Hypothesis Testing for a Population Mean

Javier Zapata
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Ex: Age of people in California

- Suppose you are studying the age of the people in California.
- After some studies you conclude that the age behaves like a Normal Distribution with some parameters μ and σ.
- Then you watch the news and a Senator of California claims that:

"The average age of people in California

is greater than 40"

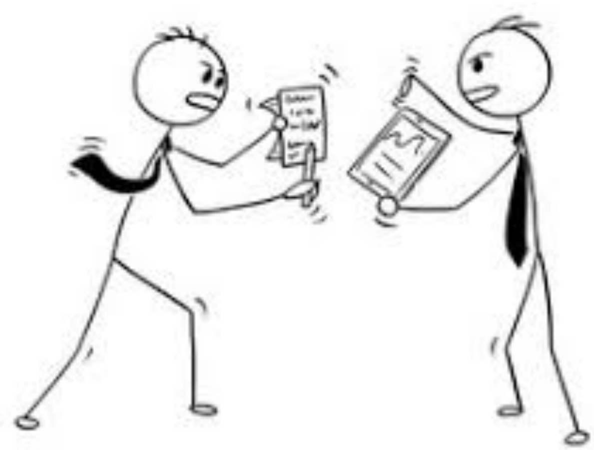
- How can you test the validity of such claim?

Figuring out the Hypothesis

- Think of this as the process of choosing between two hypothesis:

"The average age of people in California is greater than 40"

  - Someone says: $\mu \leq 40$ (and we call it the **null hypothesis H_0**)
  - And the other says: $\mu > 40$ (and we call it the **alternative hypothesis H_A**)



Figuring out the Hypothesis

- Claim: *"The average age of people in California <u>is greater than</u> 40"*

$$H_0: \mu \leq 40 \text{ vs } H_A: \mu > 40$$

- Claim : *"The average age of people in California <u>is smaller than</u> 40"*

$$H_0: \mu \geq 40 \text{ vs } H_A: \mu < 40$$

- Claim : *"The average age of people in California <u>is not</u> 40"*

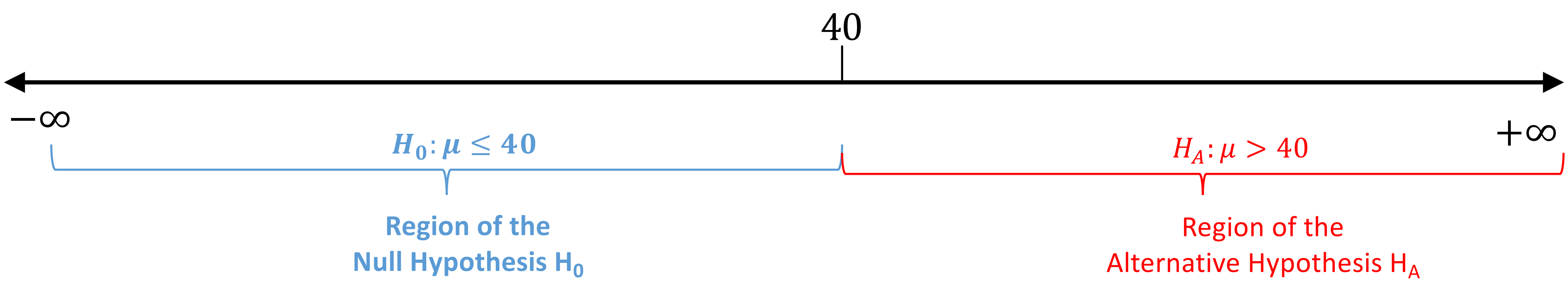
$$H_0: \mu = 40 \text{ vs } H_A: \mu \neq 40$$

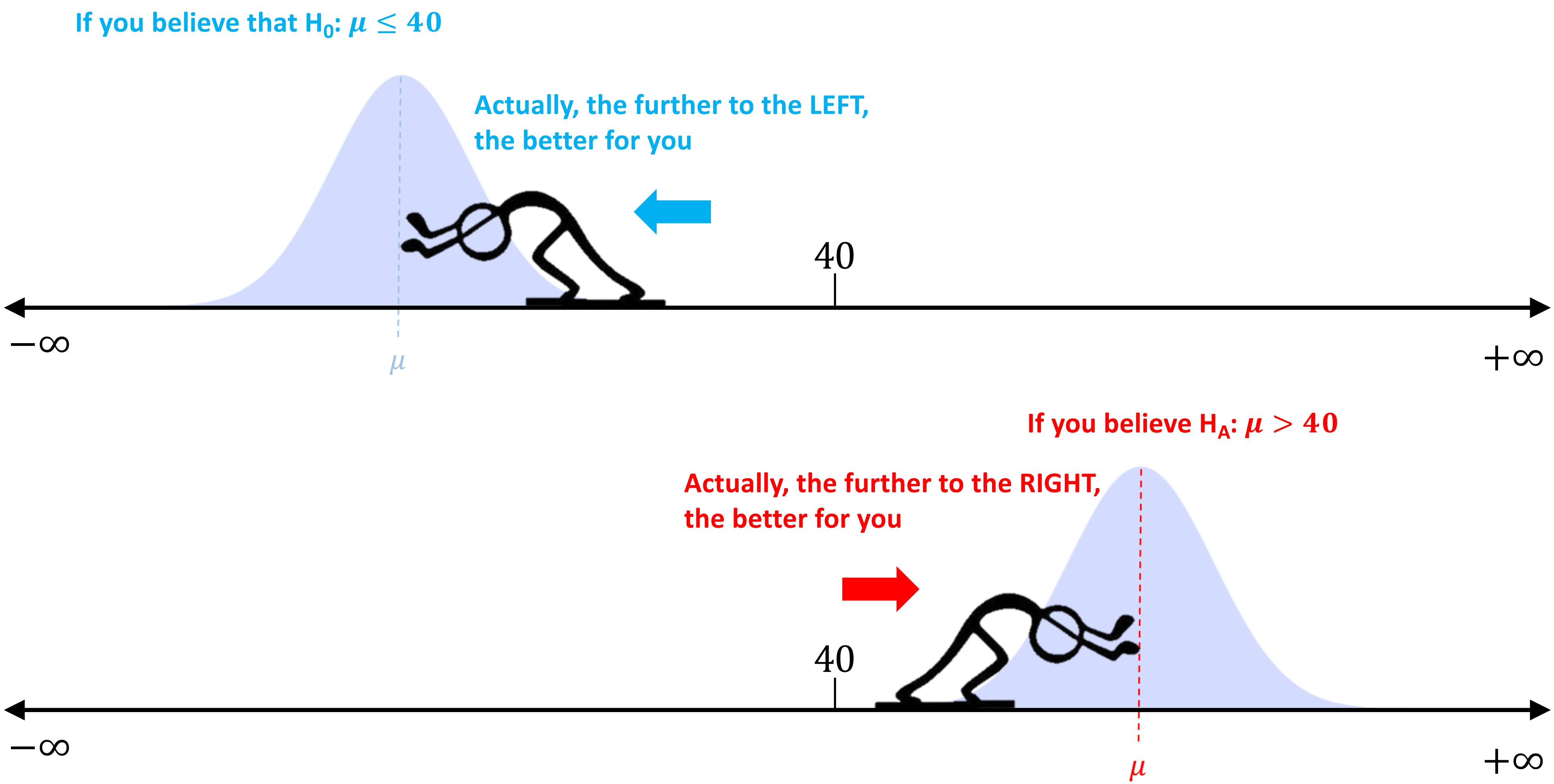
- <u>Note:</u> In all these cases, the equality ALWAYS goes into to the null hypothesis H_0

- From now on let's stick to this case:

"The average age of people in California ***is greater than*** *40"*

$$\mathbf{H_0: \mu \leq 40} \text{ vs } \mathbf{H_A: \mu > 40}$$

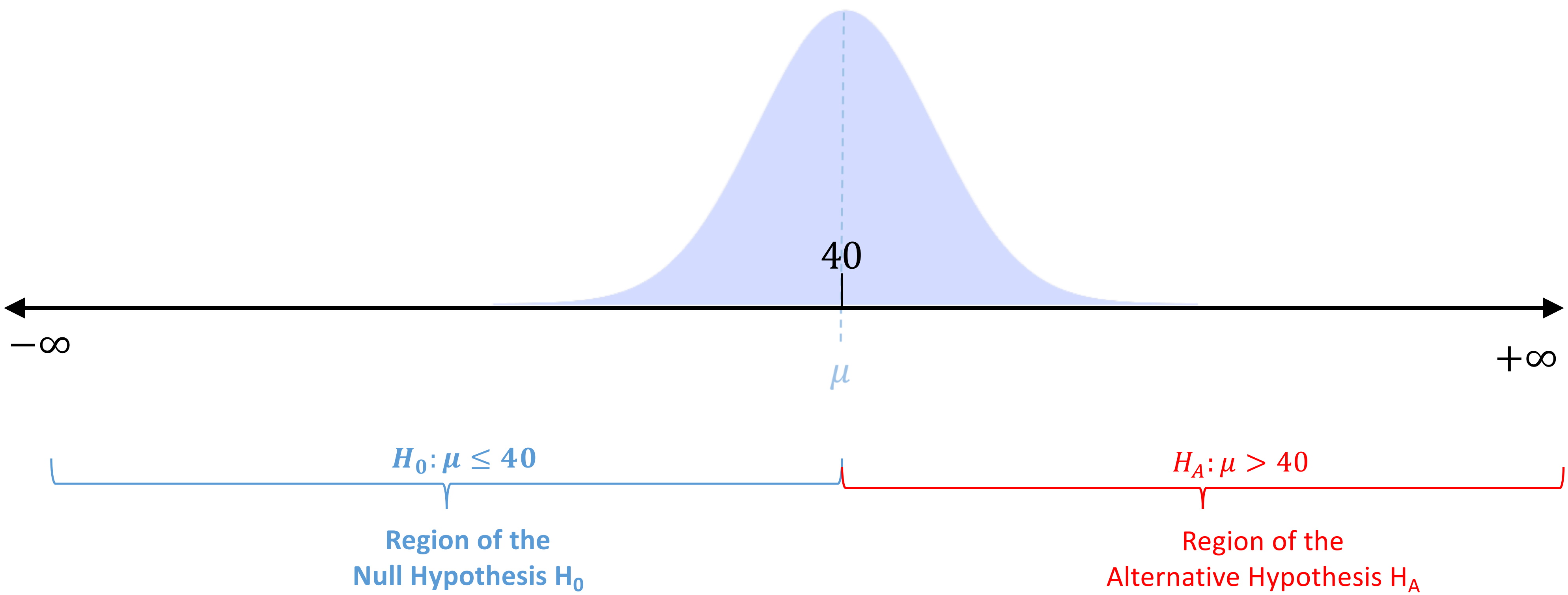




If you believe that H_0: $\mu \leq 40$
Actually, the further to the LEFT, the better for you
40
$-\infty$
μ
$+\infty$
If you believe H_A: $\mu > 40$
Actually, the further to the RIGHT, the better for you
40
$-\infty$
μ
$+\infty$

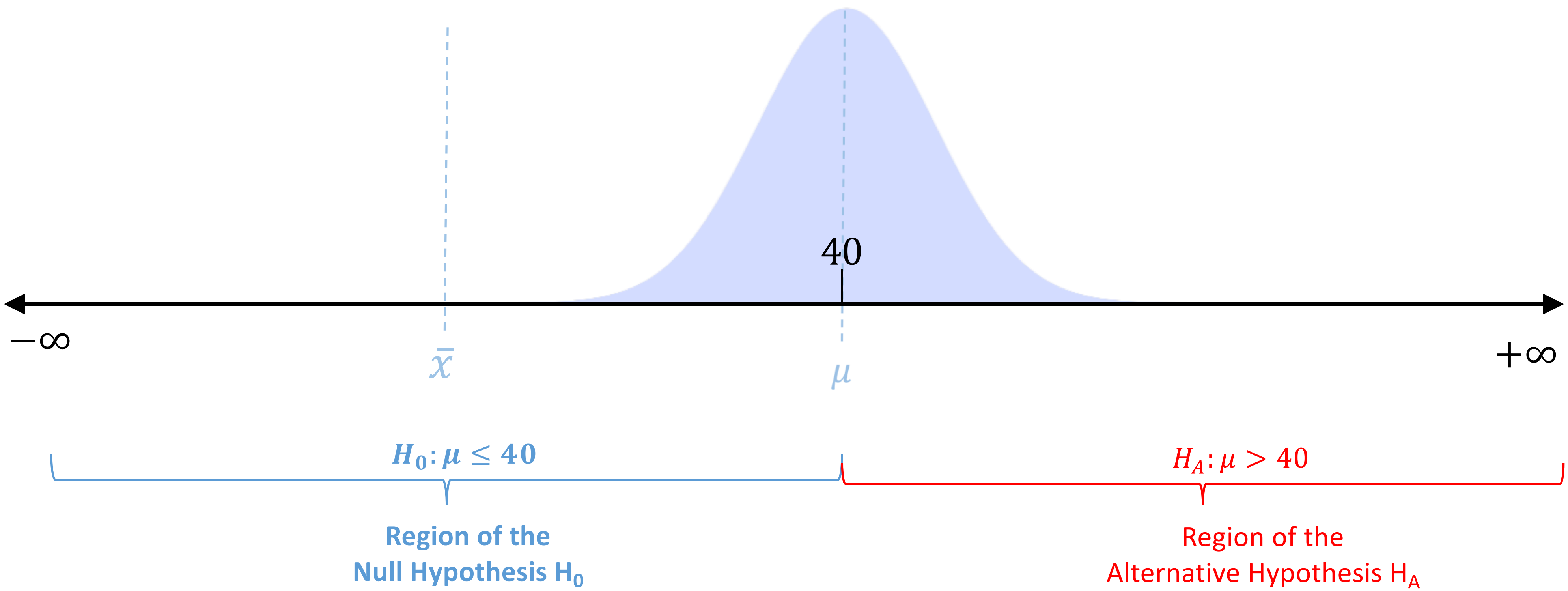
1. To settle this dispute:
   - Assume the **null hypothesis $H_0: \mu \leq 40$** is the truth
   - And give your maximum benefit of the doubt to the **alternative hypothesis $H_A: \mu > 40$**

Where would you place the Normal distribution under these two conditions?



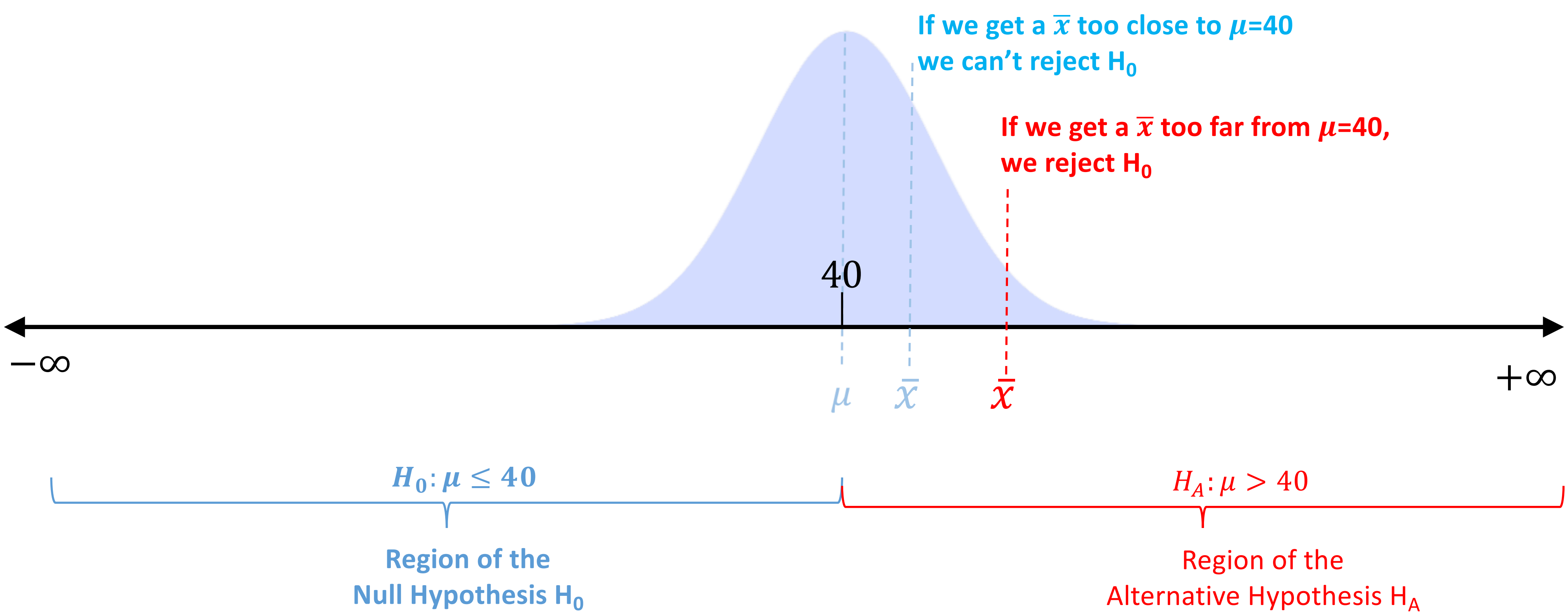
2. You sample some data and obtain the sample mean $\bar{x}$

- If $\bar{x} \leq 40$, then you can't reject **H_0**



2. You sample some data and obtain the sample mean $\bar{x}$

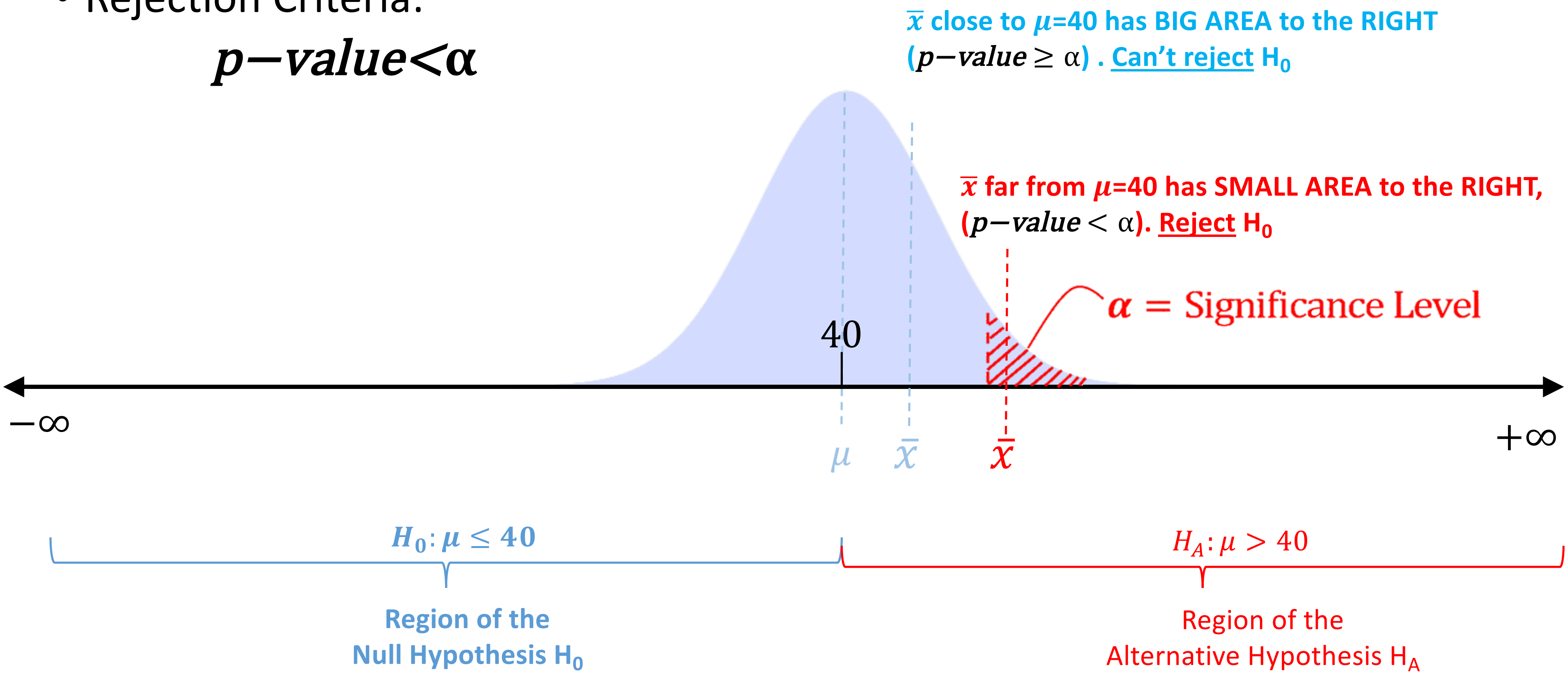
- If $\bar{x} \leq 40$, then you can't reject **H_0**
- What If $\bar{x} > 40$? You would like to reject only if its much bigger than $\mu = 40$
- How far to the right is too big? You need a rejection criterion!



2. You sample some data and obtain the sample mean $\bar{x}$

- Let the ***p−value*** be the probability of obtaining some more extreme than what you observe. In the figure below, the ***p−value*** is THE AREA TO THE RIGHT.
- Let $\boldsymbol{\alpha}$ be the significance level
- Rejection Criteria:

$$\boldsymbol{p\text{−}value < \alpha}$$



- The other two cases are analogous:

$$\mathbf{H_0}: \boldsymbol{\mu \geq 40} \ \textit{vs} \ \mathbf{H_A}: \boldsymbol{\mu < 40}$$

$$\mathbf{H_0}: \boldsymbol{\mu = 40} \ \textit{vs} \ \mathbf{H_A}: \boldsymbol{\mu \neq 40}$$

- Look at the hypothesis testing document in Gaucho Space for the rest of the details.