Toward an Understanding of Skewed Top Corridors

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Abstract and Research Questions

A **lattice** is the set of all points Z^2. The lattice paths that we study are the set of movements on a lattice with restrictions of up-right and down-right moves. The paths that we are studying reside within an upper and lower boundary. We call this structure a **corridor**.

In a classic corridor, the top and bottom boundary lines have a slope of zero. In a different model, we allow the upper boundary line to vary with a non-zero slope. We call this model the **skewed-top corridor**.

How does the data set differ in skewed-top corridors based on the variation of parameters?

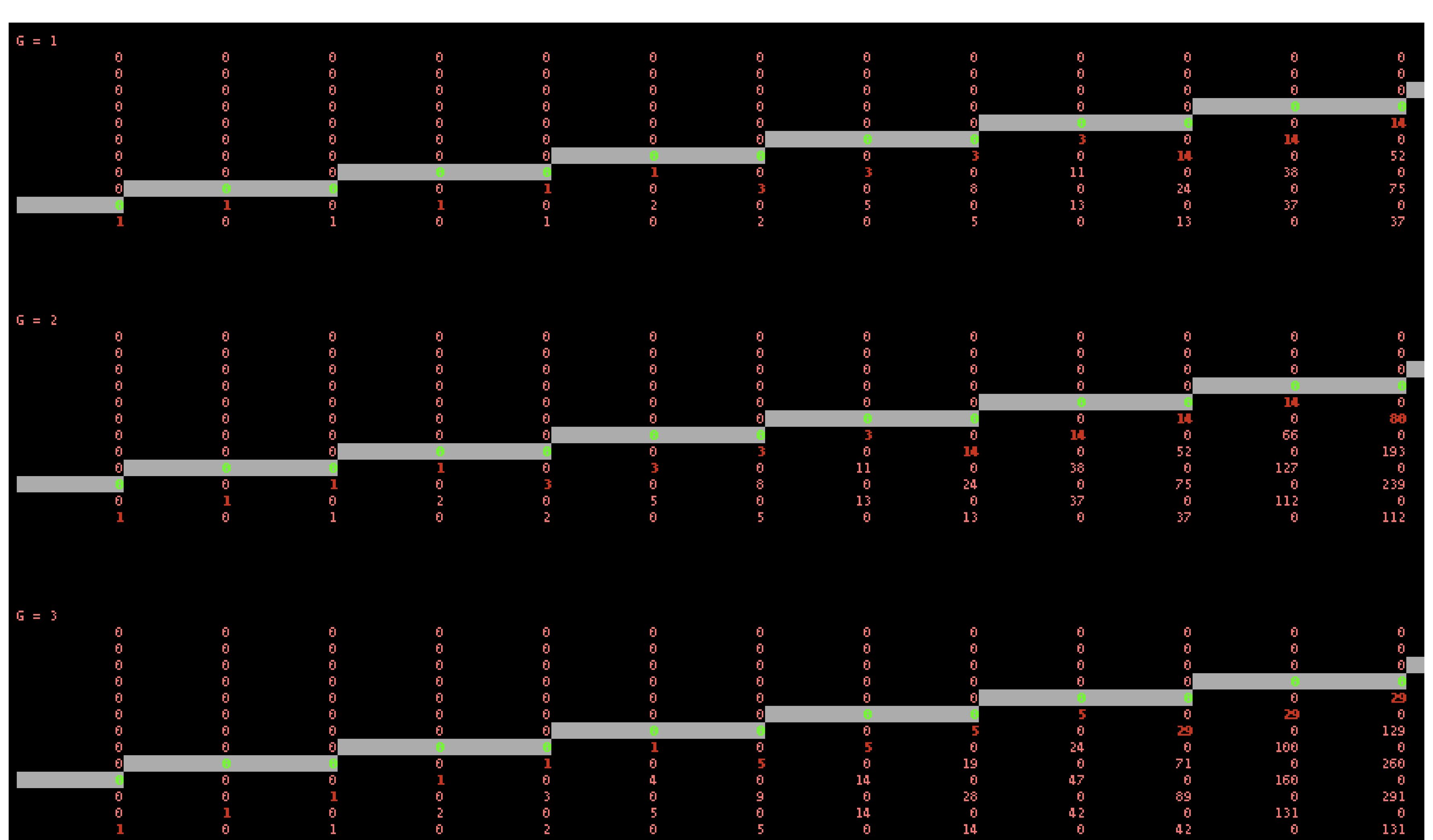
What observations are there to be made about the varying data within this skewed-top model?

What mathematical relationships exist within skewed-top corridors?

Observations

Arithmetic sequences of n^{th} degree lie within the corridors and contribute to each other in various ways.

When generating the corridors, we fix 3 out of 4 of our initial conditions. With these conditions set, we call this a **classic-case** skewed top corridor. Below are some of the corridor structures generated with various fixed parameters.



Classic-Case Skewed Top Corridors 1 - 3

Corridor Recursive Formula

Let V be a function accepting input from the set of natural numbers. It's output lies within the set of points Z^2.

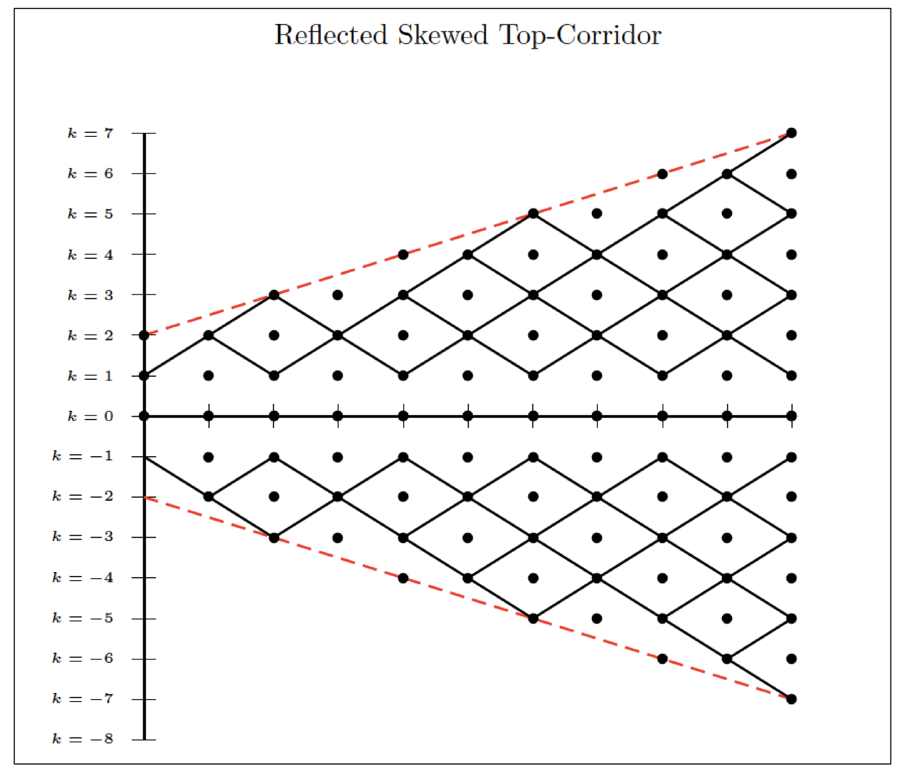
Let x and y be values that lie within the corridor.

A recursive formula for generating values in the corridor is:

$$V(x, y) = V(x-1, y-1) + V(x-1, y+1)$$

A **diagonal** in the corridor begins at some value on the lattice and moves up-right at some slope through the corridor. This formula states that the first non-zero diagonal at position (k+1, k) is k.

$$2 \leq k \leq 2g, \quad V(k+1, k) = k$$



Reflected Skewed Top-Corridor

Corridor Diagonal Sequences

Follow the diagonal sequences of numbers in a one of the corridors. One will notice various intriguing patterns that appear on the diagonal. Some patterns are obvious to spot and others are slightly more difficult to notice.

The first diagonal in the corridor can be expressed as start and end points:

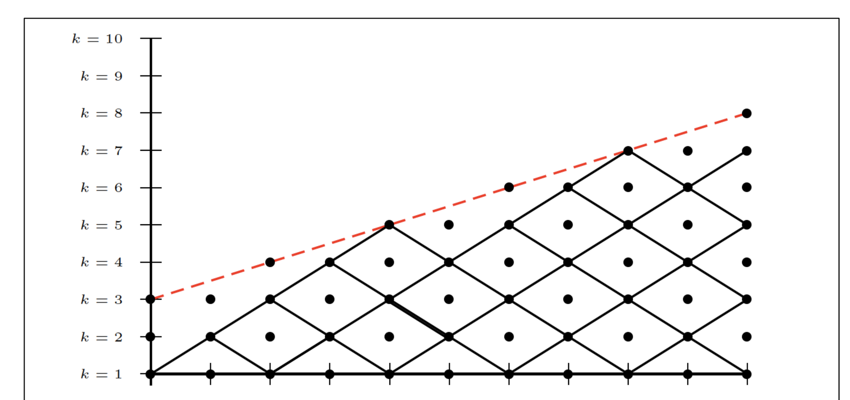
***Start:* $(0, 1)$ *End:* $(2g, 2g\text{-}1)$**

Other diagonals from the initial can be computed in the following way, where ***D*** is the diagonal number in the corridor:

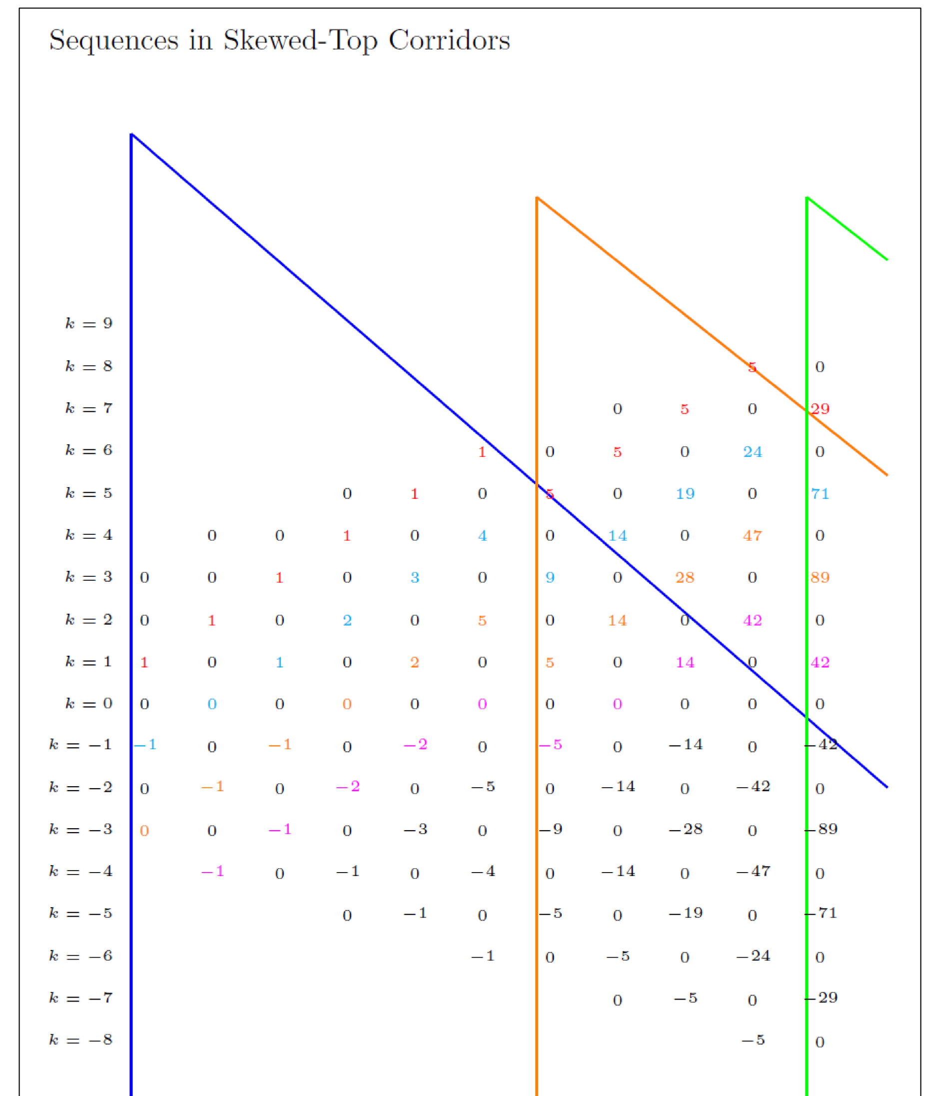
***Start:*$(2g\text{-}D, 2g\text{+}D)$ *End:*$(2g\text{-}D\text{+}4, 2g\text{+}D\text{+}4)$**

Skewed-Top Corridor State Information

							0	5
					1	0	5	0
			0	1	0	5	0	19
	0	0	1	0	4	0	14	0
0	0	1	0	3	0	9	0	28
0	1	0	2	0	5	0	14	0
1	0	1	0	2	0	5	0	14



The Classical Skewed Top Corridor



Sequences in Skewed-Top Corridors

Methodology

We wrote programs in Java and Python to generate and analyze the skewed-top corridor dataset. The following is pseudocode describing the program used to calculate the corridor numbers.

```
procedure create_grid
   create work space with row+1 * col+1 entries
   make all values zero
   place start value

####create sentinel line####
   for every column in corridor
       for every row in corridor
            if col modulo slope_run is 0
                    draw line
                    increase vertical position
                    increase horizontal position
            end if
            else
                   draw line
                   increase horizontal position
            end else

####generate values####
  for every column in corridor
        for every row in corridor
            corridor[row][col] = corridor[row-1][col-1] + corridor[row-1][col+1]
        end for
   end for
```

Discussions and Conclusions

We've concluded that there are sequences of degree n within the corridor and contribute to each other to create new sequences of degree n+1, n+2, ... When sequences are added together, their sums create new sequences of a higher degree. Similarly, sequences with a higher degree have a common difference with the previous sequence contribution and degrade into linear sequences, that is, arithmetic sequences of degree one.

This behavior has been noticed in the corridors' diagonals. If one chooses a diagonal with starting point at (0, 2k + 1), the sequence will degrade into a linear sequence by the time the diagonal reaches the upper boundary line.

In addition, comparing the linear sequences that appear near the surface of the sentinel lines of corridors as the gap size increases reveal another relationship. With a fixed slope of one-half, 4-element linear sequences appear near the sentinel lines. As the gap size increases in these fixed corridors, the values of these 4-element sequences are part of degree n sequences as well.

We hope to carry the results that we've found by fixing three of the four corridor parameters into corridors with several varying parameters.

The integer sequences that we've noticed are compelling and are not listed in the OEIS. We will further investigate the skewed-top corridor sequences and the information that they might encode.

Sources

1. Shaun Ault and Charles Kicey *Counting Lattice Paths Using Fourier Methods*

2. Kenneth H. Rosen *Discrete Mathematics and Its Applications*

3. Richard A. Brualdi *Introductory Combinatorics*

4. *The On-Line Encyclopedia of Integer Sequences*