Problem Set 4

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Take a grating with N slits. The let e be the slit width and d be the width of the opaque material between consecutive slits. Monochromatic light with a wavelength λ is shone through the grating. The slits cause each light ray to create secondary wavelets in all directions. The wavelet which doesn't deviate from its indicent path arrives at point P_0, but take a ray which deviates from its indicent path by θ. The secondary wavelets enter a lens and encounter a phase change as well as a refraction, reaching point P_1, creating diffraction and fringe patterns with varying intensity.

Approximate the secondary wavelets at each slit as a single wave with amplitude $A\frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{\pi}{\lambda}e\sin\theta$. Since there are N slits, there are N of these. The path difference between two slits is $e+d$ so the phase difference is $\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta$. We'll call this 2β. Making use of the angle of deviation formula for the resultant ampltude $a\frac{\sin\frac{1}{2}(\alpha+\delta)}{\sin\frac{\alpha}{2}}$ the amplitude at P_1 is

$$\frac{A\sin\alpha}{\alpha}\cdot\frac{\sin N\beta}{\beta}$$

and using Malus's law the intensity is

$$I = \frac{A^2\sin^2\alpha}{\alpha^2}\cdot\frac{\sin^2 N\beta}{\beta^2}.$$

Take the derivative of the intensity function and set it equal to zero to find the maxima:

$$\frac{dI}{d\beta} = 2(A\frac{\sin\alpha}{\alpha})^2\frac{\sin N\beta}{\beta}\frac{N\cos N\beta\sin\beta}{\sin^2\beta} = 0$$

which reveals that

$$N\tan\beta = \tan N\beta.$$

This relationship implies that

$$sin^2 N\beta = \frac{N^2}{1+(N^2-1)sin^2\beta}$$

and so the intensity can be rewritten as

$$I = I_0\frac{A^2\sin^2\alpha}{\alpha^2}\cdot\frac{N^2}{1+(N^2-1)sin^2\beta}.$$

As N becomes larger, the secondary term which governs secondary orders' intensity becomes neglegable, thus for the principle maxima

$$I \approx I_0\frac{N^2\sin^2\alpha}{\alpha^2}.$$