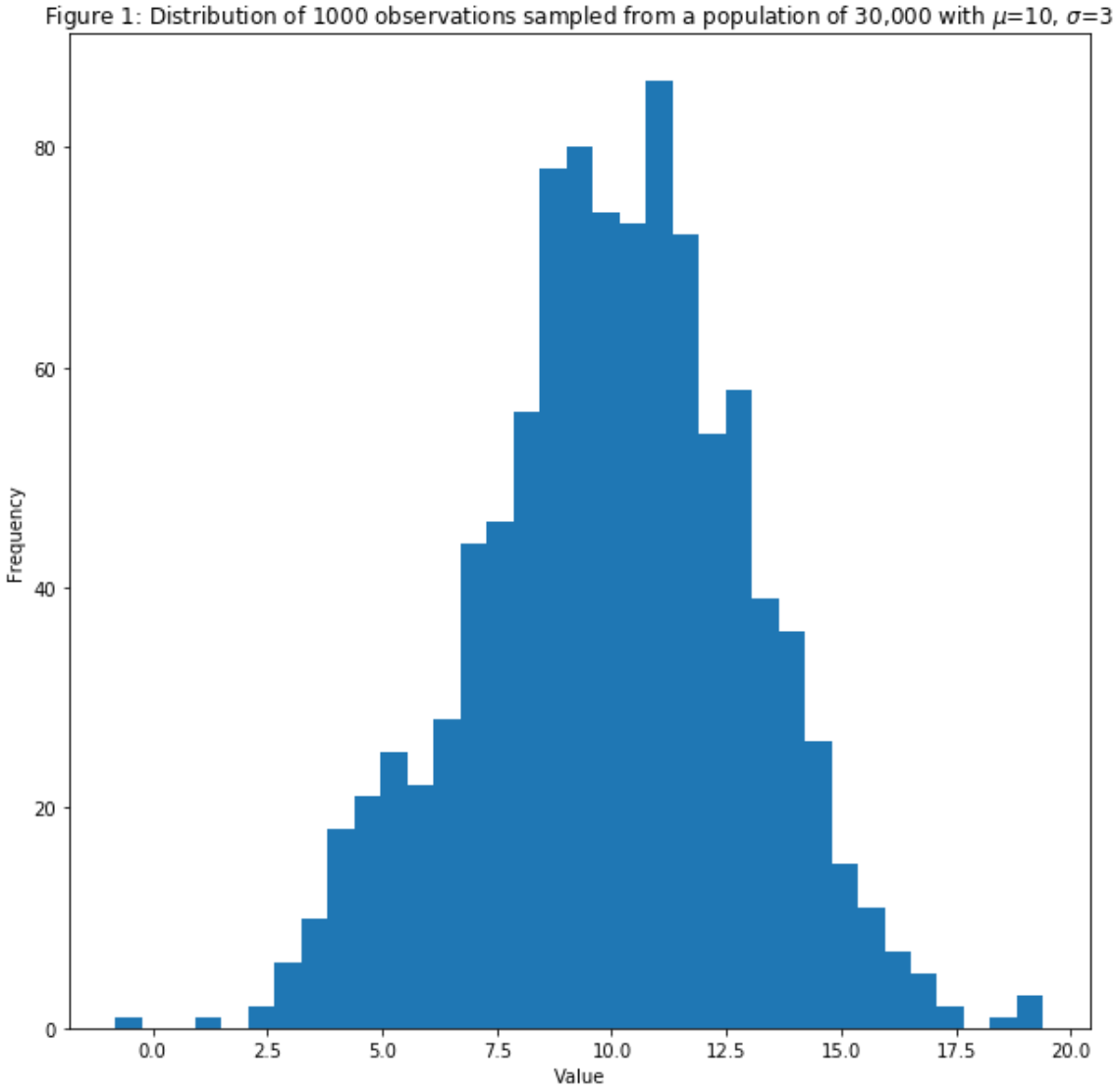
From Scratch applying Bayesian Inference, Markov Chain Monte Carlo and Metropolis Hastings and MALA in python

A common technique called Markov Chain Monte Carlo commonly referred to as MCMC. The method stated is a class of techniques for sampling from a probability distribution and can be used to estimate the distribution of parameters given a set of observations.

MCMC Metropolis Hastings - Nomal distribution (function definitions and samples)

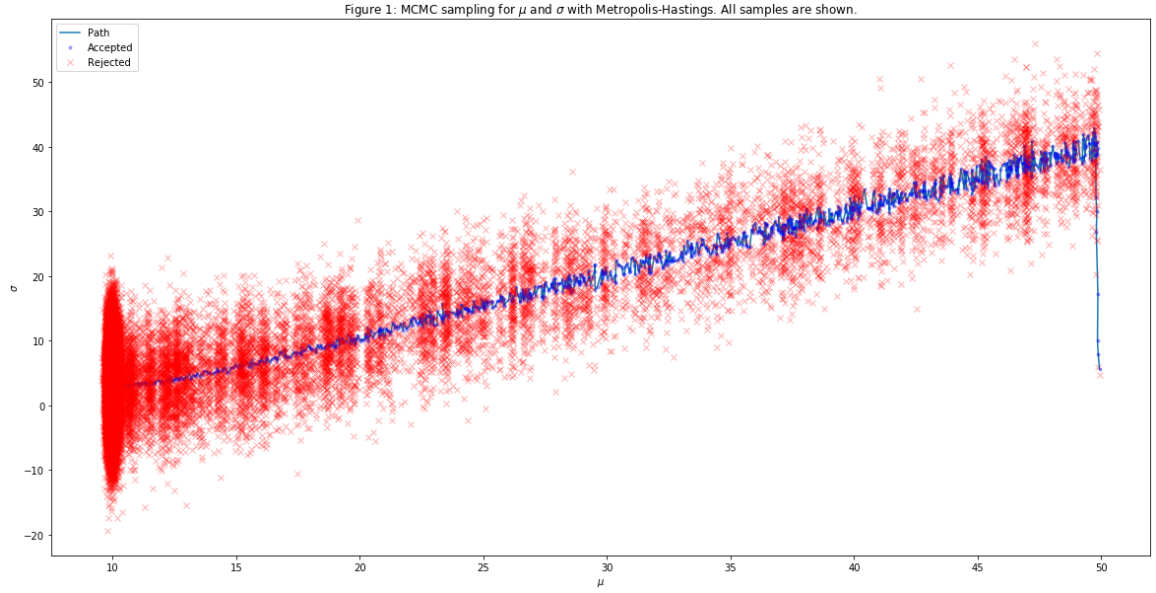
Step 1. Sample data generation
Step 2. Formula for finding distirubtion of parameter
Step 3. Define PDF and Propsoal Distribution (transition model)
Step 4. Acceptance or rejection of new parameter

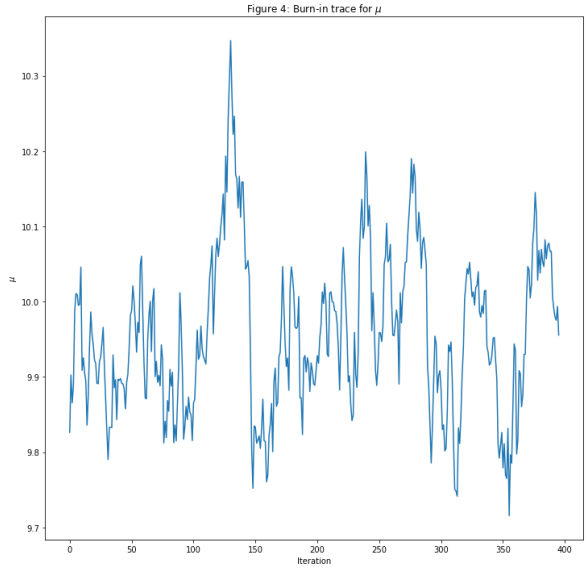


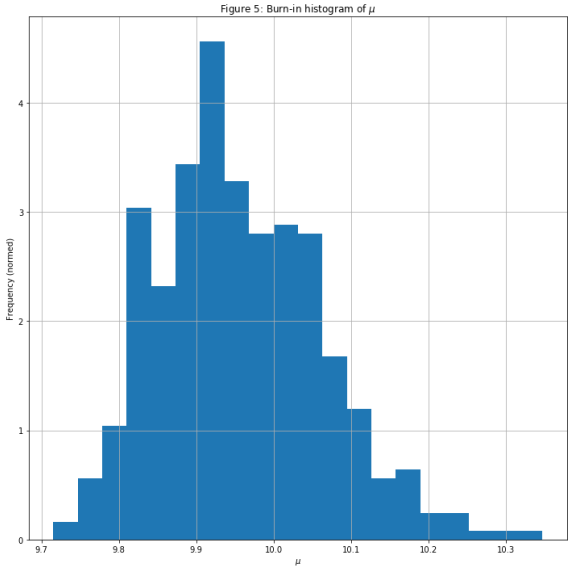
```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:21: Runti
meWarning: invalid value encountered in log
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:55: Runti
meWarning: divide by zero encountered in log
```

Out[135]:

```
Text(0.5, 1.0, '2D histogram showing the joint distribution of $\\mu$ and
$\\sigma$')
```







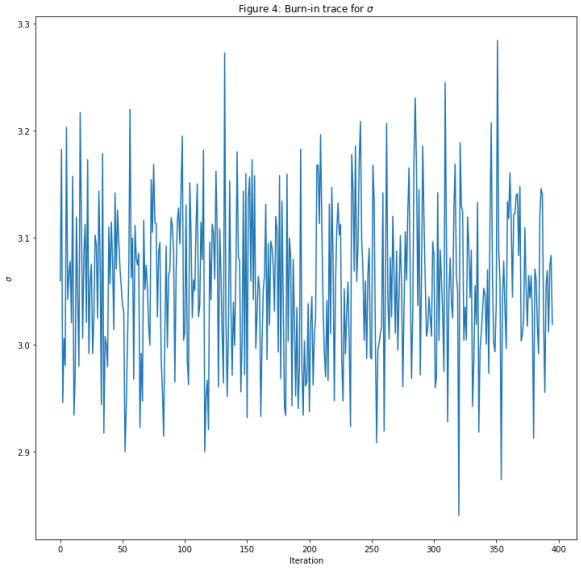


Figure 4: Burn-in trace for σ
σ
Iteration

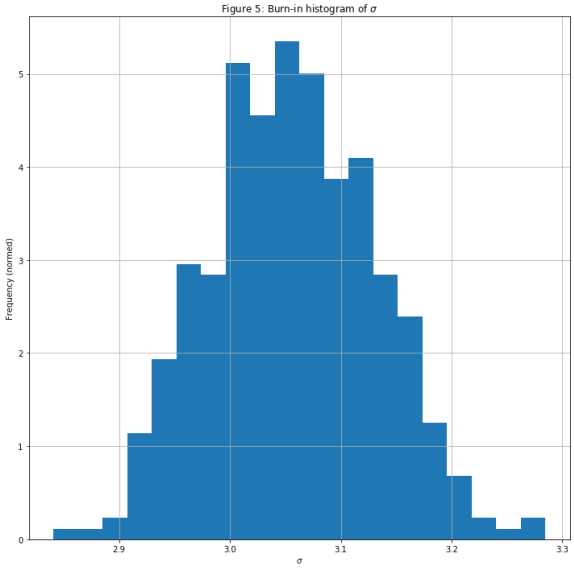
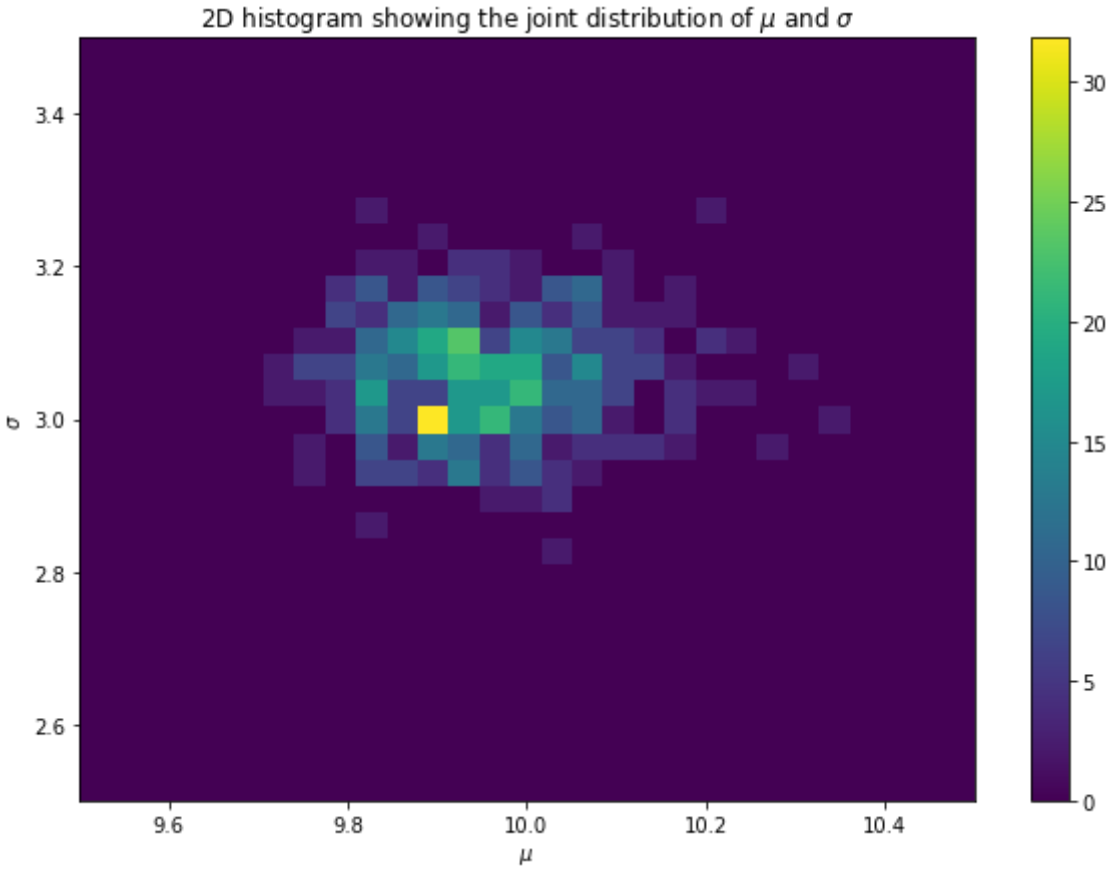


Figure 5: Burn-in histogram of σ
Frequency (normed)
σ


2D histogram showing the joint distribution of μ and σ
σ
μ

```
9.956129674255305 3.0568370400673177
```

Out[144]:

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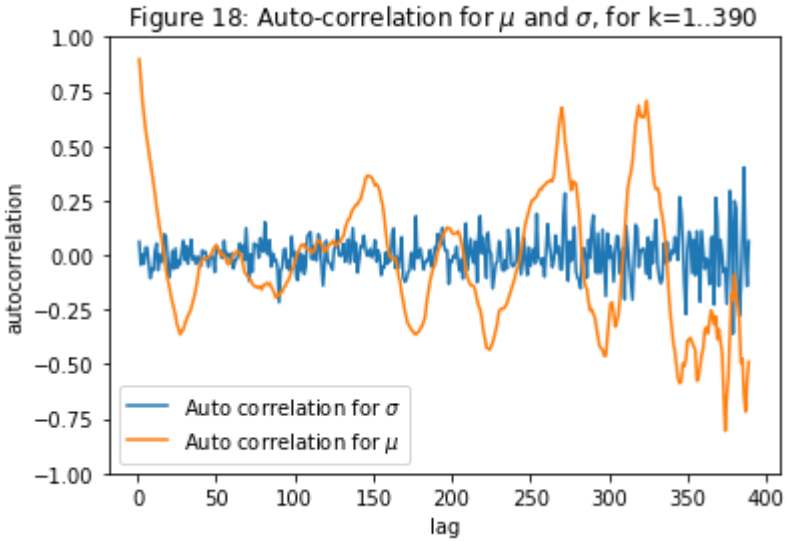
[(-1, 1), Text(0, 0.5, 'autocorrelation'), Text(0.5, 0, 'lag')]
```



MCMC Metropolis-adjusted Langevin algorithm (MALA) - Nomal distribution (function definitions and samples)

Step 1. Using previous Metropolis algorithm

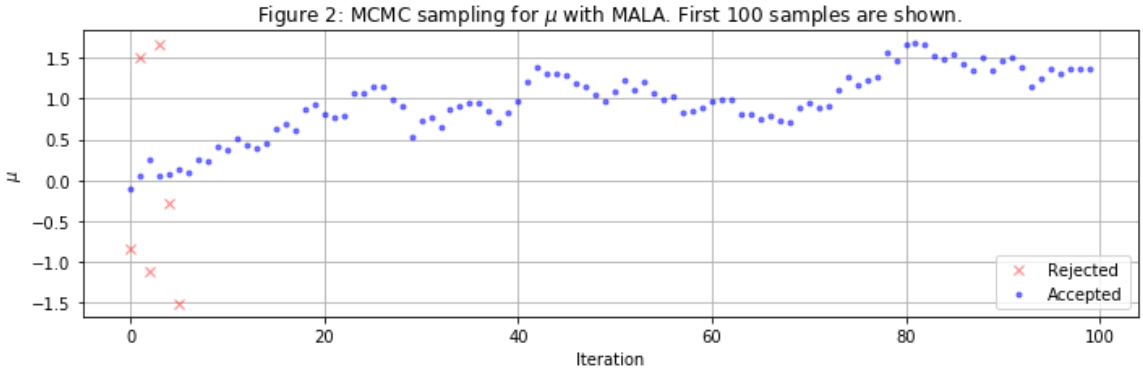
Step 2. Update new states to include Langevin dynamics, using gradient evaluations of the gradient of the target probability density function.

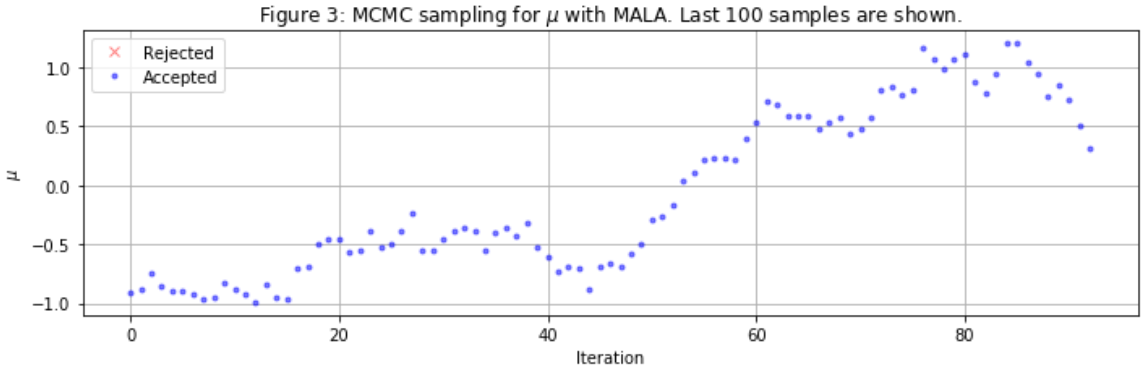
$$Y \sim \mathcal{N}(X_n + h\frac{\nabla\pi(X_n)}{\pi(X_n)}, 2hI)$$

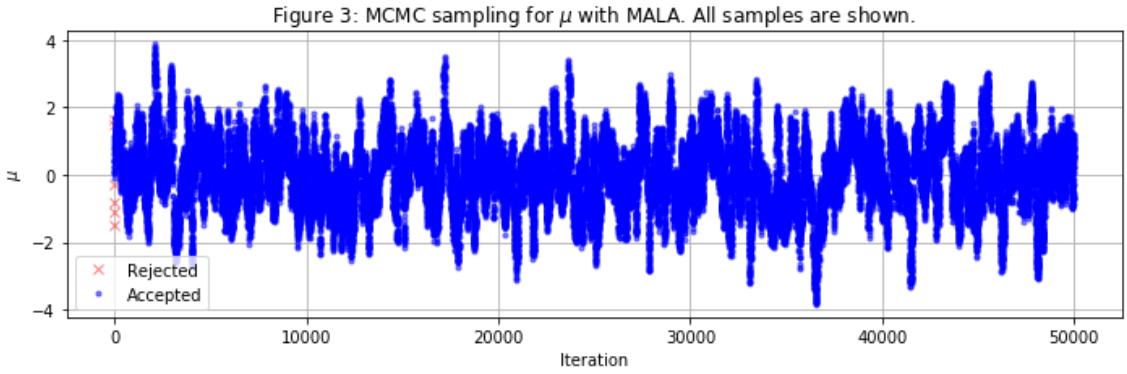
$$\alpha(X_n, Y) = \frac{\pi(Y)}{\pi(X_n)}\exp\left\{\frac{1}{4h}\left(\left|Y - X_n - h\frac{\nabla\pi(X_n)}{\pi(X_n)}\right|^2 - \left|X_n - Y - h\frac{\nabla\pi(Y)}{\pi(Y)}\right|^2\right)\right\}$$

Out[167]:

```
(49993, 1)
```







Out[151]:

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[<matplotlib.lines.Line2D at 0x2421669cb88>]
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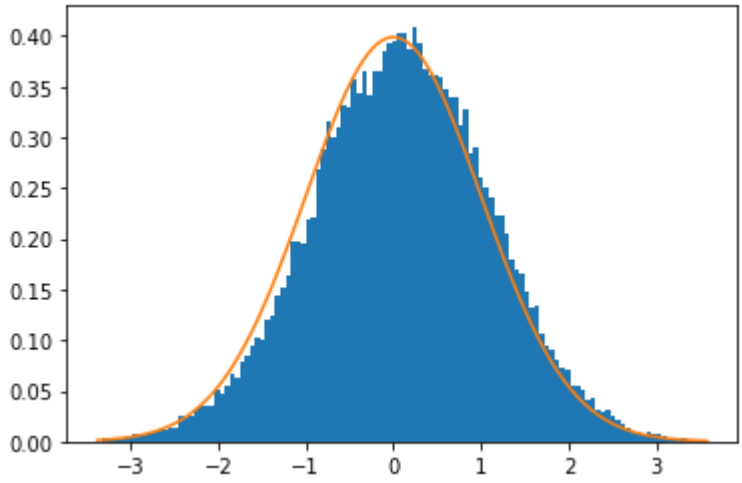

References:
https://en.wikipedia.org/wiki/Markov_chain_Monte_Carlo
(https://en.wikipedia.org/wiki/Markov_chain_Monte_Carlo)
https://en.wikipedia.org/wiki/Metropolis%E2%80%93Hastings_algorithm
(https://en.wikipedia.org/wiki/Metropolis%E2%80%93Hastings_algorithm)
https://en.wikipedia.org/wiki/Metropolis-adjusted_Langevin_algorithm
(https://en.wikipedia.org/wiki/Metropolis-adjusted_Langevin_algorithm)
Geof, H 2013, Computational Statistics, Second Edition C:\Users\User\Desktop\git_repos\MCMC-Python

```
  File "<ipython-input-168-4ece4ee89ef7>", line 1
    jupyter nbconvert C:/Users/User/Desktop/git_repos/MCMC_ver1.ipynb --to
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                    ^
SyntaxError: invalid syntax
```