CS 325 Homework 5 Solutions

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1. Converting to Standard SOP Form

(a) $F(A,B,C) = \bar{A}B + C$

$$\begin{aligned}
\bar{A}B &= \bar{A}B(C+\bar{C}) = \bar{A}BC + \bar{A}B\bar{C} \\
C &= C(A+\bar{A})(B+\bar{B}) = C(AB + A\bar{B} + \bar{A}B + \bar{A}\bar{B}) \\
&= ABC + A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C \\
F(A,B,C) &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC \\
&= \sum m(1,2,3,5,7)
\end{aligned}$$

(b) $F(A,B,C) = AB + BC + B$

$$\begin{aligned}
F(A,B,C) &= B + AB + BC = B \quad \text{(by absorption)} \\
B &= B(A+\bar{A})(C+\bar{C}) = (AB+\bar{A}B)(C+\bar{C}) \\
&= ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} \\
&= \sum m(2,3,6,7)
\end{aligned}$$

(c) $F(A,B,C) = \bar{A}B + AC + \bar{A}C$

$$\begin{aligned}
F(A,B,C) &= \bar{A}B + AC + \bar{A}C = \bar{A}B + C(A+\bar{A}) = \bar{A}B + C \\
&= \sum m(1,2,3,5,7) \quad \text{(same as part a)}
\end{aligned}$$

2. Deriving SOP, POS, and Logic Diagram

Truth table:

A	B	C	F	Minterm	Maxterm
0	0	0	1	m_0	
0	0	1	0		M_1
0	1	0	1	m_2	
0	1	1	0		M_3
1	0	0	0		M_4
1	0	1	1	m_5	
1	1	0	0		M_6
1	1	1	1	m_7	

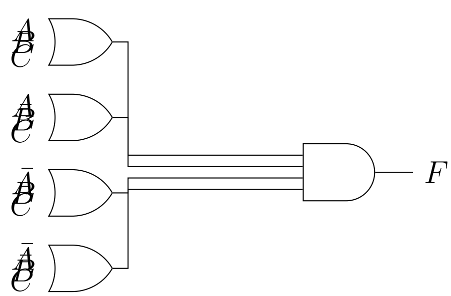
(a) Standard SOP Expression

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + ABC = \sum m(0, 2, 5, 7)$$

(b) Standard POS Expression

$$F = (A + B + \bar{C}) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C) \cdot (\bar{A} + \bar{B} + C) = \prod M(1, 3, 4, 6)$$

(c) Logic Gate Diagram for POS



3. Karnaugh Map Creation and Derivation

K-map from Truth Table

	$C = 0$	$C = 1$
$AB = 00$	1	0
$AB = 01$	1	0
$AB = 11$	0	1
$AB = 10$	0	1

Simplified SOP from K-map

Groups of 1's: m_0, m_2 group: $\bar{A}\bar{C}$ and m_5, m_7 group: AC

$$F = \bar{A}\bar{C} + AC$$

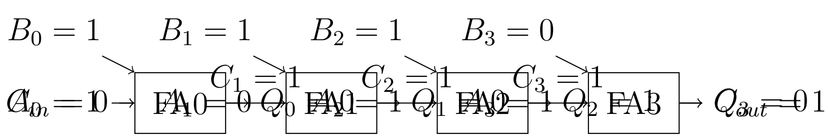
Simplified POS from K-map

Groups of 0's: M_1, M_3: $\bar{A}C$ and M_4, M_6: $A\bar{C}$

$$F = (\bar{A} + C)(A + \bar{C})$$

4. 4-Bit Ripple Carry Adder Analysis

$A = 1101$, $B = 0111$, $C_{in} = 0$



Calculation:

- LSB: $A_0 = 1, B_0 = 1, C_{in} = 0 \rightarrow Sum = 0, C_{out} = 1$
- Bit1: $A_1 = 0, B_1 = 1, C_{in} = 1 \rightarrow Sum = 0, C_{out} = 1$
- Bit2: $A_2 = 1, B_2 = 1, C_{in} = 1 \rightarrow Sum = 1, C_{out} = 1$
- Bit3: $A_3 = 1, B_3 = 0, C_{in} = 1 \rightarrow Sum = 0, C_{out} = 1$

Final: $Q = 0100$, $C_{out} = 1$

5. Binary Subtractor Circuit

To modify a 4-bit binary adder to perform subtraction $A - B$ using two's complement:

1. Invert each bit of B (find $\bar{B}$)
2. Set $C_{in} = 1$ for the LSB adder

This computes: $A + (\bar{B} + 1) = A + (\text{two's complement of } B) = A - B$