Final Exam

PSTAT 5A, Summer B, 2018

Name:

Perm #:

Section Time & TA Name (draw a circle):	A. Bernstein TW 2 pm	A. Bernstein TW 3 pm	K. Wang TW 5 pm

Instructions:

- You have 80 minutes to complete the exam.
- Read each question carefully and answer all questions.
- **Round numbers to 3 decimal places.**
- You must show your work clearly: NO WORK=NO CREDIT.
- Anyone found copying another students' work will be given an F for the course.
- You are **NOT ALLOWED** to consult any notes or textbook during this exam.
- You are **NOT ALLOWED** to consult any cellphones, smartphones, computers or electronic device of any form during this exam.
- All cellphones, smartphones and computers must be turned off.
- You may use a calculator. You cannot not use a phone as a calculator.

Questions	Points
1 (40 pts)	
2 (30 pts)	
3 (30 pts)	
TOTAL	

Question 1

You are hired to study the rental market for UCSB students. The Housing Office did a survey of 43 apartments for rent in the area. They found that the average rent was $1,873 per month with a sample standard deviation of $800. To see how the size of the apartment affects the rent, they also recorded the overall square footage for each apartment which found an average of 750 square feet with a sample standard deviation of 343.4 square feet. The sample correlation between these measurements was 0.685.

a) (10 points) Identify the explanatory and response variables in this study and estimate a simple linear regression. Give an interpretation to the slope.

b) (5 points) What would be the rent for an 800 square feet apartment?

c) (5 points) Find the coefficient of determination and give an interpretation to it.

d) (10 points) Find a 98% confidence interval for the population mean of the explanatory variable. Assume it has a normal distribution.

e) (10 points) Suppose you dig more into the details of this same survey. You find out that 35% of the surveyed apartments are infested with cockroaches. But your boss believes that at most 25% of the apartments have this issue. Would you agree with him at a 10% significance level? Check any necessary conditions.

Question 2

A scientist believes that the distribution of adults height in North Dakota is well represented by a normal random variable with mean 70 inches and standard deviation 5 inches. And a study from the US Dept. of Health found a national average height of 69.15 inches.

a) (10 pts) What' the probability that a randomly chosen North Dakotan is taller than an average American given that is shorter than 70.3 inches?

b) (10 points) If I randomly chose five North Dakotans, what is the probability that at most 2 of them are shorter than 70.4 inches?

c) (10 points) You want to make inference on the proportion of North Dakotans taller than the national average. How many North Dakotans do you need to survey to get a 97% confidence interval with a margin of error of 0.15 for this task?

Question 3

a) (10 pts) Let $\boldsymbol{X} \sim Unif(-6,7)$. What is the 60th percentile of $\boldsymbol{X}$**?**

b) (10 pts) Let $\boldsymbol{Z}$ be a standard normal random variable. You sample this variable and you record its value each time. Based on the value of $\boldsymbol{Z}$, you assign a value to $\boldsymbol{W}$ as follows:

$$\boldsymbol{W} = \begin{cases} 5 & \textbf{if } \boldsymbol{Z} > -\mathbf{0.6} \\ 10 & \textbf{if } \boldsymbol{Z} < -\mathbf{2} \\ 20 & \text{otherwise} \end{cases}$$

Find the mean of $\boldsymbol{W}$ and $P(\boldsymbol{W} > 15 | \boldsymbol{W} > 6)$

c) (10 pts) You have two trucks: one full of oranges and another full of blueberries. You take a random sample of 150 oranges and 200 blueberries. You find that 78 and 122 of them respectively were in contact with fruit flies. Test the claim that the proportion of blueberries minus the proportion of oranges in touch with fruit flies is greater than 5%. Use a 10% significance level. Check any necessary conditions.

Standard Normal Probabilities

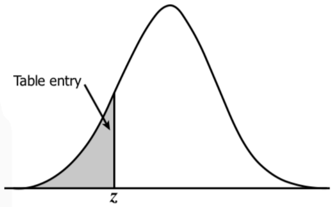

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

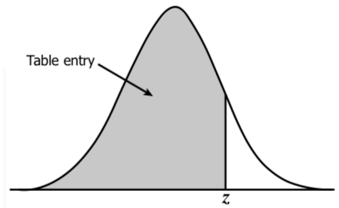

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

T-table Confidence Levels

Degrees of Freedom (*df*)	80%	90%	95%	98%	99%
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690
46	1.300	1.679	2.013	2.410	2.687
47	1.300	1.678	2.012	2.408	2.685
48	1.299	1.677	2.011	2.407	2.682
49	1.299	1.677	2.010	2.405	2.680
50	1.299	1.676	2.009	2.403	2.678
51	1.298	1.675	2.008	2.402	2.676
52	1.298	1.675	2.007	2.400	2.674
53	1.298	1.674	2.006	2.399	2.672
54	1.297	1.674	2.005	2.397	2.670
55	1.297	1.673	2.004	2.396	2.668
56	1.297	1.673	2.003	2.395	2.667
57	1.297	1.672	2.002	2.394	2.665
58	1.296	1.672	2.002	2.392	2.663
59	1.296	1.671	2.001	2.391	2.662
60	1.296	1.671	2.000	2.390	2.660
61	1.296	1.670	2.000	2.389	2.659
62	1.295	1.670	1.999	2.388	2.657
63	1.295	1.669	1.998	2.387	2.656
64	1.295	1.669	1.998	2.386	2.655
65	1.295	1.669	1.997	2.385	2.654
66	1.295	1.668	1.997	2.384	2.652
67	1.294	1.668	1.996	2.383	2.651
68	1.294	1.668	1.995	2.382	2.650
69	1.294	1.667	1.995	2.382	2.649
70	1.294	1.667	1.994	2.381	2.648
71	1.294	1.667	1.994	2.380	2.647
72	1.293	1.666	1.993	2.379	2.646
73	1.293	1.666	1.993	2.379	2.645
74	1.293	1.666	1.993	2.378	2.644
75	1.293	1.665	1.992	2.377	2.643
76	1.293	1.665	1.992	2.376	2.642
77	1.293	1.665	1.991	2.376	2.641
78	1.292	1.665	1.991	2.375	2.640
79	1.292	1.664	1.990	2.374	2.640
80	1.292	1.664	1.990	2.374	2.639
81	1.292	1.664	1.990	2.373	2.638
82	1.292	1.664	1.989	2.373	2.637
83	1.292	1.663	1.989	2.372	2.636
84	1.292	1.663	1.989	2.372	2.636
85	1.292	1.663	1.988	2.371	2.635
86	1.291	1.663	1.988	2.370	2.634
87	1.291	1.663	1.988	2.370	2.634
88	1.291	1.662	1.987	2.369	2.633
89	1.291	1.662	1.987	2.369	2.632
90	1.291	1.662	1.987	2.368	2.632
91	1.291	1.662	1.986	2.368	2.631
92	1.291	1.662	1.986	2.368	2.630
93	1.291	1.661	1.986	2.367	2.630
94	1.291	1.661	1.986	2.367	2.629
95	1.291	1.661	1.985	2.366	2.629
96	1.290	1.661	1.985	2.366	2.628
97	1.290	1.661	1.985	2.365	2.627
98	1.290	1.661	1.984	2.365	2.627
99	1.290	1.660	1.984	2.365	2.626
100	1.290	1.660	1.984	2.364	2.626
101	1.290	1.660	1.984	2.364	2.625
102	1.290	1.660	1.983	2.363	2.625
103	1.290	1.660	1.983	2.363	2.624
104	1.290	1.660	1.983	2.363	2.624
105	1.290	1.659	1.983	2.362	2.623

Probability: $P(A^c) = 1 - P(A)$ $\qquad P(B|A) = \frac{P(A\ and\ B)}{P(A)}$

$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

If A and B are independent events: $P(A\ and\ B) = P(A)(B)$

If A and B are mutually exclusive events: $P(A\ and\ B) = 0$

Discrete Random Variables:

$$\mu = \sum_{for\ all\ x} x\, P(X = x) \qquad \sigma = \sqrt{\left(\sum_{for\ all\ x} x^2\, P(X = x)\right) - \mu^2}$$

Binomial pdf: $\binom{n}{x} = \frac{n!}{(n-x)!x!}$ $\qquad P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

$\mu = np$ $\qquad \sigma = \sqrt{np(1-p)}$

Uniform pdf: pdf: $p(x) = \begin{cases} \frac{1}{b-a} \text{ if } a \le x \le b \\ 0 \text{ otherwise} \end{cases}$ $\qquad \mu = \frac{a+b}{2} \qquad \sigma = \frac{(b-a)}{\sqrt{12}}$

Normal Distribution: If $X \sim N(\mu, \sigma)$ then $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

Normal Approximation for Binomial Distribution:

$X \sim Bin(n,p) \qquad Y \sim N(\mu = np, \sigma = \sqrt{np(1-p)})$

$$P(X = x) \approx P\left(x - \frac{1}{2} \le Y \le x + \frac{1}{2}\right)$$

$P(X \le x) \approx P\left(Y < x + \frac{1}{2}\right)$ $\qquad P(X < x) \approx P\left(Y < x - \frac{1}{2}\right)$

$P(X \ge x) \approx P\left(Y > x - \frac{1}{2}\right)$ $\qquad P(X > x) \approx P\left(Y > x + \frac{1}{2}\right)$

Sample Statistics: $s = \sqrt{\frac{(\sum x^2) - n\bar{x}^2}{n-1}}$ Range=Max-Min $\quad IQR = Q_3 - Q_1$

$N(\mu, \sigma)$: $\quad \hat{\mu} = \bar{x} \quad \hat{\sigma} = s \qquad Unif(a,b)$: $\quad \hat{a} = Min \quad \hat{b} = Max$

$Bin(n,p)$: $\quad \hat{p} = \bar{x}$

Confidence Intervals for a Population Mean

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}} \qquad n_{MIN} = \left(\frac{z\sigma}{E}\right)^2 \qquad \bar{x} \pm t\frac{s}{\sqrt{n}} \qquad n_{MIN} = \left(\frac{ts}{E}\right)^2$

Hypothesis Testing for a Population Mean $\qquad z_{\text{test}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

If $H_A: \mu > \mu_0 \Rightarrow$ p-value $= P(Z > z_{\text{test}})$

If $H_A: \mu < \mu_0 \Rightarrow$ p-value $= P(Z < z_{\text{test}})$

If $H_A: \mu \ne \mu_0 \Rightarrow$ p-value $= P(Z < -|z_{\text{test}}|) + P(Z > |z_{\text{test}}|)$

Confidence Intervals for a Sample Proportion

$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad n_{MIN} = \left(\frac{z}{E}\right)^2 p(1-p)$

Hypothesis Testing for a Population Mean $\qquad z_{\text{test}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

If $H_A: p > p_0 \Rightarrow$ p-value $= P(Z > z_{\text{test}})$

If $H_A: p < p_0 \Rightarrow$ p-value $= P(Z < z_{\text{test}})$

If $H_A: p \ne p_0 \Rightarrow$ p-value $= P(Z < -|z_{\text{test}}|) + P(Z > |z_{\text{test}}|)$

Confidence Intervals for Two Sample Proportions

$$\hat{p}_1 - \hat{p}_2 \pm z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Hypothesis Testing for Two Sample Proportions

$$z_{test} = \frac{(\hat{p}_1 - \hat{p}_2) - \Delta}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\bar{p} = \frac{\#\ successes}{n_1 + n_2}$$

If $H_A: p_1 - p_2 > \Delta \Rightarrow$ p-value $= P(Z > z_{\text{test}})$

If $H_A: p_1 - p_2 < \Delta \Rightarrow$ p-value $= P(Z < z_{\text{test}})$

If $H_A: p_1 - p_2 \ne \Delta \Rightarrow$ p-value $= P(Z < -|z_{\text{test}}|) + P(Z > |z_{\text{test}}|)$

Linear Regression: $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} \quad \hat{\beta}_1 = r\frac{s_y}{s_x}$