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Section : BCS-3A

Assignment : Discrete No 4:-

Question No #1:-

which of the following are poset?

(A,R) is called a poset when the relation R is reflexive, anti-symmetric and transitive

ⓐ $(z, =)$

i) let $a \in z$, $a = a$ relation holds the reflexive, property.

ii) When $a = b \wedge b = a$, $a, b \in z$ then $a = b$, relation holds the antisymmetric property.

iii) when $a = b \wedge b = c$ with $a, b, c \in z$ Then $a = c$, the relation holds the transitive property.

$\Rightarrow (z, =)$ is a poset.

ⓑ $(z, \neq)$

i) When $a \in z$ Then $a \neq a$, relation is not reflexive.

ii) When $a \neq b \wedge b \neq a$, Then $a \neq b$, relation is not anti-symmetric.

iii) When $a \neq b \wedge b \neq c$, $a, b, c \in \mathbb{Z}$ Then $a \neq c$ which means that relation is not transitive.

So, $(\mathbb{Z}, \neq)$, is not a poset.

ⓒ $(\mathbb{Z}, \geq)$

i) $a \in \mathbb{Z}$ Then $a \geq a$, relation is reflexive.

ii) $a, b \in \mathbb{Z}$, $a \geq b \wedge b \geq a$ Then $a = b$, relation is anti-symmetric.

iii) $a, b, c \in \mathbb{Z}$, $a \geq b \wedge b \geq c$ then $a \geq c$ so relation is transitive.

So, $(\mathbb{Z}, \geq)$ is a poset.

ⓓ $(\mathbb{Z}, \leq)$

i) $a \in \mathbb{Z}$, Then $a \leq a$, so relation is reflexive

ii) $a, b \in \mathbb{Z}$, the $a \leq b \wedge b \leq a$ Then $a = b$
so, property of Anti-symmetric holds.

iii) $a, b, c \in \mathbb{Z}$ Then $a \leq b \wedge b \leq c \longrightarrow a \leq c$,
property holds.

so, $(\mathbb{Z}, \leq)$ is a poset

ⓔ $(\mathbb{Z}, |)$

i) let $a \in \mathbb{Z}$ then $a|a$, so reflexive property holds.

ii) let $a, b \in \mathbb{Z}$, $a|b \wedge b|a \longrightarrow a = b$, anti-symmetric property also holds.

iii) let $a, b, c \in \mathbb{Z}$, $a|b \wedge b|c \longrightarrow a|c$, transitive property holds by default.

So, $(\mathbb{Z}, |)$ is a poset

Question No #2:-

Is (S,R) a poset where S = set of all people in the world and $(a,b) \in R$ where a and b are people if:

ⓐ a is not shorter than b

ⓘ Reflexive property $(a,a) \in R$, $a=a$. A is not shorter than a. property holds.

ⓘ Anti symmetric property: $(a,b) \wedge (b,a) \longrightarrow a=b$ is property holds. $\boxed{T \wedge F \longrightarrow T}$

ⓘⓘⓘ Transitive property: $(a,b) \wedge (b,c) \longrightarrow (a,c)$
$\boxed{T \wedge T \longrightarrow T}$

ⓐ is a poset.

ⓑ a weights more then b.

ⓘ Reflexive property: $(a,a) \in R$ then $a=a$ "a" does not weights more then "a".

Reflexive property does not hold.

ⓑ is not a poset.

ⓒ a is a brother of b

ⓘ Reflexive property does not hold because "a" cannot be a brother of "a".

ⓒ is not a poset.

(d) a and b do not have a common friend.

(i) Reflexive property does not hold because a and a can have a common friend.

(d) is not a poset.

(e) a and b are enemies

(i) Checking Antisymmetric property:

if A ("a and "b" are enemies) $\wedge$ ("b" and "a" are enemies) $\longrightarrow a = b$

$\boxed{T \wedge T \longrightarrow F}$

because "a" and "b" are not the same persons.

So, Anti-symmetric property does not hold.

(e) is not a poset.

Question No # 3:-

Determine whether the relations represented by these matrices are partial order or not...

(a)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) Reflexive property holds because diagonal elements are 1s.

ii) Anti symmetric property does not hold because $a_{11}=1$ and $a_{11}=1$

ⓐ is not a partial order.

ⓑ

$$R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(i) Reflexive property holds because diagonal elements are 1s.

ii) Anti-symmetric property also holds because $M_{ij}=0 \vee M_{ji}=0$ a both.

iii) Transitive property does not hold because

$$R^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ which does not main R.

so, ⓑ is not a partial order.

Question No #4:-

let $S=\{1,2,3,4\}$. with respect to the lexicographic order based on the usual 'less than' relation,

ⓐ Find all the pairs in $S*S$ less than $(2,3)$

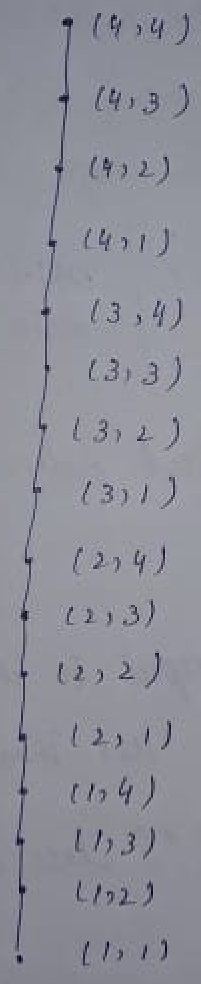
(a,b) is considered less than (c,d) in lexocographic order if $(a<c) \vee (a=c) \wedge (b<d)$

All pairs less than (2,3) if $(a<2) \vee (a=2 \wedge b<3)$

: {(1,1),(1,2),(1,3),(1,4),(2,1),(2,2)} : All pairs in

$S \times S$ = {(1,1),(1,2),(1,3)
(1,4),(2,1),(2,2)
(2,3),(2,4), (~~3,1~~)
(3,1),(3,2)(3,3)
(3,4),(4,1),(4,2)
(4,3),(4,4)}

b) Draw the hasse Diagram of the poset $(S \times S, \preceq)$



Question No #5:-

Draw the Hisse diagram for the subset relation on the power set $p(S)$ where $S = \{a,b,c,d\}$.

$(P(\{a,b,c,d\}), \subseteq)$

$\{\{\phi\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\},$
$\{b,c\}, \{b,d\}$

$\{a,d\}, \{c,d\}, \{a,b,c\},$
$\{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$

Hasse Diagram:-