$h(\vec{x}) = (2x_1, x_1+x_2)^T$ $\quad u_1 = \begin{bmatrix}1\\1\end{bmatrix}, u_2 = \begin{bmatrix}-1\\1\end{bmatrix}$ Linear Algeb. #1

$e_1 = \begin{bmatrix}1\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\end{bmatrix}$

$A = \{h(e_1), h(e_2)\} = \{(2(1), 1+0), (2(0), 0+1)\} = \{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\}$

1a

$A = \begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

1b

$h(u_1) = (2(1), 1+1)^T = \begin{bmatrix}2\\2\end{bmatrix}$ $\quad L(u_2) = (2(-1), -1+1)^T = \begin{bmatrix}-2\\0\end{bmatrix}$

1c $\{u_1, u_2\}$ to $\{e_1, e_2\}$ $\quad$ 1d

$U = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$, $U^{-1} = \dfrac{Adj(U)}{det(U)} = \dfrac{1}{(1+1)}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix} = \dfrac{1}{2}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix} = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}$

$\{e_1, e_2\}$ to $\{u_1, u_2\}$

1e get $h(u_1), h(u_2)$ to $\{u_1, u_2\}$

from ~~[illegible]~~ std basis in $\mathbb{R}^2$.

$U^{-1}h(u_1) = U^{-1}Au_1 = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$

$= \begin{bmatrix}1+1/2 & 0+1/2\\-1+1/2 & 0+1/2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3/2 & 1/2\\-1/2 & 1/2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3/2+1/2\\-1/2+1/2\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

$U^{-1}h(u_2) = U^{-1}Au_2 = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}$

$= \begin{bmatrix}1+1/2 & 0+1/2\\-1+1/2 & 0+1/2\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}3/2 & 1/2\\-1/2 & 1/2\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}-3/2+1/2\\1/2+1/2\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$

1g $\begin{bmatrix}2 & -1\\0 & 1\end{bmatrix}$ So in $\{u_1, u_2\}$,

$h(u_1) = 2u_1 + 0u_2$ 1f

$h(u_2) = -1u_1 + 1u_2$

1h

$B = (U^{-1}Au_1, U^{-1}Au_2) = U^{-1}(Au_1, u_2) = U^{-1}A(u_1, u_2) = U^{-1}AU$