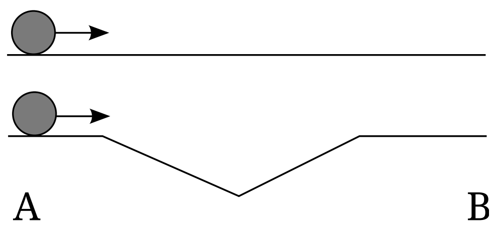
University Physcis Problems 1

23rd January 2013

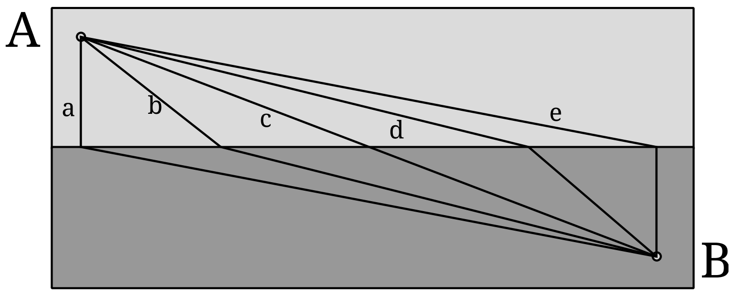
1 Rolling

Two balls of identical mass and shape with coefficient of friction $\mu = 0$, are set rolling at the same initial velocity from point A. Ignoring air resistance, which reaches the end of the track, point B, first and why?



2 Running

Runners race from point A to point B. The first section is smooth tarmac where there is good grip, the second is soft sand. Which of the five possible paths, a to e, is the fastest and why? Explain your answer.



Which area of physics is the applicable to? How would you solve this numerically?

3 Dropping

A slinky is suspended at the top and hangs under its own weight. The top is released. Does the bottom initially:

a) move up,

b) move down,

c) stay still?

4 Molecules in the Atmosphere

Calculate the number of molecules in each of your breaths that was inhaled by Julius Ceasar in his final breath. What assumptions have you made? Use the radius of the earth to be roughly 6 300 km.

5 Floating

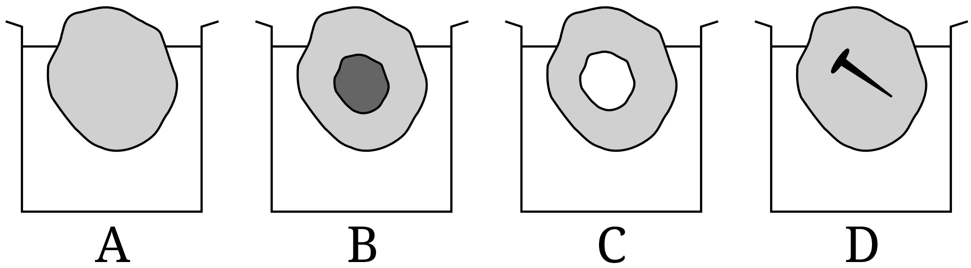
Four vessels each contain a block of floating ice in water. In the first vessel, the ice is solid; in the second, the ice has a large air buble trapped inside; the third has a region of water trapped and the fourth contains a large iron nail.

What happens to the level of the water in each vessel when all the water has melted?

a) move up,

b) move down,

c) no change?



University Physcis Answers 1

23rd January 2013

1 Rolling

The first ball reaches B first.

The total displacement is the same in each case, but it is clear that the second ball will travel a greater distance. We must consider the velocity changes. For the flat sections in both cases, the velocity is the same since there is not friction so no way to loose energy. The downill and uphill sections however must change the velocity.

Looking only at the second case, if we examine the average velocity, we have two sections at the initial velocity, one secion of positive acceleration and one section of negative acceleration. Since, over the range of the dip, the ball must begin and end at the same velocity, these must cancel out. Thus, the total average velocity of the second path is the same as the velocity of the first.

Finally we look at the time taken. The time to cover a distance s is

$$t = \frac{s}{v}$$

Since both paths have the same average velocity, but the second travels further, this must take longer and so the first ball reaches B first.

2 Running

d is the fastest route.

This is a problem of minimising the time to travel a given distance. We must take into account the fact that the runner will be able to travel faster on the tarmac than the sand, but that the longer they spend on the tarmac, the further they must travel. A compromise, then, must be made.

c (the shortest overall distance), means the runner spends too long in the sand and so is slowed down too much, however, e (the longest time on the tarmac) means the runner travels to far overall. d then is the best compromise.

This is applicable to light travelling through mediums of different density. It is the same reason that light bends when moving from air to water, or the other way round, making a straw appear to bend. The tarmac is the air in this case, where light travels faster, it then slows down at the boundary with the water, the sand.

The mathematical basis for calculating the exact route taken by the light, or runner, is called Fermat's Principle of Least Time and involves integrating the time taken given the velocity in each medium. This would be covered in first year optics.

3 Dropping

The bottom of the slinky will stay still.

There are several different explations for this effect and whole accademic papers published on the subject. Simply put, it takes a finite amount of time for information to be passed from one place to another. In this case, the information is regarding the state of the top of the slinky (has it been dropped or not) and this has to be passed from the top of the sliky to the bottom before the bottom "knows" what state it should be in. This information travels at a maximum speed that is determined by the tension in the material of the slniky. This means that if the gravity is higher, then the tension is higher, so the information travels faster, but also the slinky falls faster.

So we have a situation where the time for the information, in the form of a pressure wave, to travel the length of the slinky, and the top of the slinky to fall the same distance are the same. This means the end stays where it is until the information that the top has been released, and the rest of the slinky, reach that point, and then the whole slinky falls together. See a recent paper, August 2012, here http://arxiv.org/pdf/1208.4629v1

4 Molecules in the Atmosphere

Assume the following:

- The volume of an average breath is 0.5 litres (actual value 0.3 to 2.0 litres depending on physical activity, gender etc.)
- The atmosphere is roughly 10 km high (actual value is considerably larger but majority of mass contained within 10-15 km)
- The density of the atmosphere is $1.2\,\mathrm{kg\,m^{-3}}$ and composed entirely of nitrogen (actual value at sea level but then decreases steadily to zero, and nitrogen is 78%)
- Julius Caesar lived to 50 years (actually 55)

First we need to work out the number of molecules in the whole atmosphere, so we need the volume of the atmosphere. This is approximated as the difference between the volume of the sphere of the earth plus the atmosphere and the sphere of just the earth.

$$\begin{aligned}
V &= \frac{4}{3}\pi r^3 \\
V_{atmos} &= \frac{4}{3}\pi \times (6\,300 \times 10^3 + 10 \times 10^3) - \frac{4}{3}\pi \times (6\,300 \times 10^3) \\
V_{atmos} &= 5 \times 10^{18}\,\mathrm{m}^3
\end{aligned}$$

This means the mass of the atmosphere is easy to calculate,

$$\begin{aligned}
M_{atmos} &= \text{density} \times \text{volume} \\
M_{atmos} &= 1.2 \times 5 \times 10^{18} \\
&= 6 \times 10^{18}\,\mathrm{kg}
\end{aligned}$$

The average atomic mass of nitrogen is 14, so

$$\begin{aligned}
14\,\mathrm{g} &= 1\,\mathrm{mol} \\
14 \times 10^{-3}\,kg &= N_A\,\text{molecules} \\
\Rightarrow 1\,\mathrm{kg} &= \frac{N_A}{14 \times 10^{-3}}\,\text{molecules} \\
&= 4 \times 10^{25}\,\text{molecules per kilogram} \\
\Rightarrow N_{atmos} &= (4 \times 10^{25}) \times (6 \times 10^{18}) = 2.5 \times 10^{44}\,\text{molecules}
\end{aligned}$$

For the number of molecules in each breath, we know that 1 litre is $10^{-3}\,\mathrm{m}^3$, so the number of molecules in each breath is N_{breath}.

$$M_{breath} = 0.5 \times 10^{-3} \times 1.2 = 6 \times 10^{-4}\,\mathrm{kg}$$

We know the number of molecules of nitrogen per kilogram, so

$$\begin{aligned} N_{breath} &= 4 \times 10^{25}\,\text{molecules per kilogram} \times 6 \times 10^{-4}\,\mathrm{kg} \\ &= 2.4 \times 10^{22}\,\text{molecules} \end{aligned}$$

Assuming that, in the intervening time, the molecules from Caesar's last breath are now evenly distributed throughout the atmosphere, when randomly selecting a molecule from the air, there is a small chance that is was one of Caesar's last, given by

$$P = \frac{2.4 \times 10^{22}}{2.5 \times 10^{44}}$$

But since there are such a large number of molecules in each of your breaths, this probability becomes quite high. So for each lungfull of air, there is, on average,

$$\begin{aligned} M &= \frac{2.4 \times 10^{22}}{2.5 \times 10^{44}} \times 2.4 \times 10^{22} \\ &\approx 2 \end{aligned}$$

molecules from Caesar's final breath.