An Exploration of Skewed Top Corridors

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2017-2018

Lattice paths have been studied extensively over the course of several centuries. For the purpose of this study, a lattice consists of points $\mathbb{Z}^2$ with certain restrictions and with only two allowable moves, up-right and down right. Movements in various directions on the lattice are called paths.

We are studying a lattice path model which consists of a point at which all paths begin, upper and lower bounds which mark areas beyond which non-zero paths cease to exist, and an ending point which marks the end of non-zero path values. The area of the lattice contained by the upper and lower bound is referred to as a corridor. The number of paths within the corridor depend on the initial value which is placed at the starting position of the corridor, the nature of the upper and lower bounds, and the position of the starting value in the corridor. The upper and lower boundary lines mark the areas where non-zero path values cease to exist. Paths can exist on the two boundary lines, but they cannot exist beyond the lines. The lower boundary is essentially a horizontal line with zero slope. The upper boundary line is a line with variable slope. These conditions seem to present a problem when attempting to systematically generate values contained in the corridors. Due to the nature of the upper bound, which is a line with slope not parallel to that of the lower bound, the paths bounce off of the upper diagonal line, rippling into and distorting the data below it. One might propose that calculating the error caused by each interruption would be somewhat intuitive, but the impacts of each diagonal-boundary disturbance grow larger as time in the corridor progresses. Although the data changes because of the upper bound's slope, intriguing patterns and characteristics have been observed in the configurations of this environment. The corridor model which has varying boundary lines is referred to as a skewed top corridor.

We interpret the corridor, for now, on the two dimensional Cartesian coordinate plane which is the span of unit vectors $\hat{\imath} = \vec{(}1,0)$ and $\hat{\jmath} = \vec{(}0,1)$. The elapse of time is represented along the x-axis. The amount of paths is represented along the y-axis which is perpendicular to the x-axis. At a glance, the structure of the corridor consists of an initial value placed at a starting position in the corridor. A gap of predetermined size exists in the same column as the starting position. The size of the gap is equal to the number of lattice points extending vertically from the starting position and includes the starting path value. The gap, for now, extends vertically in the positive direction of the y-axis, for which our convention is upward. Values in this gap are zeros except for the starting value which is also contained in the gap. Our exploration consists of manipulating the initial conditions of the corridors, observing and quantifying the results, and assembling various formulas to express relationships that we notice. We want to learn as much as we can about these corridor structures and what the relationships within them might encode.

Symbols and Definitions

We start our journey of exploring skewed top corridors by establishing our definitions. We also need a symbolic method for referring to specific aspects of the corridors. We will use these specific symbols and definitions not only to discuss corridors but to also make mathematical conclusions about the structures.

First off, we will be using the set of numbers which are integers larger than one.

Definition 1. Let the set of natural numbers be represented by $\mathbb{N}$.

We will also discuss integers which are positive and greater than zero.

Definition 2. Let $\mathbb{N}_0$ represent the set of natural numbers including zero.

At times, we will need to utilize the set of integer values which are positive, negative, and include zero.

Definition 3. Let $\mathbb{Z}$ represent the set of integers.

We can now begin defining some of the parameters for establishing the skewed-top corridor. Let us begin by defining the corridor gap. This will be an important feature later in our journey.

Definition 4. Let $g \in \mathbb{N}$ equal the gap size in the corridor.

This value g is the number of points in the column of the initial starting path value in the corridor. It simply counts the number of points in the starting column including the starting value. The gap is one of the parameters which is required to establish a skewed top corridor. We will allow the gap to vary over time and will later discuss this in more detail.

Another important parameter for the skewed top corridor is the value that is placed at the beginning of the structure. This value exsits at a position in the gap. We will define this position shortly.

Definition 5. Let $v \in \mathbb{N}$ represent the value which is placed at the beginning of the gap.