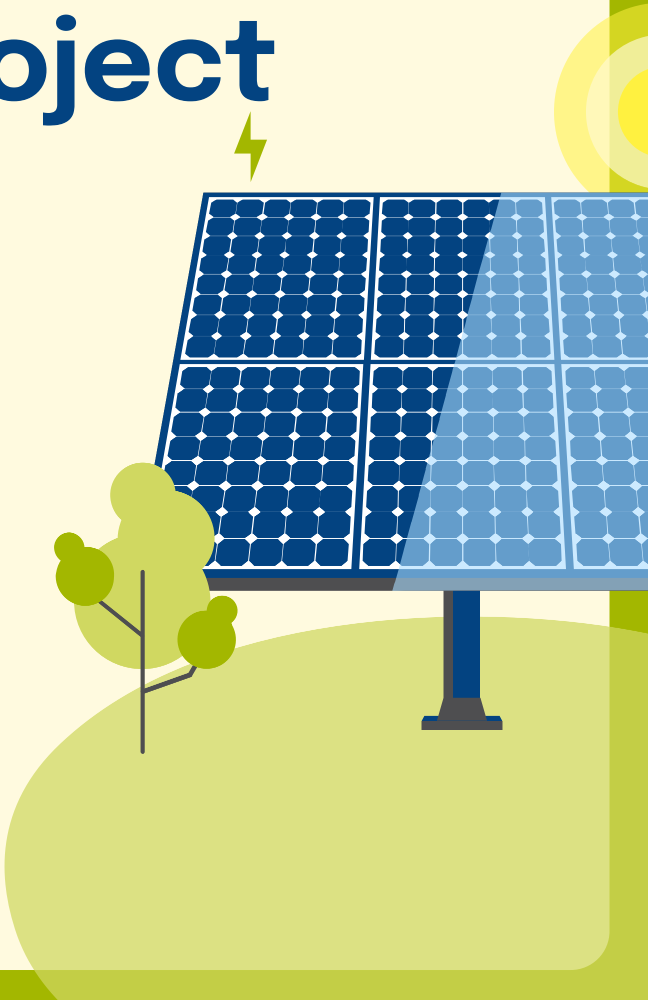
Photovoltaic project

On the Parameter Extraction of a Five-Parameter Double-Diode Model of Photovoltaic Cells and Modules



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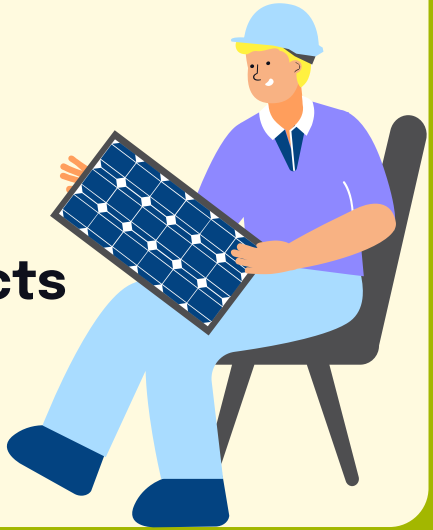


Abstract

- **Main Contribution**:
  - New set of approximate analytical solutions for PV five-parameter double-diode model.
  - Initial values for numerical solutions based on Newton–Raphson method.
- **Proposed Formulations**:
  - Developed using limited information from PV manufacturers:
    - Open-circuit voltage (Voc)
    - Short circuit current (Isc)
    - Current and voltage at maximum power point (Im and Vm)
- **Comparison with Existing Techniques**:
  - Does not require entire experimental I–V curve or additional information (e.g., slope of I–V curves).
  - Low cost and fast parameter extraction method.
- **Accuracy Evaluation**:
  - Theoretical I–V curves compared with simulation results and experimental data.
  - Results show accuracy and validity of the proposed analytical-numerical method.

Introduction

- The accurate modeling of photovoltaic (PV) cells and modules is essential for the design, analysis, and optimization of PV systems.
- Among the various models available, the single-diode and double-diode models are the most widely used to simulate the current-voltage (I-V) characteristics of PV devices.
- While the single-diode model is simpler, the double-diode model provides greater accuracy, particularly under low irradiance conditions, by accounting for recombination effects in the space-charge region.
- However, a key challenge in utilizing these models is the extraction of their parameters, which are typically implicit and nonlinear.
- Conventional methods often rely on detailed experimental I-V curve data or require additional slope information that is not always provided in manufacturer datasheets.
- These requirements make traditional parameter extraction methods time-consuming, computationally intensive, and less practical for large-scale or industrial applications.

Solving for Double Diode

fitting theoretical I–V curves to some of the experimental ones.

In the other method, the parameters are determined using a few selected key points of the experimental data. In this technique, to solve the resulting nonlinear equations, one needs a suitable initial point to make sure that the numerical iterations will converge.

In [6], [7], and [20], some analytical solutions for the parameters of the double-diode model have been derived. However, these solutions need the slope of the I–V curves at the open-circuit point which is not normally given by the PV manufacturers.

This paper : 5 parameters using the just 3 coordinates, the open-circuit (0,Voc), the short circuit (Isc, 0) the maximum power point (MPP) (Im,Vm)

Methodology overview

1. **Data Utilization**:
   - The model relies on the following three points from the manufacturer's data, The goal is to determine the parameters ($I_{ph}, I_{s1}, I_{s2}, R_s, R_{sh}$) using only the limited data provided in manufacturer datasheets:
     - Open-circuit voltage (V_{oc})
     - Short-circuit current (I_{sc})
     - Maximum power point (V_m, I_m)
     - An additional equation is derived from the slope of the power curve ($dP/dV=0$) at the MPP.
     - These lead to five nonlinear equations for the five unknown parameters.
2. **Formulation of Equations**:
   - The I-V relationship is evaluated at the three points mentioned above to derive nonlinear equations. Additionally, the slope of the power curve at the maximum power point (MPP) is used.
3. **Simplification Assumptions**:
   - Diode ideality factors ($n_1=1$, $n_2=2$).
   - Approximate exponential terms for practical computational simplification.
4. **Initial Analytical Solutions**:
   - Using the simplified equations, approximate analytical solutions for R_s and R_{sh} are derived to serve as initial guesses.
   - It is assumed that $R_{sh} \gg R_s$, which is reasonable for most PV modules.

Derivation of nonlinear equations

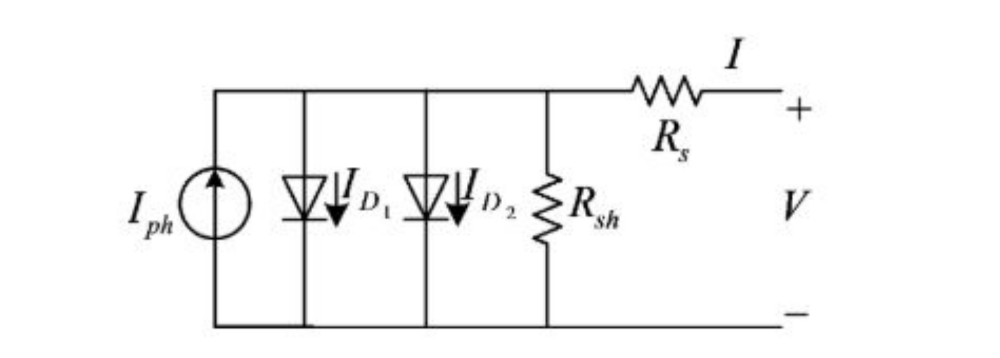
5 unknowns - Iph , Rs , Rsh , Is1, Is2 .

Known variables - Isc , Voc , Vm, Im and Vt equation

$$V_t = \frac{kT}{q}$$

$$I = I_{ph} - I_{s1}\left[\exp\left(\frac{V + R_s I}{n_1 N_s V_t}\right) - 1\right]$$

$$-I_{s2}\left[\exp\left(\frac{V + R_s I}{n_2 N_s V_t}\right) - 1\right] - \frac{V + R_s I}{R_{sh}}$$



Method

1.Approximation

Diode ideality factors approximated
n1 = 1 and n2 = 2

$$I = I_{ph} - I_{s1}\left[\exp\left(\frac{V + R_s I}{n_1 N_s V_t}\right) - 1\right] - I_{s2}\left[\exp\left(\frac{V + R_s I}{n_2 N_s V_t}\right) - 1\right] - \frac{V + R_s I}{R_{sh}}$$

We get

$$I = I_{ph} - I_{s1}\left[\exp\left(\frac{V + R_s I}{N_s V_t}\right) - 1\right] - I_{s2}\left[\exp\left(\frac{V + R_s I}{2 N_s V_t}\right) - 1\right] - \frac{V + R_s I}{R_{sh}}. \quad (3)$$

2. I-V curve evaluation

We get Voc , Isc and Im,Vm

$$0 = I_{ph} - I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - 1\right] - I_{s2}\left[\exp\left(\frac{V_{oc}}{2 N_s V_t}\right) - 1\right] - \frac{V_{oc}}{R_{sh}} \quad (4)$$

$$I_{sc} = I_{ph} - I_{s1}\left[\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right) - 1\right] - I_{s2}\left[\exp\left(\frac{R_s I_{sc}}{2 N_s V_t}\right) - 1\right] - \frac{R_s I_{sc}}{R_{sh}} \quad (5)$$

$$I_m = I_{ph} - I_{s1}\left[\exp\left(\frac{V_m + R_s I_m}{N_s V_t}\right) - 1\right] - I_{s2}\left[\exp\left(\frac{V_m + R_s I_m}{2 N_s V_t}\right) - 1\right] - \frac{V_m + R_s I_m}{R_{sh}}. \quad (6)$$

Mathematical manipulations

3. Take derivatives

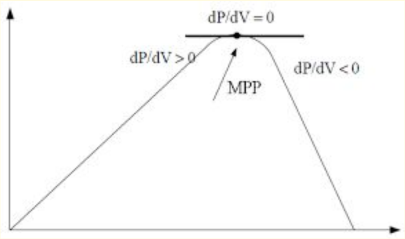
We take various derivatives to get from our initial equation to **3 final independent equation** with **4 unknowns Rs Rsh Is1 Is2**

We take the **Power Equation**: **P=VI**
Use its derivative and the results from the there to formulate the equations

$$\frac{I_m}{V_m} = \frac{I_{s1}}{N_s V_t}\left(1 - R_s\frac{I_m}{V_m}\right)\exp\left(\frac{V_m + R_s I_m}{N_s V_t}\right) + \frac{I_{s2}}{2N_s V_t} \times \left(1 - R_s\frac{I_m}{V_m}\right)\exp\left(\frac{V_m + R_s I_m}{2N_s V_t}\right) + \frac{1}{R_{sh}}\left(1 - R_s\frac{I_m}{V_m}\right)$$

$$I_{sc} = I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - \exp\left(\frac{R_s I_{sc}}{N_s V_t}\right)\right] + I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_s V_t}\right) - \exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] + \frac{V_{oc} - R_s I_{sc}}{R_{sh}}$$

$$I_m\left(1 + \frac{R_s}{R_{sh}}\right) = I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - \exp\left(\frac{V_m + R_s I_m}{N_s V_t}\right)\right] + I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_s V_t}\right) - \exp\left(\frac{V_m + R_s I_m}{2N_s V_t}\right)\right] + \frac{V_{oc} - V_m}{R_{sh}}.$$



$$\frac{dP}{dV} = \left(\frac{dI}{dV}\right)V + I. \qquad (8)$$

The derivative of the power with respect to the voltage at the MPP is zero. Thus,

$$\frac{dI}{dV} = -\frac{I_m}{V_m}. \qquad (9)$$

New variable creation – Rsho

At the short-circuit point on the I–V curve, $I = I_{sc}, V = 0, \frac{dI}{dV}\Big|_{V=0} = -\frac{1}{R_{sho}}$. Substituting these values into (10) and after some mathematical manipulations, one can obtain

$$(R_{sho} - R_s)\left[\frac{1}{R_{sh}} + \frac{I_{s1}}{N_s V_t}\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right) + \frac{I_{s2}}{2N_s V_t}\exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] - 1 = 0. \quad (15)$$

Rewriting equation 15

As shown in [6], assuming $R_{sho}, R_{sh} \gg R_s$, and $\frac{I_{s1}}{N_s V_t} \exp(\frac{R_s I_{sc}}{N_s V_t}), \frac{I_{s2}}{2N_s V_t} \exp(\frac{R_s I_{sc}}{2N_s V_t}) \ll \frac{1}{R_{sh}}$, from (15) one can conclude that $R_{sho} \approx R_{sh}$. Therefore, (15) can be rewritten as

$$(R_{sh} - R_s)\left[\frac{1}{R_{sh}} + \frac{I_{s1}}{N_s V_t} \exp\left(\frac{R_s I_{sc}}{N_s V_t}\right) + \frac{I_{s2}}{2N_s V_t} \exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] - 1 = 0. \quad (16)$$

This is our 4th independent equation with 4 unknown variables
These equation are further solved with the Newton-Raphson Method

Newton-Raphson method:

Once the initial guesses (Rs,Rsh,Is1,Is2) are derived, they are used in the Newton-Raphson method to iteratively solve the original nonlinear equations for more accurate parameter values.

1. Begin with an **initial guess** for the unknown parameter(s)
2. The parameter is updated iteratively using the following equation
3. Continue iterations until the change in Xn is very small.

$$|x_{n+1} - x_n| < \epsilon$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

x_n: Current estimate of the parameter.

$f(x_n)$: Value of the equation being solved at x_n.

$f'(x_n)$: Derivative of the equation at x_n.

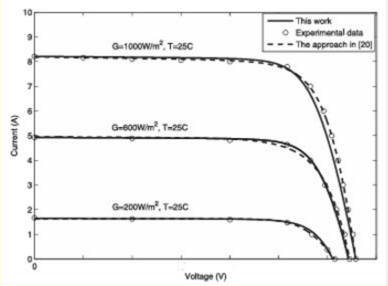
Final result

$$a = (1 + \frac{R_s}{R_{sh}})I_{sc} - \frac{V_{oc}}{R_{sh}} \quad \text{and} \quad b = (1 + \frac{R_s}{R_{sh}})(I_{sc} - I_m) - \frac{V_m}{R_{sh}}$$

$$\left[\frac{1}{R_{sh}}\left(1 - \frac{R_s I_m}{V_m}\right) - \frac{I_m}{V_m}\right]\left[2 - \exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right) - \exp\left(\frac{V_m - V_{oc} + R_s I_m}{2N_s V_t}\right)\right] + \frac{1}{N_s V_t}\left(1 - \frac{R_s I_m}{V_m}\right) \times \left[-\left(\frac{a}{2} + b\right)\exp\left(\frac{-V_{oc} + V_m + R_s I_m}{2N_s V_t}\right)\right.$$
$$\left. + \frac{a}{2}\exp\left(\frac{-V_{oc} + V_m + R_s I_m}{N_s V_t}\right) - \frac{b}{2}\exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right) + \frac{3b}{2}\right] = 0$$

$$\frac{R_{sh} - R_s}{N_s V_t}\left[a\exp\left(\frac{R_s I_{sc} - V_{oc}}{N_s V_t}\right) - (a+b)\exp\left(\frac{R_s I_{sc}}{N_s V_t} - \frac{V_m + V_{oc} + R_s I_m}{2N_s V_t}\right) + b\exp\left(\frac{R_s I_{sc}}{N_s V_t} - \frac{V_m + R_s I_m}{N_s V_t}\right)\right.$$
$$+ \frac{a}{2}\exp\left(\frac{R_s I_{sc} - V_{oc}}{2N_s V_t}\right) - \frac{b}{2}\exp\left(\frac{V_{oc} + R_s I_{sc}}{2N_s V_t} - \frac{V_m + R_s I_m}{N_s V_t}\right) - \frac{a}{2}\exp\left(\frac{V_m + R_s I_m + R_s I_{sc}}{2N_s V_t} - \frac{V_{oc}}{N_s V_t}\right)$$
$$\left. + \frac{b}{2}\exp\left(\frac{R_s I_{sc} - V_m - R_s I_m}{2N_s V_t}\right)\right] - \frac{R_s}{R_{sh}}\left[2 - \exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right) - \exp\left(\frac{V_m - V_{oc} + R_s I_m}{2N_s V_t}\right)\right] = 0.$$

Simulation 1-



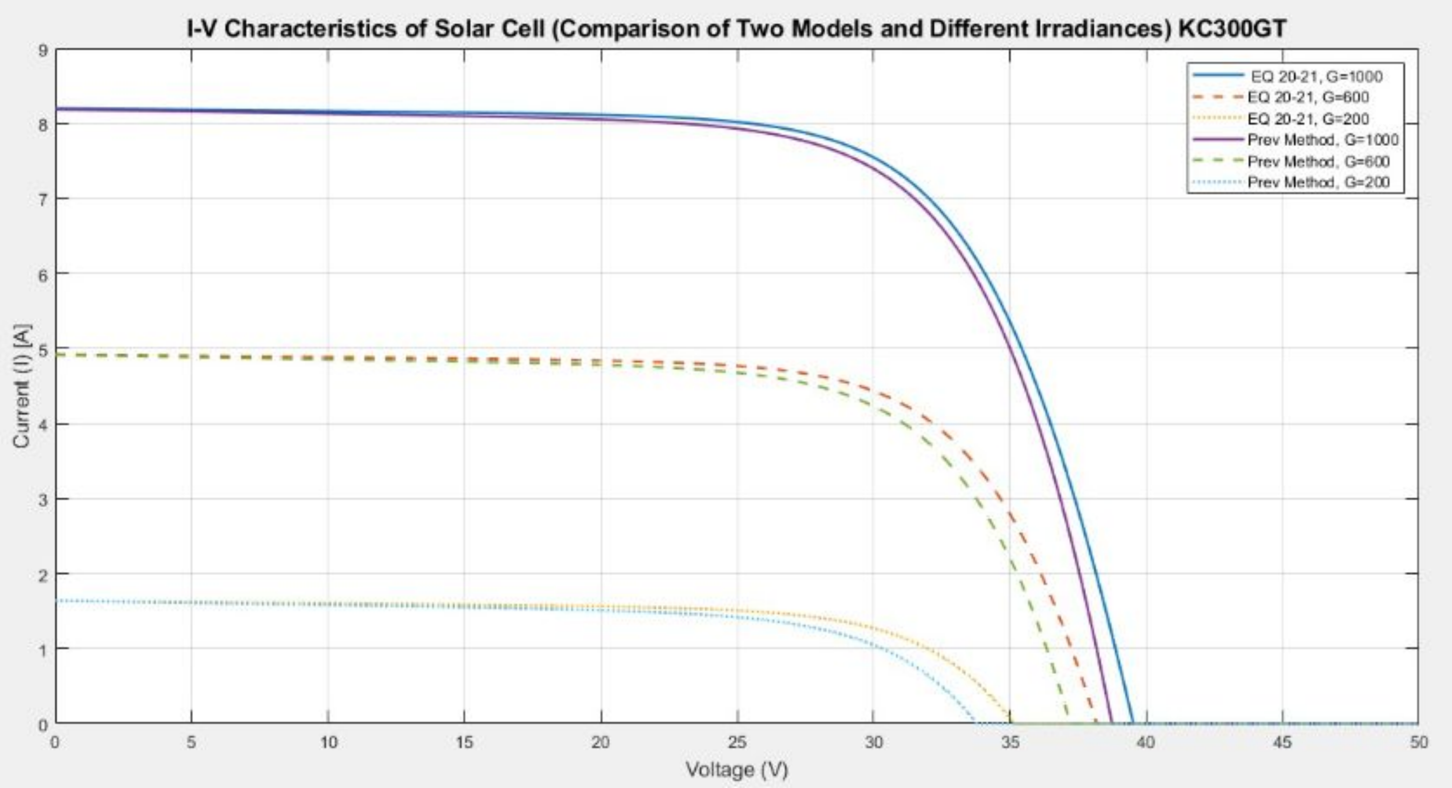


TABLE II

IDENTIFIED PARAMETERS FOR THE PV MODULE KC200GT VIA THE NUMERICAL SOLUTION OF (20), (21) AND THE METHOD IN [20]

Parameters	(20-21)	The approach in [20]
$R_s(\Omega)$	0.3181	0.1797
$R_{sh}(\Omega)$	278.9255	177.0234
$I_{s1}(nA)$	0.3795	0.3032
$I_{s2}(\mu A)$	4.4330	13.9731
$I_{ph}(A)$	8.2193	8.1959

TABLE III

EVALUATION OF NRMSE(%) FOR THE PV MODULE KC200GT VIA THE NUMERICAL SOLUTION OF (20), (21) AND THE METHOD IN [20]

	Irradiation level $G(W/m^2)$ and cell temperature $T(°C)$		
	$G=1000$ $T=25$	$G=600$ $T=25$	$G=200$ $T=25$
$nRMSE(\%)$ This work	6.35	4.36	6.55
$nRMSE(\%)$ The method in [20]	1.12	2.15	1.29

in STC. Table III gives the corresponding normalized root mean square error percentage [nRMSE(%)] calculated by

$$\mathrm{nRMSE}(\%) = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(E_i - M_i)^2}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N} M_i^2}} \times 100 \qquad (40)$$

TABLE V

IDENTIFIED PARAMETERS FOR THE PV MODULE GEPV110 VIA THE NUMERICAL SOLUTION OF (20), (21) AS THE INITIAL VALUE FOR (11), (13), (14), AND (16)

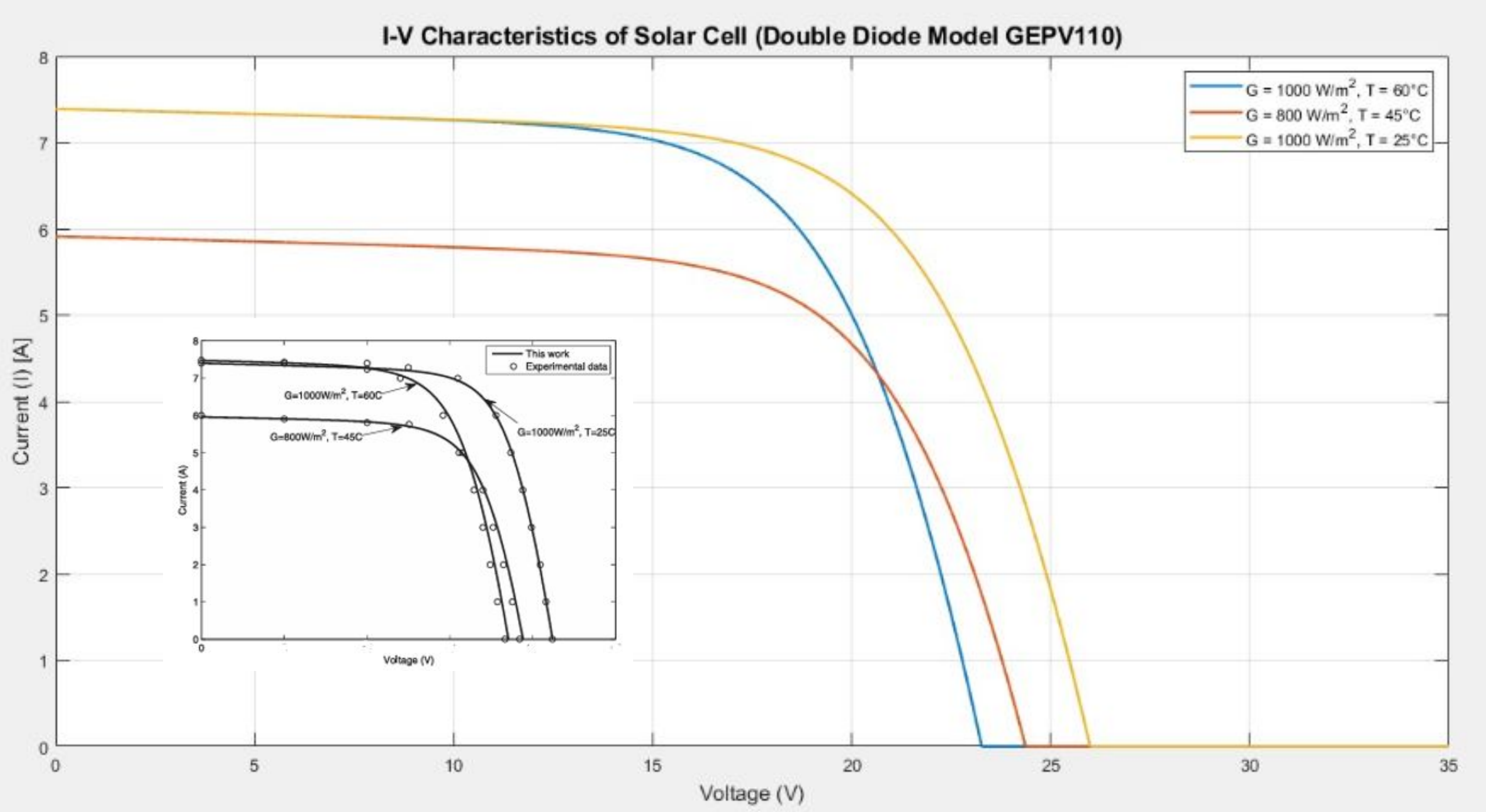
Parameters	GEPV110
$R_s(\Omega)$	0.2141
$R_{sh}(\Omega)$	83.7598
$I_{s1}(nA)$	0.5486
$I_{s2}(\mu A)$	24.0505
$I_{ph}(A)$	7.4189

TABLE VI

EVALUATION OF NRMSE(%) FOR THE PV MODULE GEPV110 VIA THE NUMERICAL SOLUTION OF (20), (21)

	Irradiation level $G(W/m^2)$ and cell temperature $T(°C)$		
	$G=1000$ $T=25$	$G=800$ $T=45$	$G=1000$ $T=60$
$nRMSE(\%)$ This work	1.29	6.05	6.73

Simulation 2-



Simulation:

The calculate for PV module for high value of resistance ie KC200GT and another low value of resistance GPEV110.

As we compare the results, we find significant similarity.

Table I. Comparing the results of Tables I and II, one can see that there is a good agreement between the numerical solution of the set of equations in (20), (21) and (11), (13), (14), (16). This confirms the reasonable approximations $\exp(\frac{V_{oc}}{N_s V_t}) \gg \exp(\frac{R_s I_{sc}}{N_s V_t})$ and $\exp(\frac{V_{oc}}{2N_s V_t}) \gg \exp(\frac{R_s I_{sc}}{2N_s V_t})$ which are used to derive (20), (21) from (11), (13, (14), and (16). This means that one can

This means we are reducing the number of parameters
Along with obtaining an acceptable degree of accuracy and convergence reliability.

The curve fitting technique utilizes Levenberg–Marquardt method, which needs to determine the appropriate initial points that are not mentioned in the manufacturer's catalogue. This method provides the calculation for the same.

During testing of the proposed parameter identification technique on different PV module datasheets, it has been observed that for some PV modules, the algebraic equations in (25) do not result in any suitable value for Rs, i.e., no feasible initial point was found to solve the set of equations in (11), (13), (14), (16) or in (20), (21). In tha

```
Iph1 = 8.21;
Rs1 = 0.3181;
Rsh1 = 278.9;
Is1_1 = 0.3795e-9;
Is2_1 = 4.4330e-6;
n1_1 = 1;
n2_1 = 2;
Ns1 = 54;
T1 = 298;

Iph2 = 8.1959;
Rs2 = 0.1797;
Rsh2 = 177;
Is1_2 = 0.3032e-9;
Is2_2 = 13.97e-6;
n1_2 = 1;
n2_2 = 2;
Ns2 = 54;
T2 = 298;

q = 1.602e-19;
k = 1.381e-23;
Vt1 = k * T1 / q;
Vt2 = k * T2 / q;
Voc1limit = 35;
Voc2limit = 35;

G_values = [1000, 600,

Vpv = linspace(0, max(V
```

```
G_values = [1000, 600, 200];
Vpv = linspace(0, max(Voc1limit, Voc2limit), 1000);

Ipv_curve1_1000 = zeros(size(Vpv));
Ipv_curve1_600 = zeros(size(Vpv));
Ipv_curve1_200 = zeros(size(Vpv));

Ipv_curve2_1000 = zeros(size(Vpv));
Ipv_curve2_600 = zeros(size(Vpv));
Ipv_curve2_200 = zeros(size(Vpv));

for i = 1:length(G_values)
    G = G_values(i);
    Iph1_G = Iph1 * (G / 1000);
    Iph2_G = Iph2 * (G / 1000);

    for j = 1:length(Vpv)
        V = Vpv(j);
        I_old = Iph1_G;
        max_iter = 10000;
        tol = 1e-6;
```

```
        for iter = 1:max_iter
            I_new = Iph1_G - Is1_1 * (exp((V + Rs1 * I_old) / (n1_1 * Ns1 * Vt1)) - 1) ...
                    - Is2_1 * (exp((V + Rs1 * I_old) / (n2_1 * Ns1 * Vt1)) - 1) ...
                    - (V + Rs1 * I_old) / Rsh1;

            if abs(I_new - I_old) < tol
                if G == 1000
                    Ipv_curve1_1000(j) = max(I_new, 0);
                elseif G == 600
                    Ipv_curve1_600(j) = max(I_new, 0);
                elseif G == 200
                    Ipv_curve1_200(j) = max(I_new, 0);
                end
                break;
            end
            I_old = I_new;
        end
end
```

```
    for j = 1:length(Vpv)
        V = Vpv(j);
        I_old = Iph2_G;
        max_iter = 1000;
        tol = 1e-6;

        for iter = 1:max_iter
            I_new = Iph2_G - Is1_2 * (exp((V + Rs2 * I_old) / (n1_2 * Ns2 * Vt2)) - 1) ...
                    - Is2_2 * (exp((V + Rs2 * I_old) / (n2_2 * Ns2 * Vt2)) - 1) ...
                    - (V + Rs2 * I_old) / Rsh2;

            if abs(I_new - I_old) < tol
                if G == 1000
                    Ipv_curve2_1000(j) = max(I_new, 0);
                elseif G == 600
                    Ipv_curve2_600(j) = max(I_new, 0);
                elseif G == 200
                    Ipv_curve2_200(j) = max(I_new, 0);
                end
                break;
            end
            I_old = I_new;
        end
    end
nd
```

```
figure;
plot(Vpv, Ipv_curve1_1000, 'LineWidth', 1.5, 'DisplayName', 'EQ 20-21, G=1000');
hold on;
plot(Vpv, Ipv_curve1_600, '--', 'LineWidth', 1.5, 'DisplayName', 'EQ 20-21, G=600');
plot(Vpv, Ipv_curve1_200, ':', 'LineWidth', 1.5, 'DisplayName', 'EQ 20-21, G=200');

plot(Vpv, Ipv_curve2_1000, 'LineWidth', 1.5, 'DisplayName', 'Prev Method, G=1000');
plot(Vpv, Ipv_curve2_600, '--', 'LineWidth', 1.5, 'DisplayName', 'Prev Method, G=600');
plot(Vpv, Ipv_curve2_200, ':', 'LineWidth', 1.5, 'DisplayName', 'Prev Method, G=200');

grid on;
xlabel('Voltage (V)', 'FontSize', 12);
ylabel('Current (I) [A]', 'FontSize', 12);
title('I-V Characteristics of Solar Cell (Comparison of Two Models and Different Irradiances) KC
legend('show');
```

```
for iter = 1:max_iter
    I_new = Iph2_G - Is1_2 * (exp((V + Rs2 * I_old) / (n1_2 * Ns2 * Vt2)) - 1) ...
            - Is2_2 * (exp((V + Rs2 * I_old) / (n2_2 * Ns2 * Vt2)) - 1) ...
            - (V + Rs2 * I_old) / Rsh2;
```

```
Rs = 0.2141;
Rsh = 83.75;
Is1 = 0.5486e-9;
Is2 = 24.05e-6;
n1 = 1;
n2 = 2;
Ns = 36;
Voclimit = 35;
q = 1.602e-19;
k = 1.381e-23;

conditions = [1000, 298; 800, 318; 1000, 333];

figure;

for i = 1:size(conditions, 1)
    G = conditions(i, 1);
    T = conditions(i, 2);

    Iph = 7.41 * (G / 1000);

    Vt = k * T / q;

    Vpv = linspace(0, Voclimit, 10000);

    Ipv_curve = zeros(size(Vpv));
```

```
for j = 1:length(Vpv)
    V = Vpv(j);

    I_old = Iph;

    max_iter = 100000;
    tol = 1e-8;

    for iter = 1:max_iter
        I_new = Iph - Is1 * (exp((V + Rs * I_old) / (n1 * Ns * Vt)) - 1) ...
                - Is2 * (exp((V + Rs * I_old) / (n2 * Ns * Vt)) - 1) ...
                - (V + Rs * I_old) / Rsh;

        if abs(I_new - I_old) < tol
            Ipv_curve(j) = I_new;
            break;
        end

        I_old = I_new;
    end

    if Ipv_curve(j) < 0
        Ipv_curve(j) = 0;
    end
end
```

```
    plot(Vpv, Ipv_curve, 'LineWidth', 1.5);
    hold on;
end

xlabel('Voltage (V)', 'FontSize', 12);
ylabel('Current (I) [A]', 'FontSize', 12);
title('I-V Characteristics of Solar Cell (Double Diode Model GEPV110)', 'FontSize', 14);
legend('G = 1000 W/m^2, T = 60°C', 'G = 800 W/m^2, T = 45°C', 'G = 1000 W/m^2, T = 25°C');
grid on;
```

Advantages

1. **Cost-Effectiveness and Speed**:
    - The method eliminates the need for complete experimental I-V curves or additional slope data, making it faster and cheaper.
2. **Independence from Full Curve Data**:
    - Unlike curve-fitting techniques that require extensive data, this approach relies solely on three key points from the I-V curve.
3. **Generality**:
    - The method can be applied to various PV modules with high accuracy.
4. **Robustness**:
    - The solutions are reliable across a range of operating conditions and PV module types, including modules with low series resistance.

Datasheet values → Analytical solutions → Numerical refinement → Accurate I-V curves.

⚠️Limitations :

1. **Computational Intensity:**
   The model is computationally intensive due to the non-linear nature of the equations, requiring iterative numerical methods for solving.

2. **Challenging Parameter Extraction:**
   Extracting precise parameters is complex and can significantly impact the accuracy of the model.

3. **Assumptions of Uniformity:**
   The model assumes **uniform temperature and irradiance**, which may not accurately represent real-world conditions, such as partial shading or temperature gradients.

Improvements :

1. **Optimization Algorithms:**
   Implement advanced techniques like **genetic algorithms** or **Machine Learning** to improve parameter extraction efficiency and accuracy.
2. **Integration with Real-Time Data:**
   Use real-time environmental data (e.g., varying temperature and irradiance) to enhance the model's adaptability and accuracy.
3. **Partial Shading Analysis:**
   Extend the model to account for **partial shading effects** and include bypass diodes for better representation of real-world scenarios.
4. **Model Generalization:**
   Adapt the model for **emerging PV technologies** like perovskite and bifacial modules to broaden its applicability.
5. **Reduced Computational Complexity:**
   Simplify the equations or develop faster iterative solvers to make the model computationally efficient for large-scale PV systems.

Conclusion:

1. **Approximate Analytical Solutions**:
   - The authors developed a set of approximate analytical solutions for the five parameters of the double-diode model (Rs,Rsh,Iph,Is1,Is2).
   - These solutions are derived using the open-circuit voltage (Voc), short-circuit current (Is), and maximum power point (MPP: Vm,Im)—data commonly available in datasheets.
2. **Suitability for Numerical Analysis**:
   - The analytical solutions provide reliable initial guesses for iterative numerical methods like Newton–Raphson, ensuring convergence to accurate results even for complex nonlinear equations.
3. **Validation**:
   - The proposed method was validated against experimental data for different PV modules. The theoretical I-V curves closely matched the experimental curves, demonstrating the method's accuracy and reliability.

The proposed analytical-numerical method strikes a balance between simplicity, efficiency, and accuracy, making it a practical solution for industrial applications in PV modeling and simulation. It offers a valuable tool for improving the design, evaluation, and performance prediction of PV systems.

Methodology:

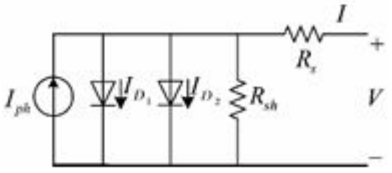


Fig. 1. Equivalent circuit of a double-diode model of a PV cell.

$$I = I_{ph} - I_{s1}\left[\exp\left(\frac{V+R_sI}{n_1N_sV_t}\right)-1\right] - I_{s2}\left[\exp\left(\frac{V+R_sI}{n_2N_sV_t}\right)-1\right] - \frac{V+R_sI}{R_{sh}} \quad (1)$$

$$V_t = \frac{kT}{q}$$

$$I = I_{ph} - I_{s1}\left[\exp\left(\frac{V+R_sI}{N_sV_t}\right)-1\right] - I_{s2}\left[\exp\left(\frac{V+R_sI}{2N_sV_t}\right)-1\right] - \frac{V+R_sI}{R_{sh}}.$$

$$0 = I_{ph} - I_{s1}\left[\exp\left(\frac{V_{oc}}{N_sV_t}\right)-1\right] - I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_sV_t}\right)-1\right] - \frac{V_{oc}}{R_{sh}} \quad (4)$$

$$I_{sc} = I_{ph} - I_{s1}\left[\exp\left(\frac{R_sI_{sc}}{N_sV_t}\right)-1\right] - I_{s2}\left[\exp\left(\frac{R_sI_{sc}}{2N_sV_t}\right)-1\right] - \frac{R_sI_{sc}}{R_{sh}} \quad (5)$$

$$I_m = I_{ph} - I_{s1}\left[\exp\left(\frac{V_m+R_sI_m}{N_sV_t}\right)-1\right] - I_{s2}\left[\exp\left(\frac{V_m+R_sI_m}{2N_sV_t}\right)-1\right] - \frac{V_m+R_sI_m}{R_{sh}}. \quad (6)$$

$$P = VI.$$

$$\frac{dP}{dV} = \left(\frac{dI}{dV}\right)V + I.$$

$$\frac{dI}{dV} = -\frac{I_m}{V_m}.$$

$$\frac{dI}{dV} = -\frac{I_{s1}}{N_sV_t}\left(1+R_s\frac{dI}{dV}\right)\exp\left(\frac{V+R_sI}{N_sV_t}\right) - \frac{I_{s2}}{2N_sV_t} \times \left(1+R_s\frac{dI}{dV}\right)\exp\left(\frac{V+R_sI}{2N_sV_t}\right) - \frac{1}{R_{sh}}\left(1+R_s\frac{dI}{dV}\right). \quad (10)$$

By substituting (10) into (9), the following equation is obtained:

$$\frac{I_m}{V_m} = \frac{I_{s1}}{N_sV_t}\left(1-R_s\frac{I_m}{V_m}\right)\exp\left(\frac{V_m+R_sI_m}{N_sV_t}\right) + \frac{I_{s2}}{2N_sV_t} \times \left(1-R_s\frac{I_m}{V_m}\right)\exp\left(\frac{V_m+R_sI_m}{2N_sV_t}\right) + \frac{1}{R_{sh}}\left(1-R_s\frac{I_m}{V_m}\right). \quad (11)$$

Using (4), one can write

$$I_{ph} = \frac{V_{oc}}{R_{sh}} + I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - 1\right] + I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_s V_t}\right) - 1\right].$$

Using (4), one can write

$$I_{ph} = \frac{V_{oc}}{R_{sh}} + I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - 1\right] + I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_s V_t}\right) - 1\right]. \qquad (12)$$

Substituting (12) into (5) and (6) yields

$$I_{sc} = I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - \exp\left(\frac{R_s I_{sc}}{N_s V_t}\right)\right] + I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_s V_t}\right) - \exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] + \frac{V_{oc} - R_s I_{sc}}{R_{sh}} \qquad (13)$$

$$I_m\left(1 + \frac{R_s}{R_{sh}}\right) = I_{s1}\left[\exp\left(\frac{V_{oc}}{N_s V_t}\right) - \exp\left(\frac{V_m + R_s I_m}{N_s V_t}\right)\right] + I_{s2}\left[\exp\left(\frac{V_{oc}}{2N_s V_t}\right) - \exp\left(\frac{V_m + R_s I_m}{2N_s V_t}\right)\right] + \frac{V_{oc} - V_m}{R_{sh}}. \qquad (14)$$

Equations (11), (13), and (14) are three independent equations with four unknown variables R_s, R_{sh}, I_{s1}, and I_{s2}. Therefore, one further equation is needed.

At the short-circuit point on the I–V curve, $I = I_{sc}$, $V = 0$, $\frac{dI}{dV}|_{V=0} = -\frac{1}{R_{sho}}$. Substituting these values into (10) and after some mathematical manipulations, one can obtain

$$(R_{sho} - R_s)\left[\frac{1}{R_{sh}} + \frac{I_{s1}}{N_s V_t}\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right) + \frac{I_{s2}}{2N_s V_t}\exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] - 1 = 0. \qquad (15)$$

Now, a new unknown variable R_{sho} is created.

As shown in [6], assuming $R_{sho}, R_{sh} \gg R_s$, and $\frac{I_{s1}}{N_s V_t}\exp(\frac{R_s I_{sc}}{N_s V_t}), \frac{I_{s2}}{2N_s V_t}\exp(\frac{R_s I_{sc}}{2N_s V_t}) \ll \frac{1}{R_{sh}}$, from (15) one can conclude that $R_{sho} \approx R_{sh}$. Therefore, (15) can be rewritten as

$$(R_{sh} - R_s)\left[\frac{1}{R_{sh}} + \frac{I_{s1}}{N_s V_t}\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right) + \frac{I_{s2}}{2N_s V_t}\exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] - 1 = 0. \qquad (16)$$

Equations (11), (13), (14), and (16) are four independent equations with four unknown variables R_s, R_{sh}, I_{s1}, and I_{s2}. These equations can be solved by the Newton–Raphson method. However, as it will be shown in Section IV, because of the very small terms of I_{s1} and I_{s2} in the Jacobian matrix, this matrix is close to singularity and for some PV modules the Newton–Raphson method may not converge. To overcome this problem, I_{s1} and I_{s2} have been eliminated from (11), (13), (14), and (16) by applying some mathematical manipulations which result in a set of equations with only unknown variables of R_s, R_{sh}.

For PV modules, the approximations $\exp(\frac{V_{oc}}{N_s V_t}) \gg \exp(\frac{R_s I_{sc}}{N_s V_t})$ and $\exp(\frac{V_{oc}}{2N_s V_t}) \gg \exp(\frac{R_s I_{sc}}{2N_s V_t})$ are valid [7]. Therefore, (13) can be rewritten as

$$I_{sc} = I_{s1} \exp\left(\frac{V_{oc}}{N_s V_t}\right) + I_{s2} \exp\left(\frac{V_{oc}}{2N_s V_t}\right) + \frac{V_{oc} - R_s I_{sc}}{R_{sh}}. \tag{17}$$

Solving the set of equations in (17) and (14) with respect to the unknown variables I_{s1} and I_{s2} yields

$$I_{s1} = \frac{a\exp(-\frac{V_{oc}}{2N_s v_t}) - b\exp(-\frac{V_m + R_s I_m}{2N_s v_t})}{\exp(\frac{V_{oc}}{2N_s v_t}) - \exp(\frac{V_m + R_s I_m}{2N_s v_t})} \tag{18}$$

$$I_{s2} = \frac{a\exp(-\frac{V_{oc}}{N_s v_t}) - b\exp(-\frac{V_m + R_s I_m}{N_s v_t})}{\exp(-\frac{V_{oc}}{2N_s v_t}) - \exp(-\frac{V_m + R_s I_m}{2N_s v_t})} \tag{19}$$

where, $a = (1 + \frac{R_s}{R_{sh}})I_{sc} - \frac{V_{oc}}{R_{sh}}$ and $b = (1 + \frac{R_s}{R_{sh}})(I_{sc} - I_m) - \frac{V_m}{R_{sh}}$.

By substituting (18) and (19) into (11) and (16) and after some mathematical manipulations one can obtain

$$\begin{aligned}
&\left[\frac{1}{R_{sh}}\left(1 - \frac{R_s I_m}{V_m}\right) - \frac{I_m}{V_m}\right]\left[2 - \exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right)\right. \\
&\quad \left. - \exp\left(\frac{V_m - V_{oc} + R_s I_m}{2N_s V_t}\right)\right] + \frac{1}{N_s V_t}\left(1 - \frac{R_s I_m}{V_m}\right) \\
&\quad \times \left[-\left(\frac{a}{2} + b\right)\exp\left(\frac{-V_{oc} + V_m + R_s I_m}{2N_s V_t}\right)\right. \\
&\quad + \frac{a}{2}\exp\left(\frac{-V_{oc} + V_m + R_s I_m}{N_s V_t}\right) \\
&\quad \left. - \frac{b}{2}\exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right) + \frac{3b}{2}\right] = 0
\end{aligned} \tag{20}$$

$$\begin{aligned}
&\frac{R_{sh} - R_s}{N_s V_t}\left[a\exp\left(\frac{R_s I_{sc} - V_{oc}}{N_s V_t}\right) - (a+b)\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right.\right. \\
&\quad \left. - \frac{V_m + V_{oc} + R_s I_m}{2N_s V_t}\right) + b\exp\left(\frac{R_s I_{sc}}{N_s V_t} - \frac{V_m + R_s I_m}{N_s V_t}\right) \\
&\quad + \frac{a}{2}\exp\left(\frac{R_s I_{sc} - V_{oc}}{2N_s V_t}\right) - \frac{b}{2}\exp\left(\frac{V_{oc} + R_s I_{sc}}{2N_s V_t}\right. \\
&\quad \left. - \frac{V_m + R_s I_m}{N_s V_t}\right) - \frac{a}{2}\exp\left(\frac{V_m + R_s I_m + R_s I_{sc}}{2N_s V_t} - \frac{V_{oc}}{N_s V_t}\right) \\
&\quad \left. + \frac{b}{2}\exp\left(\frac{R_s I_{sc} - V_m - R_s I_m}{2N_s V_t}\right)\right] \\
&\quad - \frac{R_s}{R_{sh}}\left[2 - \exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right)\right. \\
&\quad \left. - \exp\left(\frac{V_m - V_{oc} + R_s I_m}{2N_s V_t}\right)\right] = 0.
\end{aligned} \tag{21}$$

III. Derivation of Analytical Solutions as an Initial Point

Considering $R_{sh} \gg R_s$ [7], $1 + \frac{R_s}{R_{sh}}$ is approximated by 1. In addition, according to the typical available values for the PV module parameters in the datasheets and literatures [6], the approximations $I_{sc} \gg \frac{V_{oc}}{R_{sh}}$, $I_{sc} - I_m \gg \frac{V_m}{R_{sh}}$ are valid. According to these reasonable assumptions, the terms $a = (1 + \frac{R_s}{R_{sh}})I_{sc} - \frac{V_{oc}}{R_{sh}}$ and $b = (1 + \frac{R_s}{R_{sh}})(I_{sc} - I_m) - \frac{V_m}{R_{sh}}$ are approximated with $a = I_{sc}$ and $b = I_{sc} - I_m$. Thus, they become independent of R_s and R_{sh}. The term $\frac{1}{R_{sh}}(1 - \frac{R_s I_m}{V_m}) - \frac{I_m}{V_m}$ in (20) is also estimated as $-\frac{I_m}{V_m}$. According to these simplifications, (18) and (19) can be rewritten as

$$I_{s1} = \frac{I_{sc}\exp(-\frac{V_{oc}}{2N_s v_t}) - (I_{sc} - I_m)\exp(-\frac{V_m + R_s I_m}{2N_s v_t})}{\exp(\frac{V_{oc}}{2N_s v_t}) - \exp(\frac{V_m + R_s I_m}{2N_s v_t})} \quad (22)$$

$$I_{s2} = \frac{I_{sc}\exp(-\frac{V_{oc}}{N_s v_t}) - (I_{sc} - I_m)\exp(-\frac{V_m + R_s I_m}{N_s v_t})}{\exp(-\frac{V_{oc}}{2N_s v_t}) - \exp(-\frac{V_m + R_s I_m}{2N_s v_t})}. \quad (23)$$

Also, (20) is simplified as

$$\begin{aligned} &-\frac{I_m}{V_m}\left[2 - \exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right)\right. \\ &\left.- \exp\left(\frac{V_m - V_{oc} + R_s I_m}{2N_s V_t}\right)\right] + \frac{1}{N_s V_t}\left(1 - \frac{R_s I_m}{V_m}\right) \\ &\times\left[-\left(\frac{3I_{sc}}{2} - I_m\right)\exp\left(\frac{-V_{oc} + V_m + R_s I_m}{2N_s V_t}\right)\right. \\ &+ \frac{I_{sc}}{2}\exp\left(\frac{-V_{oc} + V_m + R_s I_m}{N_s V_t}\right) - \frac{I_{sc} - I_m}{2} \\ &\left.\times\exp\left(\frac{V_{oc} - V_m - R_s I_m}{2N_s V_t}\right) + \frac{3(I_{sc} - I_m)}{2}\right] = 0. \quad (24) \end{aligned}$$

We use the first-, second-, and third-order approximations $\exp(kR_s) = 1 + kR_s, \exp(kR_s) = 1 + kR_s + k^2R_s^2/2$, and $\exp(kR_s) = 1 + kR_s + k^2R_s^2/2 + k^3R_s^3/6$ to transform the exponential terms in (24) to polynomial ones. Therefore, the following quadratic, cubic, and quartic equations are obtained:

$$\begin{aligned} A_2R_s^2 + B_2R_s + C_2 &= 0 \\ A_3R_s^3 + B_3R_s^2 + C_3R_s + D_3 &= 0 \\ A_4R_s^4 + B_4R_s^3 + C_4R_s^2 + D_4R_s + E_4 &= 0. \end{aligned} \quad (25)$$

Coefficients $A_i, B_i, C_i, i \in \{2,3,4\}, D_3, D_4, E_4$ are provided in Appendix. Comparing the calculated coefficients to each other, one can obtain

$$\begin{aligned} E_4 &= D_3 = C_2 \\ D_4 &= C_3 = B_2 \\ C_4 &= B_3. \end{aligned} \quad (26)$$

After analytical solving of the proposed classic equations in (25) and obtaining a feasible solution for R_s, one can substitute it into (22) and (23) to compute the saturation currents I_{s1} and I_{s2}. Next, an estimation of R_{sh} is calculated by (16). This equation can be rewritten in the following form:

$$\begin{aligned} R_{sh}(R_{sh} - R_s)&\left[\frac{1}{R_{sh}} + \frac{I_{s1}}{N_s V_t}\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right)\right. \\ &\left.+ \frac{I_{s2}}{2N_s V_t}\exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right)\right] - R_s = 0. \quad (27) \end{aligned}$$

Since $R_{sh} \gg R_s$, from (27), one can obtain

$$R_{sh} = \sqrt{\frac{R_s}{\frac{I_{s1}}{N_s V_t}\exp(\frac{R_s I_{sc}}{N_s V_t}) + \frac{I_{s2}}{2N_s V_t}\exp(\frac{R_s I_{sc}}{2N_s V_t})}} \quad (28)$$

where I_{s1} and I_{s2} are calculated via (22) and (23). Therefore, (25), (22), (23), and (28) provide approximate analytical solutions, and at the same time they are suitable initial points for the implicit nonlinear equations of (11), (13), (14), and (16) or (20), (21).

A comprehensive discussion on the analytic solution of the cubic and quartic equations in (25) can be found in any classic book on the theory of equations [30]. Based on the Abel–Ruffini theorem, the quartic or the fourth-order equation is the highest degree of a general polynomial for which the analytical solutions based on radicals can be found [31].

Note that in (11), (13), (14), and (16) or (20), (21), the parameter I_{ph} is eliminated, and therefore, it needs no initial value. However, one can find a good estimation of this parameter via (5). This equation can be rewritten as

$$\begin{aligned} I_{ph} = I_{sc} + I_{s1}&\left[\exp\left(\frac{R_s I_{sc}}{N_s V_t}\right) - 1\right] \\ &+ I_{s2}\left[\exp\left(\frac{R_s I_{sc}}{2N_s V_t}\right) - 1\right] + \frac{R_s I_{sc}}{R_{sh}}. \quad (29) \end{aligned}$$

Now, since I_{sc} is much greater than the other terms on the right-hand side of (29), therefore, the value of I_{ph} is estimated as I_{sc}.