Toward an Understanding of Skewed Top Corridors

Joanne Wardell

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Abstract

Lattice paths have been studied extensively over the course of several centuries. For the purpose of this study, a lattice consists of points $\mathbb{Z}^2$ with certain restrictions and with only two allowable moves, up-right and down-right. Movements in various directions on the lattice are called paths.

The lattice path enumeration model that we propose consists of a starting point, an upper and lower bound, and all possible paths from said starting point to some end point. The area in which the paths are propagated is referred to as a corridor. The number of paths within the corridor depend on the initial values of the starting point, the nature of the upper and lower bounds, and the value placed at the starting point. In our model, the lower bound is a line with zero slope and the upper bound is a line with a variable slope. These conditions seem to present a problem when attempting to systematically generate the values contained in the corridors. Due to the nature of the upper bound, which is a line with slope not parallel to that of the lower bound, the paths bounce off of the upper diagonal line, rippling into and distorting the data below it. One would think that calculating the error caused by each interruption would be somewhat intuitive, but the impacts of each diagonal-boundary disturbance grow larger as time in the corridor progresses. Although the data changes because of the upper bound's slope, intriguing patterns and characteristics have been observed in the configurations of this environment. We call the model which has been briefly introduced a skewed top corridor.

The corridor exists on a two dimensional plane which involves the elapse of time on the horizontal axis and the amount of paths on the vertical axis. A gap extends vertically from the starting point and contains the initial value which is placed at the starting point. The gap contains all zeros except for the initial value which resides at the bottom of the gap. A diagonal line, which the paths can touch but not surpass, begins just above the gap and extends to a finite point on the grid. Paths can move in an upward or downward direction in the corridor.

Our exploration consists of manipulating the initial conditions of the corridors, observing and quantifying the results, assembling formulae to express and generate values within the corridor, and proving that said formulae are true.