Toward an Understanding of Skewed-Top Corridors

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Abstract and Research Questions

A lattice is the set of all points Z^2. The lattice paths that we study are the set of movements on a lattice with restrictions of up-right and down-right moves. The paths that we are studying reside within an upper and lower boundary. We call this structure a corridor.

In a classic corridor, the top and bottom boundary lines have a slope of zero. In a different model, we allow the upper boundary line to vary with a non-zero slope. We call this model the skewed-top corridor.

How does the data set differ in skewed-top corridors based on the variation of parameters?

What observations are there to be made about the varying data within this skewed-top model?

What mathematical relationships exist within the skewed-top corridors?

Methodology

We wrote programs in Java and Python to generate and analyze the skewed-top corridor dataset.

```python
var('g, s, D')

def start_diag(g, s, D):
    y0 = 1 - 2*D
    L0 = 2*g + D
    if s == 0:
        x = 0
        y = y0
    elif s > 0:
        x = 2*g
        y = 2*g - 2*D - 1
    else:
        print("Error")
        return
    return (x, y)

def end_diag(g, s, D):
    y0 = 1 - 2*D
    L0 = 2*g + D
    if s == 0:
        x = L0 - 1
        y = y0 + L0 - 1
    elif s > 0:
        x = 2*g + D + 3
        y = 2*g - D + 2
    else:
        print("Error")
        return
    return (x, y)

g = 2; s = 1
[ [start_diag(g, s, D), end_diag(g, s, D)] for D in range(5)]

end_diag(1, 0, 0)
```

```java
public static int[][] makeArray(int n, int g, int a, int m) {
    int[][] array = new int[g + ((n + 1) / 2)][n];
    array[array.length - a][0] = 1;
    array[(array.length - 1)][0] = -1;
    int rowneg = (array.length - g) - 2;
    for (int col = 1; col < array[0].length; col++) {
        if (rowneg == -1) {
            break;
        }
        array[rowneg][col] = -1;
        if (col % m == 0) {
            rowneg -= 1;
        }
    }
    for (int col = 1; col <= array[0].length - 1; col++) {
        for (int row = array.length - 1; (array[row][col] != -1); row--) {
            if ((row == 0)) {
                array[row][col] = 0;
                break;
            }
            if (row == array.length - 1) {
                array[row][col] = array[row - 1][col - 1];
            } else if (array[row - 1][col - 1] == -1) {
                array[row][col] = array[row + 1][col - 1];
            } else {
                array[row][col] = array[row - 1][col - 1]
                    + array[row + 1][col - 1];
            }
        }
    }
    return array;
}
```
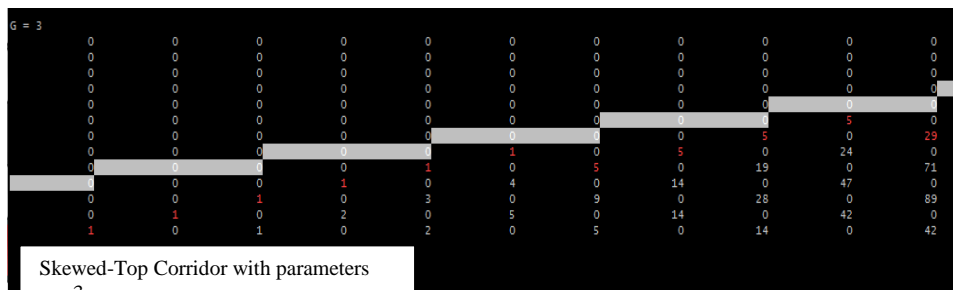
Observations

Fix the following parameters:

    upper boundary line slope = ½
    starting value = 1
    starting position = (0,1)

Allow the gap parameter to vary.

Arithmetic sequences of nth degree lie within the corridors and contribute to each other in various ways.

Skewed-Top Corridor with parameters
g = 3
slope = ½
starting value = 1
starting position = (0,1)

Discussions and Conclusions

We've concluded that …

We hope to carry the results that we've found by fixing three of the four corridor parameters into corridors with several varying parameters.

The integer sequences that we've noticed are compelling and are not listed in the OEIS. We will further investigate the skewed-top corridor sequences and the implications that they encode.