



8 Probability

Probability in Statistics (Imkanaat in Shumariyat)

Probability, ya Imkanaat, statistics mein ek bunyadi aur ahem concept hai jo ke kisi event ke hone ki likelihood ya chances ko measure karta hai.

- **Tafseel:** Probability ye batati hai ke koi khaas event ya natija kitna mumkin hai. Is ki range 0 se 1 tak hoti hai, jahan 0 ka matlab hai ke event bilkul bhi mumkin nahi aur 1 ka matlab hai ke event yaqeenan hone wala hai.
- **Ahmiyat:** Probability statistics ke har shobe mein istemal hoti hai aur ye data analysis, predictions, aur decision-making mein madad karta hai. Ye humein uncertain situations mein rational decisions lene mein madad karta hai.
- **Misal:** Jaise, ek sikke ko uchhalne par heads ya tails aane ka probability 0.5 (ya 50%) hota hai. Ya phir, mausam ki peshgoi karte waqt barish hone ka probability batana.

8.1 Probability ke Types (Iqsaam)

1. **Theoretical Probability (Nazari Imkanaat):** Ye woh probability hoti hai jo theory ya basic principles par mabni hoti hai. Jaise, sikke ki misal mein heads ya tails aane ka theoretical probability hamesha 0.5 hota hai.
2. **Experimental Probability (Tajurbaati Imkanaat):** Ye woh probability hoti hai jo actual experiments ya observations se calculate ki jati hai. Jaise, agar aap 100 dafa sikka uchhalte hain aur 60 dafa heads aata hai, to heads aane ki experimental probability 0.6 ho jayegi.
3. **Subjective Probability (Subjective Imkanaat):** Ye individual judgment ya anubhav par based hoti hai. Jaise, kisi doctor ka ye kehna ke kisi mareez ko kisi bimari hone ki kitni probability hai, based on unke past experiences.

Probability

Probability, statistics ka woh hissa hai jo humein uncertain events aur outcomes ko quantify karne mein madad karta hai. Is ke zariye, hum risk assessment karte hain, predictions karte hain, aur complex data sets ko better samajh sakte hain. Ye statistical analysis ka aham juz hai aur is ka istemal kai different fields mein hota hai, jaise finance, engineering, science, aur social sciences.

Let's explore a real-life example of probability and understand how to calculate it. I'll present this in an engaging and informative style:

8.1.1 Real-Life Example of Probability

Example: Weather Forecasting (Mausam ki Peshgoi)

Imagine you're a meteorologist trying to predict the probability of rain tomorrow in your city. Based on historical weather data, you know that in your city, out of 365 days, it typically rains on 100 days.

8.1.1.1 Calculating Probability

1. Identify the Total Number of Outcomes (Mumkin Natijaat ki Tadaad):

- In this case, the total number of outcomes is 365 days (a year).

2. Identify the Number of Favorable Outcomes (Sazgaar Natijaat ki Tadaad):

- The number of days it rains is 100.

3. Use the Probability Formula (Imkanaat ka Formula):

- The formula for probability is:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

- Applying this to our example:

$$P(\text{Rain}) = \frac{100}{365} \approx 0.274$$

8.1.1.2 Interpretation (Tashreeh)

- The probability of rain tomorrow, based on historical data, is approximately 0.274 or 27.4%.
- This means there's a 27.4% chance that it will rain tomorrow.

8.1.2 Real-Life Importance (Asal Zindagi Mein Ahmiyat)

Probability calculations like this are used every day in weather forecasting. By analyzing historical weather data, meteorologists can predict future weather patterns and provide valuable information to the public and various industries. Similarly, probability is used in many other fields like finance for risk assessment, in healthcare for disease prediction, and in sports for game strategy.

Calculating and understanding probabilities helps in making informed decisions based on statistical evidence rather than guesswork or intuition. This is crucial in fields where decisions can have significant consequences.

8.2 Basic Probability Concepts and Definitions

1. Probability:

- **Definition:** Probability kisi event ke hone ke chances ya likelihood ko measure karta hai.
- **Example:** Sikka uchalne par 'heads' aane ki Probability 0.5 hai.
- **Formula:** Probability ka formula $P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$

2. Experiment (Tajurba):

- **Definition:** Koi bhi process ya activity jise perform kiya ja sakta hai aur jiska measurable outcome ho.
- **Example:** Sikka uchalna ek tajurba hai jiska outcome 'heads' ya 'tails' ho sakta hai.

3. Outcome (Natija):

- **Definition:** Kisi tajurbe ke natije mein se har ek mumkin result.

- **Example:** Sikka uchalne mein ‘heads’ ya ‘tails’ aana ek natija hai.

4. Sample Space:

- **Definition:** Kisi tajurbe ke sab mumkin natijaat ka majmua.
- **Example:** Sikka uchalne mein sample space hai {‘heads’, ‘tails’}.

5. Event (Waqaat):

- **Definition:** Sample space mein se kisi specific group of outcomes ka selection.
- **Example:** Ek card draw karne mein, ‘king’ card nikalna ek event hai.

6. Random Variable:

- **Definition:** Kisi tajurbe ke natijat ko numeric values se represent karta hai.
- **Example:** Sikka uchalne par, assign karen ‘1’ for ‘heads’ aur ‘0’ for ‘tails’.

7. Independent Events (Azad Waqeaat):

- **Definition:** Jab ek event ke natije dusre event par asar nahi daalte.
- **Example:** Alag alag sikke uchalne ke natije ek dusre par asar nahi daalte.

8. Dependent Events (Mutassir Waqeaat):

- **Definition:** Jab ek event ka natija dusre event par asar daalta hai.
- **Example:** Agar ek bag se bina wapas daale hue ek card nikala jaye, to dusra card nikalne ke natije pehle card par depend karte hain.

9. Mutually Exclusive Events:

- **Definition:** Aise events jo ek sath nahi ho sakte.
- **Example:** Ek toss mein ek waqt mein ‘heads’ aur ‘tails’ dono nahi aa sakte.

10. Collectively Exhaustive Events:

- **Definition:** Aise events jo ek sath ho sakte hain aur jin ka union sample space ke barabar ho.
- **Example:** Ek toss mein ek waqt mein ‘heads’ aur ‘tails’ dono aa sakte hain.

11. Complement of an Event:

- **Definition:** Ek event ke sath us ke bilkul opposite event.
- **Example:** Ek card draw karne mein, ‘king’ card nikalne ka complement ‘not king’ card hai.
- **Formula:** Complement ka formula $P(\text{Event}) + P(\text{Complement of Event}) = 1$

12. Trial (Aazmaish):

- **Definition:** Imkanaat mein, aazmaish se murad woh individual execution ya performance hota hai jahan ek ya zyada outcomes mumkin hote hain.
- **Example:** Ek dice ko roll karna ek aazmaish hai, jahan outcomes 1 se 6 tak ho sakte hain.

13. Venn Diagram (Venn Diagram):

- **Definition:** Ye ek graphic organizer hai jo sets aur unke relationships ko visually show karta hai.
- **Example:** Do circles jo ek dusre ko overlap karte hain, un sets ke common elements ko show karte hain.

► Code

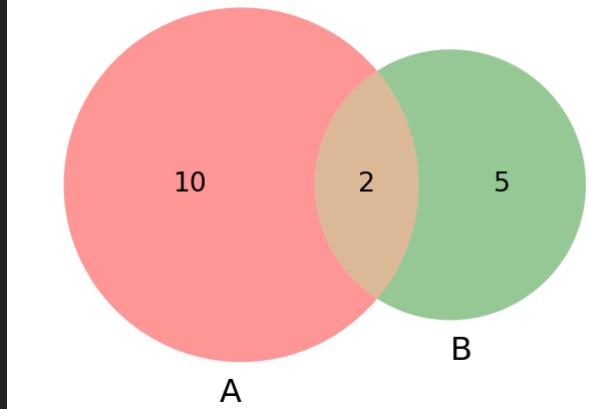


Figure 8.1: Ven Diagram

14. Union (Ittihad):

- o **Definition:** Do ya zyada sets ka union woh set hota hai jo un sabhi sets ke elements ko include karta hai.
- o **Example:** Agar Set A = {1, 2, 3} aur Set B = {3, 4, 5}, to A Union B = {1, 2, 3, 4, 5}.

15. Intersection (Goondh):

- o **Definition:** Do ya zyada sets ka intersection woh set hota hai jo un sabhi sets ke common elements ko include karta hai.
- o **Example:** Agar Set A = {1, 2, 3} aur Set B = {3, 4, 5}, to A Intersection B = {3}.

► Code

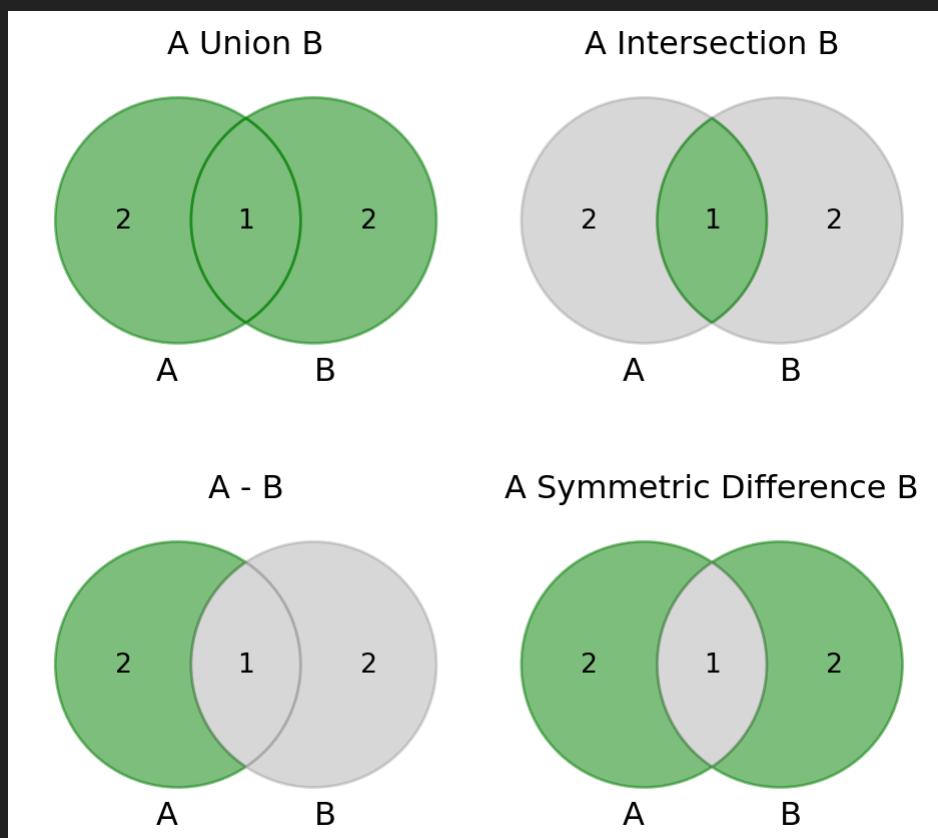


Figure 8.2: Set operations on Venn Diagram

16. Complement (Takmeel):

- o **Definition:** Ek set ka complement woh set hota hai jo universal set ke elements ko include karta hai lekin pehle set ke elements ko nahi include karta.

- Example: Agar Set A = {1, 2} aur universal set = {1, 2, 3, 4}, to A ka complement = {3, 4}.

► Code

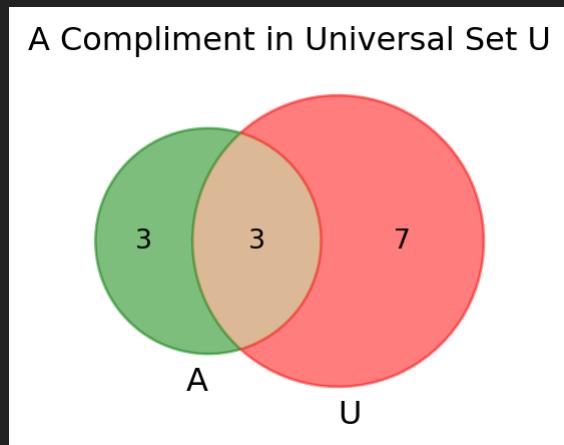


Figure 8.3: Set complement on Venn Diagram

17. Permutations (Tarteebat):

- Definition:** Permutations se murad woh tamam mumkin ways hoti hain jin mein kisi set ke elements ko tarteeb di ja sakti hai.
- Example:** Agar 3 alag numbers ko 2 ki sequence mein arrange karna hai, to $3P2 = 6$ ways hote hain.

18. Combinations (Imtiazajat):

- Definition:** Combinations woh tamam mumkin ways hoti hain jin mein kisi set ke elements ko group kiya ja sakta hai, lekin yahan tarteeb matter nahi karti.
- Formula:** Combinations ka formula $nCr = n!/[r! * (n - r)!]$
- Where n = total number of elements in the set, r = number of elements in the group and ! = factorial.
- Example:** Agar 3 alag numbers mein se kisi 2 ko select karna hai, to $3C2 = 3$ ways hote hain.

8.3 Probability

There are several technical ways to define probability, but a definition useful for statistics is that **probability tells us how often something is likely to occur when an experiment is repeated**.

Probability humein batati hai ke koi cheez kitni baar ho sakti hai jab koi experiment repeat kiya jata hai. Masalan, ye kehna ke sikke ka toss karne par 'heads' aane ka probability kitna hai, isey hum sikke ko bohot baar uchaal kar dekh sakte hain aur note karte hain ke kitni dafa 'heads' aaya hai. Probability ke baare mein shayad sab se important baat yeh hai: Kisi bhi event ka probability hamesha 0 aur 1 ke darmiyan hota hai. Agar kisi event ka probability 0 hai, toh iska matlab hai ke uska hone ka koi chance nahi hai, jabke agar kisi event ka probability 1 hai, toh yeh zaroor hone wala hai. Mathematics mein probability ko decimals mein specify karna conventional hai, isliye hum kehte hain ke kisi event ka probability 0 aur 1 ke beech hota hai, lekin percentages mein baat karna bhi acceptable hai (aur rozmarra ki baat-cheet mein zyada common hai), toh yeh bhi bilkul theek hai ke kisi event ka probability hamesha 0% aur 100% ke beech hota hai. Decimals se percent mein jaane ke liye, 100 se multiply karen (per cent = har 100 mein se), toh 0.4 probability bhi 40% probability hai ($0.4 \times 100 = 40$), aur 0.85 probability ko bhi 85% probability ke taur par keh sakte hain. Negative probability aur 100% se zyada ki probabilities logical impossibilities hain jo sirf baat-cheet ke

taur par exist karti hain. Probability ka 0 aur 1 ke beech bounded hona mathematical implications rakhta hai jo hum is chapter mein baad mein explore karenge.

Tip

This fact also provides a useful check on your calculations. If you come up with a probability lower than 0 or greater than 1, you have certainly made a mistake somewhere along the way. Furthermore, if someone tells you there is a 200% chance that you will make a killing in the stock market if you follow his system, you should probably look for a new investment advisor.

Important Facts about Probability

1. The probability of an event is always between 0 and 1.
2. The probability of the sample space is always 1.
3. The probability of an event and its complement is always 1.
4. The probability of an event that cannot occur is 0.
5. The probability of an event that must occur is 1.

Probability Formula

Probability ka formula ye hai:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

8.3.1 Expressing the Probability of an Event

Probability ko express karne ke liye, hum is ka formula use karte hain. Is formula mein, humein total number of outcomes aur number of favorable outcomes ko calculate karna hota hai. Total number of outcomes, ya sample space, humein experiment ko perform karne se pehle hi pata hota hai. Lekin number of favorable outcomes ko calculate karne ke liye, humein experiment ko perform karna hota hai. Is liye, hum probability ko calculate karne ke liye experiment ko perform karte hain. Is ke baad, hum probability ko express karne ke liye is ka formula use karte hain.

8.3.1.1 Example: Probability of Rolling a Die

Question: Ek fair die ko roll karne par 3 aane ka probability kya hai?

Solution:

1. **Identify the Total Number of Outcomes (Mumkin Natijaat ki Tadaad):**
 - o In this case, the total number of outcomes is 6 (1, 2, 3, 4, 5, 6).
 - o Is ko hum sample space kehte hain.
 - o Is ke baad, humein number of favorable outcomes ko calculate karna hota hai.
2. **Identify the Number of Favorable Outcomes (Sazgaar Natijaat ki Tadaad):**
3. **Use the Probability Formula (Imkanaat ka Formula):**
 - o The formula for probability is:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

- Applying this to our example:

$$P(\text{Rolling a 3}) = \frac{1}{6} \approx 0.167$$

8.3.1.2 Example: Probability of Drawing a Card

Question: Ek standard deck of cards mein se ek card draw karne par 'king' card aane ka probability kya hai?

Solution:

1. **Identify the Total Number of Outcomes (Mumkin Natijaat ki Tadaad):**

- In this case, the total number of outcomes is 52 (4 suits × 13 ranks).
- Is ko hum sample space kehte hain.
- Is ke baad, humein number of favorable outcomes ko calculate karna hota hai.

2. **Identify the Number of Favorable Outcomes (Sazgaar Natijaat ki Tadaad):**

3. **Use the Probability Formula (Imkanaat ka Formula):**

- The formula for probability is:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

- Applying this to our example:

$$P(\text{Drawing a King}) = \frac{4}{52} \approx 0.077$$

8.3.2 Conditional Probability

Conditional Probability, wo probability hoti hai jisme ek event ke hone ki likelihood ko dosre event ke hone ya na hone ke context mein dekha jata hai.

- **Tafseel:** Yeh basically batata hai ke agar humein pata ho ke koi specific condition ya event pehle hi occur ho chuka hai, to is ke baad kisi dusre event ke hone ki kya probability hogi.
- **Formula:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Yahan, $P(A|B)$ ka matlab hai probability of A given B.

- **Misal:** Maan lijiye, ek bag mein 5 red aur 5 blue balls hain. Agar ek ball randomly nikali jati hai aur wo red hoti hai (Event B), to dobara red ball nikalne ki Imkanaat (Event A) kya hogi?

8.3.2.1 Conditional Probability ka Ahmiyat

1. **Complex Situations Mein Samajh:** Conditional probability complex situations ko samajhne mein madad karti hai, jahan ek event dusre se kis tarah linked hota hai.
2. **Decision Making:** Ye concept especially decision-making mein useful hota hai, jaise medical diagnosis ya financial forecasting mein. 
3. **Statistics aur Data Analysis:** Conditional probability statistics aur data analysis ke advanced concepts jaise Bayes' Theorem mein istemal hoti hai.

8.3.2.2 Real-Life Application (Asal Zindagi Mein Istemal)

Conditional probability ka istemal rozmarra ki zindagi ke kayi decisions mein hota hai. Jaise, doctors ye jan'ne ke liye istemal karte hain ke agar koi patient kisi symptom ko show karta hai, to usay koi specific bimari hone ki kitni imkanaat hai. Isi tarah, business analysts market trends aur consumer behavior ko samajhne ke liye conditional probability ka istemal karte hain.

For example, we have a bag with 5 red and 5 blue balls, and we want to find the probability of drawing a red ball again (Event A) given that a red ball has already been drawn (Event B), the calculation is as follows:

After drawing the first red ball, there are now 4 red balls left in a total of 9 balls. So, the conditional probability is calculated as:

$$P(A \text{ given } B) = \frac{4}{9} \approx 0.444$$

This means there's approximately a 44.4% chance of drawing another red ball given that one red ball has already been drawn. Jaisay jaisay zada red baal nikaltay ayan gay probability kam hoti jayegi, kyun ke red balls ki tadaad kam hoti jayegi.

8.3.3 Calculating the probability of multiple events

Multiple events ka probability calculate karne ke liye, hamen pata hna chahyeay k event independent hain ya dependent.

- **Independent Events:** Independent events woh events hoti hain jo ek dusre par asar nahi daalte. Jaise, ek coin ko uchhalne par 'heads' ya 'tails' aane ka probability 0.5 hai. Agar hum ek coin ko 10 dafa uchhalte hain aur 5 dafa 'heads' aur 5 dafa 'tails' aata hai, to ye independent events hain.
- **Dependent Events:** Dependent events woh events hoti hain jo ek dusre par asar daalte hain. Jaise, agar hum ek bag mein jis main 5 red and 5 blue balls hain se ek ball nikalte hain aur wo red hoti hai, to dobara se ball nikalne par red ball aane ka probability kam ho jata hai. Kyun ke ab bag mein red balls ki tadaad kam ho jati hai. Is liye, ye dependent events hain.

8.3.3.1 1. Union of Mutually Exclusive Events

Example: Tossing a Coin - Events: Event A = Getting heads, Event B = Getting tails - **Mutually Exclusive:**

These events are mutually exclusive because you can't get both heads and tails on a single coin toss. -

Union: $P(A \text{ or } B) = P(A) + P(B)$. Since the probability of heads ($P(A)$) and tails ($P(B)$) is each 0.5, the union is $0.5 + 0.5 = 1$. This means either heads or tails is certain to occur.

8.3.3.2 2. Union of Events That Are Not Mutually Exclusive

Example: Drawing a Card from a Deck - Events: Event A = Drawing a heart, Event B = Drawing a queen -

Not Mutually Exclusive: These events are not mutually exclusive as one card (the Queen of Hearts)

satisfies both events. - **Union:** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. The probability of drawing a heart ($P(A)$) is $1/4$, a queen ($P(B)$) is $1/13$, and the Queen of Hearts ($P(A \text{ and } B)$) is $1/52$. So, $P(A \text{ or } B) = 1/4 + 1/13 - 1/52$.

8.3.3.3 3. Intersection of Independent Events

Example: Rolling Two Dice - Events: Event A = First die shows a 6, Event B = Second die shows a 6 -

Independent Events: The result of the first die doesn't affect the second die. - **Intersection:** $P(A \text{ and } B) =$

$P(A) \times P(B)$. The probability of each die showing a 6 is $1/6$, so $P(A \text{ and } B) = 1/6 \times 1/6$.

8.3.3.4 4. Intersection of Nonindependent Events

Example: Drawing Two Cards Successively Without Replacement - Events: Event A = First card is an ace, Event B = Second card is an ace - **Nonindependent Events:** The outcome of the first draw influences the second. - **Intersection:** For $P(A \text{ and } B)$, first calculate $P(A)$, which is $4/52$. Then, given A has occurred, there are now 3 aces left in a deck of 51 cards, so $P(B \text{ given } A) = 3/51$. Thus, $P(A \text{ and } B) = P(A) \times P(B \text{ given } A) = 4/52 \times 3/51$.

8.4 Bayes' Theorem

Bayes' Theorem, ya Bayes Ka Usool, probability theory mein ek powerful formula hai jo conditional probabilities (shartia imkaaniyat) ko calculate karta hai.

Bayes' Theorem was named after Thomas Bayes, an English statistician who lived in the 18th century. It is also known as Bayes' Rule, Bayes' Law, or Bayes' Formula.

- **Tafseel:** Ye theorem humein yeh samajhne mein madad karta hai ke agar humein kisi event ke hone ki prior knowledge ho, to kaise hum us information ko use kar ke kisi dusre related event ki updated probability ko calculate kar sakte hain.
- **Formula:**

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Yahan, $P(A|B)$ ka matlab hai probability of A given B.

- **Components:**
 - $P(A|B)$: Event A ke hone ki probability given ke event B ho chuka hai.
 - $P(B|A)$: Event B ke hone ki probability given ke event A ho chuka hai.
 - $P(A)$ aur $P(B)$: Event A aur B ke hone ki independent probabilities.

8.4.1 Bayes' Theorem Ki Ahmiyat

1. **Decision Making:** Bayes' Theorem ko decision making, especially under uncertainty mein use kiya jata hai. Ye especially medical field, finance, aur machine learning mein bohot useful hai.
2. **Updating Beliefs:** Ye formula humein nayi information milne par apne beliefs ko update karne mein madad karta hai, jaise diagnosis mein new test results ki basis par disease ki likelihood ko update karna. 
3. **Predictive Modelling:** Machine learning aur data science mein, Bayes' Theorem ko predictive models banane aur unhe fine-tune karne ke liye istemal kiya jata hai.

8.4.2 Real-Life Application (Asal Zindagi Mein Istemal)

Example: Medical Diagnosis - Maan lijiye, aap ek doctor hain aur aapko ye maloom hai ke kisi bimari ka overall prevalence ($P(B)$) bohot kam hai, lekin aapke patient mein kuch specific symptoms (A) hain. Bayes' Theorem ki madad se, aap calculate kar sakte hain ke given yeh symptoms, is patient ko woh bimari hone ki kitni probability hai ($P(A|B)$).

Bayes' Theorem, probability ke theory mein aik jadeed aur mufeed tool hai. Ye na sirf humein complex data sets ko samajhne mein madad karta hai, balki uncertain situations mein informed decisions lene mein bhi hamari rehnumai karta hai. Ye approach humein prior knowledge aur new evidence dono ko effectively combine karne ki taqat deta hai.

8.5 Sensitivity (Hassasiyat)

Sensitivity, ya hassasiyat, medical testing mein istemal hone wala ek metric hai jo ye measure karta hai ke kisi test ka disease ko detect karne ki kitni capability hai, especially jab disease present ho.

- Formula:

$$\text{Sensitivity} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$

- **Real-Life Example:** Maan lijiye, aap ek medical researcher hain aur aap ek naye cancer detection test ka study kar rahe hain. Agar aapka test 95% cancerous cases ko sahi taur par identify karta hai, to iska sensitivity 95% hai. Iska matlab hai ke ye test cancer patients ko identify karne mein high degree of accuracy rakhta hai.

8.5.1 Specificity (Tafseelat)

Specificity, ya tafseelat, ek aur important metric hai jo measure karta hai ke kisi test ka disease na hone par kitni accurately negative result dena ka capability hai.

- Formula:

$$\text{Specificity} = \frac{\text{True Negatives}}{\text{True Negatives} + \text{False Positives}}$$

- **Real-Life Example:** Agar wahi cancer test, non-cancerous cases ko 90% cases mein sahi taur par identify kar raha hai ke un mein cancer nahi hai, to iska specificity 90% hai. Iska matlab hai ke is test ka galat taur par cancer ka indication dene ka imkaan kam hai.

8.5.2 Probability of Disease in the Population (Aabadi Mein Bimari ki Imkaaniyat)

Ye metric population level par kisi specific bimari ke hone ki overall probability ko represent karta hai.

- **Importance:** Ye understanding epidemiologists aur public health officials ko help karti hai disease patterns ko samajhne aur health resources ko allocate karne mein.
- **Real-Life Example:** Maan lijiye, aapke shehar mein aik lakh log rehte hain, aur us mein se 1,000 logon ko diabetes hai. To, diabetes ki prevalence ya aabadi mein bimari ki imkaaniyat 1% hai (1,000 divided by 100,000). Ye data health planning aur resource allocation decisions ke liye crucial hai.

Sensitivity, Specificity, aur Disease ki Probability

Sensitivity, specificity, aur population mein disease ki probability ki understanding medical professionals ko accurate diagnosis, treatment planning, aur health policy making mein madad karta hai. Ye metrics na sirf individual patient care mein important hain, balki broader public health initiatives aur disease control strategies ko shape karne mein bhi key role play karte hain.

8.6 Follow us

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