a) (10 pts) Relying only on the data and your point estimates, what is the 80<sup>th</sup> percentile of people attendance for a single match?
- b) (10 pts) You would like to have a confidence interval with a margin of error of 3000 at a 95% confidence level. According to theory, how many more observations of World Cup matches would you need to achieve that?

- c) (10 pts) Can the mean match attendance be 48000 at a 5% significance level?  
Assume the standard deviation is actually 10900.