Problems Wrong

2, 13, 14, 15

Straight up wrong

correct answer, circled wrong one

Straight up wrong

Dr Killpack Look @

13, 14

I had the right answers, but I circled the wrong ones

EC En 483 / ME En 431
Introduction to Feedback Control

Winter 2024, Midterm 1

February 21-26, 2024
Professor: Marc Killpack
Extension 2-6342

Time Limit: 3 hours 45 minutes.
After the allotted time, Learning Suite will automatically save your answers and end your test.

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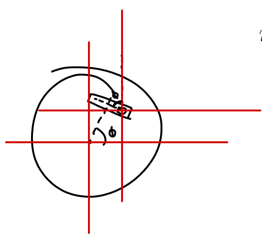
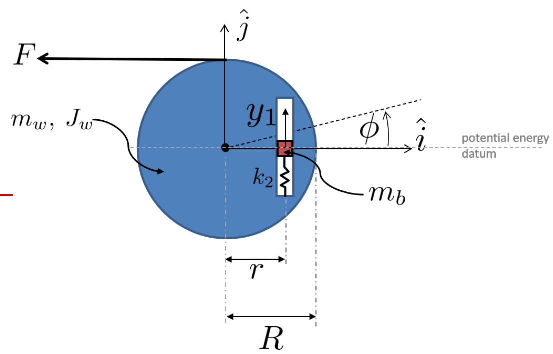
Instructions:

- Work all problems and make sure to show all work, even though the test is multiple choice. The point of having multiple choice problems is not to allow the process of elimination, but to help in grading the exam. Therefore, **please remember that "none of the above" is a valid option**, and that you may get some partial credit after the exam is graded if you show your work.
- Closed book, closed notes, closed homework and lab solutions, except...
  - You are allowed one 8.5×11 sheet of notes (front and back).
  - Open calculator, but accessing the Internet is not allowed.
  - You can use Python (with Numpy or SymPy) libraries, but NOT the control library (and NO past code).
  - You may use MATLAB, but NONE of the built-in functions beyond "syms" or "roots"

1. (5 pts) Consider the physical system shown in Figure 1 which consists of a large drum (with mass m_w and rotational inertia J_w) which can rotate about its center of mass, but cannot translate. A force is applied at the top of the drum and always acts horizontally. Inside the drum is a small mass block (m_b) that can slide in a track with fluid inside. The damping from the fluid can be modeled using viscous friction with coefficient b. The drum sits on a low friction axle or bearing, so we can neglect the friction due to rotating the drum. The displacement of the small mass inside the track can be described by the variable y_1, while the rotation of the drum can be described using ϕ. At the initial configuration ($\phi = 0$, and $y_1 = 0$), the smaller mass lines up with the x-axis which is also the potential energy datum where $P_0 = 0$, and the spring is not loaded at all (in compression or tension).





PE → gravity or spring

spring

$= \frac{1}{2} k_2 y_1^2$

gravity

m_w → alway @ datum = 0

m_b → $r m_b g \sin(\phi)$

Figure 1: Rolling Barrel. To be used for problem 1–2.

What is the potential energy of the system?

~~**(a)** $P = m_w g y_1 + m_b g y_1 + \frac{1}{2} k_2 y_1^2$~~

(b) $P = m_b g(r sin(\phi) + y_1) + \frac{1}{2} k_2 y_1^2$

(c) $P = m_b g(r sin(\phi) + y_1 cos(\phi)) + \frac{1}{2} k_2 y_1^2$

~~**(d)** $P = \frac{1}{2} k_2 y_1^2 + m_w g R$~~

(e) None of the above.

2. (5 pts) For the system in Figure 1, if $q = [\phi, y_1]^T$, what are the generalized forces?

~~**(a)** $\boldsymbol{\tau} = \begin{bmatrix} F \\ 0 \end{bmatrix}$~~

~~**(b)** $\boldsymbol{\tau} = \begin{bmatrix} 0 \\ FR\sin(\phi) \end{bmatrix}$~~

(c) $\boldsymbol{\tau} = \begin{bmatrix} 0 \\ F \end{bmatrix}$

~~**(d)** $\boldsymbol{\tau} = \begin{bmatrix} FR \\ 0 \end{bmatrix}$~~

(e) None of the above.

$\tau = \begin{bmatrix} \text{Torque} \\ \text{Force} \end{bmatrix} = \begin{bmatrix} FR\sin(\phi) \\ 0 \end{bmatrix}$

$\cancel{= \begin{bmatrix} 0 \\ F \end{bmatrix}}$

3. (5 pts) Consider the physical system shown in Figure 2. A ball of mass m_2 (that can be treated as a point mass) is rolling inside a channel in a disk with mass m_1 and rotational inertia J_1. The disk is pinned at its center of mass (CM), but can be accelerated angularly with a torque τ. There is compressed air on either side of mass m_2 that acts like linear rotary springs with stiffness k. The variable θ describes the angular position of the disk relative to the base or inertial frame. While ϕ describes the position of the motion of the ball relative to the disk. Both values are at zero in the current configuration shown.

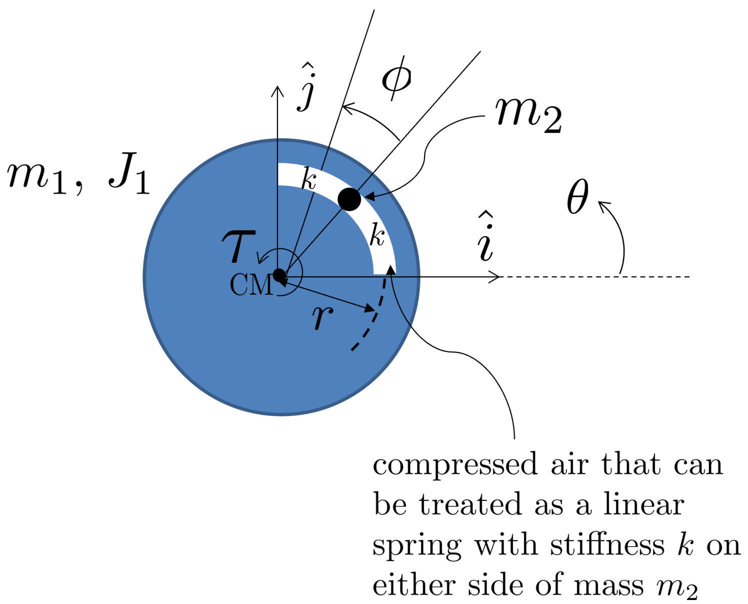


Figure 2: Rotating disk with point mass (m_2) inside inner track. To be used for problem 3.

What is the kinetic energy of the system? ✓

$P_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$P_2 = \begin{bmatrix} r\cos(\phi+\theta) \\ r\sin(\phi+\theta) \\ 0 \end{bmatrix} \quad V_1 = \begin{bmatrix} -r\sin(\phi+\theta)(\dot\phi+\dot\theta) \\ r\cos(\phi+\theta)(\dot\phi+\dot\theta) \\ 0 \end{bmatrix}$

$V_1 = 0$

(a) $K = \frac{1}{2}J_1\dot{\theta}^2 + \cancel{\frac{1}{2}m_1 r^2\dot{\theta}^2} + \frac{1}{2}m_2 r^2(\dot{\theta}+\dot{\phi})^2$

(b) $K = \frac{1}{2}J_1\dot{\theta}^2 + \cancel{\frac{1}{2}m_1 r^2\dot{\theta}^2} + \frac{1}{2}m_2 r^2(\dot{\theta}-\dot{\phi})^2$

(c) $K = \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}m_2 r^2(\dot{\theta}+\dot{\phi})^2$

$\frac{1}{2}m_2 V_2^T V_2$

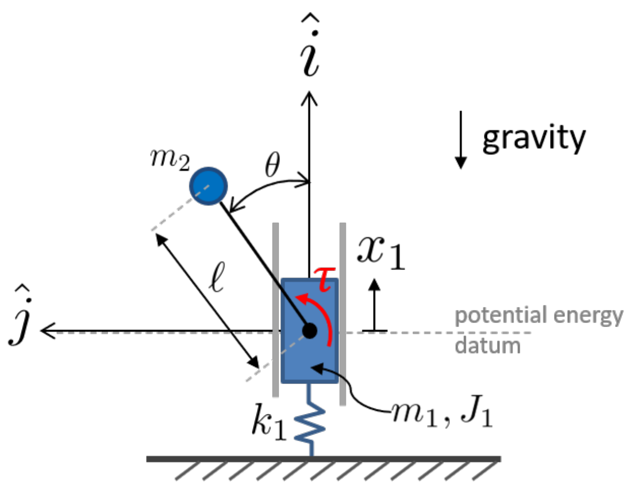
(d) $K = \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}m_2 r^2(\dot{\phi})^2$

$V^2 = r^2 \cancel{\sin^2(\theta+\phi)}(\dot\theta+\dot\phi)^2 + r^2\cancel{\cos^2(\theta+\phi)}(\dot\theta+\dot\phi)^2$

$= r^2(\dot\theta+\dot\phi)^2$

(e) None of the above.

4. (5 pts) Consider the physical system shown in Figure 3. A block of mass m_1 can only slide up and down and is attached to a spring with stiffness k_1. The displacement of the mass in the x-direction can be described using the variable x_1. There is also a pendulum (with no rotational inertia) and a point mass m_2 at a length ℓ from the center of mass of block m_1. The pendulum configuration can be described using the variable θ. The system has no external force that can be applied to the block, but we can apply a torque τ to the pendulum. The channel for block m_1 has very little friction, which can be neglected, but the pendulum is attached with a bearing that can be modeled with viscous friction, using a coefficient of b_1. By definition of our variables, when $x_1 = 0$, and $\theta = 0$, the pendulum is upright and the spring is unloaded. The potential energy datum where $P = 0$ is a horizontal line that coincides with the y-axis.



PE Gravity/Springs

Gravity

m_1: $m_1 g x_1$

m_2: $m_2 g (x_1 + \ell \cos(\theta))$

Spring

$\frac{1}{2} k_1 x_1^2$

Figure 3: Bouncing Mass (m_1) with Pendulum (m_2). To be used for problems 4–6.

Consider the physical system shown in Figure 3. What is the potential energy of the system?

(a) $P = m_2 g(\ell \sin(\theta) + x_1) + m_1 g x_1 + \frac{1}{2} k_1 x_1^2$

(b) $P = m_2 g(\ell \cos(\theta) + x_1) + m_1 g x_1 + \frac{1}{2} k_1 x_1^2$

~~**(c)**~~ $\cancel{P = (m_2 + m_1) g x_1 + \frac{1}{2} k_1 x_1^2}$

~~**(d)**~~ $\cancel{P = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} m_2 \ell^2 \dot{\theta}^2}$

(e) None of the above.

5. (5 pts) Consider the physical system shown in Figure 3.

If $q = [x_1, \theta]^T$, what are the generalized forces?

(a) $\boldsymbol{\tau} = [\tau \cos\theta, \tau \sin\theta]^\top$

(b) $\boldsymbol{\tau} = [0, \tau]^\top$

(c) $\boldsymbol{\tau} = [\tau/r, 0]^\top$

(d) $\boldsymbol{\tau} = [\tau, 0]^\top$

(e) None of the above.

$$\tau = \begin{bmatrix} 0 \\ \tau \end{bmatrix}$$

No Rotational.... point mass (m_2) no rotation (m_1)

6. (5 pts) Consider the physical system shown in Figure 3. What is the kinetic energy of the system?

$P_1 = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix}$ $V_1 = \begin{bmatrix} \dot{x}_1 \\ 0 \\ 0 \end{bmatrix}$

$P_2 = \begin{bmatrix} x_1 + l\cos(\theta) \\ l\sin(\theta) \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} \dot{x}_1 - l\sin(\theta)\dot{\theta} \\ l\cos(\theta)\dot{\theta} \\ 0 \end{bmatrix}$

(a) $K = \cancel{\frac{1}{2}J_1\dot{\theta}^2} + \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\ell^2\dot{\theta}^2 + m_2\ell\dot{\theta}\dot{x}_1(\cos(\theta) - \sin(\theta))$

(b) $K = \frac{1}{2}m_2\ell^2\dot{\theta}^2 + \frac{1}{2}(m_1 + 2m_2)\dot{x}_1^2 + m_2\ell\dot{\theta}\dot{x}_1(\cos(\theta) - \sin(\theta))$

(c) $K = \cancel{\frac{1}{2}J_1\dot{\theta}^2} + \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{\theta}^2$

(d) $K = \frac{1}{2}m_2\ell^2\dot{\theta}^2 + \frac{1}{2}(m_1 + m_2)\dot{x}_1^2$

(e) None of the above.

$\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2 V_2^2$

$V_2^2 = \dot{x}_1^2 + l^2\sin(\theta)^2\dot{\theta}^2 - 2\dot{x}_1 l\sin(\theta)\dot{\theta} + l^2\cos(\theta)^2\dot{\theta}^2$

$\dot{x}_1^2 + l^2\dot{\theta}^2 - 2\dot{x}_1 l\sin(\theta)\dot{\theta}$

$K = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_1^2 + \frac{1}{2}m_2 l^2\dot{\theta}^2 - m_2 l\sin(\theta)\dot{x}_1\dot{\theta}$

7. (5 pts) Now consider a different system with the Lagrangian that can be described as follows:

$$L = 0.5 J_w \dot{\phi}^2 + 0.5 m_b \dot{y_1}^2 + r\dot{\phi}\dot{y}_1 - y_1 cos(\phi)$$

Given that $q = [\phi, y_1]^T$, find the left-hand side of the Euler-Lagrange equations (i.e. do not include friction or generalized forces).

$$\frac{\partial L}{\partial q} = \begin{bmatrix} y_1 \sin(\phi) \\ \cos(\phi) \end{bmatrix}$$

$$\frac{\partial L}{\partial \dot{q}} = \begin{bmatrix} J_w \dot{\phi} + r\dot{y}_1 \\ m_b \dot{y}_1 + r\dot{\phi} \end{bmatrix}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) = \begin{bmatrix} J_w \ddot{\phi} + r\ddot{y}_1 \\ m_b \ddot{y}_1 + r\ddot{\phi} \end{bmatrix}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \begin{bmatrix} J_w \ddot{\phi} + r\ddot{y}_1 - y_1 \sin(\phi) \\ m_b \ddot{y}_1 + r\ddot{\phi} - \cos(\phi) \end{bmatrix}$$

(a)

$$J_w\ddot{\phi} + r\ddot{y}_1 - y_1 \sin(\phi)$$
$$r\ddot{\phi} + m_b\ddot{y}_1 + \cos(\phi)$$

(b)

$$J_w\ddot{\phi} + m_b r\ddot{y}_1 + m_b g\,(r\cos(\phi) - y_1\sin(\phi))$$
$$m_b r\ddot{\phi} + m_b\ddot{y}_1 + m_b g\cos(\phi)$$

(c)

$$J_w\ddot{\phi} + m_b r\ddot{y}_1 + m_b g\,(r\cos(\phi) - y_1\sin(\phi))$$
$$m_b r\ddot{\phi} + m_b\ddot{y}_1 + m_b g\cos(\phi)$$

(d)

$$J_w\ddot{\phi} + m_b r\ddot{y}_1 + m_b g\,(r\cos(\phi) - y_1\sin(\phi))$$
$$m_b r\ddot{\phi} + m_b\ddot{y}_1 + m_b g\cos(\phi)$$

(e) None of the above.

$$\begin{array}{c} \dot{z} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{\theta} \end{array} \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -1.5 & -2 & -2.5 \\ 0 & 3.2 & -0.25 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \\ 0.5 & 1.5 \end{bmatrix}$$

8. (5 pts) Given the equations of motion

$$\ddot{z} = -4z - 1.5\theta - 2\dot{z} - 2.5\dot{\theta} + 3F$$

$$\cancel{\ddot{z} + 2\dot{z} + 4z = 3F - 1.5\theta - 2.5\dot{\theta}}$$

$$\cancel{\ddot{\theta} = 3.2\theta = 1.5\tau + 0.5F - 0.25\dot{z},}$$

$$\ddot{\theta} = 3.2\theta - 0.25\dot{z} + 0.5F + 1.5\tau$$

If the state is defined as $x = (z, \theta, \dot{z}, \dot{\theta})^\top$, the input as $u = (F, \tau)^\top$, and the output is defined as $y = (z, \theta)^\top$, what are the state space equations?

(a) $$\dot{x} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -1.5 & -2 & -2.5 \\ 0 & 3.2 & -0.25 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \\ 0.5 & 1.5 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x$$

(b) $$\dot{x} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3.2 & -0.25 & 0 \\ -4 & -1.5 & -2 & -2.5 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \\ 0.5 & 1.5 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x$$

(c) $$\dot{x} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -2.5 & -4 & -1.5 \\ 0 & -0.25 & 3.2 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \\ 0.5 & 1.5 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x$$

(d) $$\dot{x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -4 & -1.5 & -2 & -2.5 \\ 0 & 3.2 & -0.25 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \\ 1.5 & 0.5 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x$$

(e) None of the above.

9. (5 pts) The differential equation shown below describes a nonlinear system known as a “Van der Pol” oscillator. This is a circuit where voltage (v) and its derivatives describe a nonlinear oscillatory behavior based on an initial condition.

$$\ddot{v} - \sqrt{\frac{L}{C}}(1 - v^2)\dot{v} - v = 0 \tag{1}$$

where v is the voltage of interest, $\dot{v}$ and $\ddot{v}$ are the first and second time derivatives of v, L is an inductance and C a capacitance. *You do not need to derive these equations in any way to solve this problem.*

Find the states for the equilibrium point in terms of any system parameters (i.e. constants). Hint: start by putting the system in state variable form to help you with this problem and the next.

(a) $\dot{v}_e = 0$, $\ddot{v}_e = 0$

(b) $v_e = -\sqrt{\frac{L}{C}}(1 - v_e^2)\dot{v}_e$

(c) $v_e = 0$, $\dot{v}_e = 0, \ddot{v}_e = 0$

(d) $v_e = 0$, $\dot{v}_e = 0$

(e) None of the above.

$$X = \begin{bmatrix} v \\ \dot{v} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{v} \\ \ddot{v} \end{bmatrix} = \begin{bmatrix} \dot{v} \\ \cancel{\sqrt{\frac{L}{C}}\dot{v}} - \cancel{\sqrt{\frac{L}{C}}v^2\dot{v}} + v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\dot{V}_e = 0$

$V_e = 0$

10. (5 pts) Given the equations for the previous problem and the equilibrium point, (remember that you can always substitute the equilibrium point into the nonlinear state equations to verify it is the correct equilibrium point), find the linearized equations of motion in **state space form**. We are only looking for the $\dot{\tilde{x}}$ part of the equations (not the output equation).

SVf

(a) ~~$\dot{\tilde{x}} = \left[\sqrt{\frac{L}{C}}(1 - v_e^2)\dot{v}_e - v_e \right]$~~

if −1 was 1

(b) $\dot{\tilde{x}} = \begin{bmatrix} \sqrt{\frac{L}{C}} & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{v}} \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) $\dot{\tilde{x}} = \begin{bmatrix} \sqrt{\frac{L}{C}}(1 - v^2) & -2\sqrt{\frac{L}{C}}\dot{v}v - 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{v}} \\ \tilde{v} \end{bmatrix}$

(d) $\dot{\tilde{x}} = \begin{bmatrix} \sqrt{\frac{L}{C}} & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{v}} \\ \tilde{v} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [v_e]$

(e) None of the above.

$$\dot{\tilde{X}} = \begin{bmatrix} \ddot{\tilde{V}} \\ \dot{\tilde{V}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{L}{C}} - \cancelto{0}{\sqrt{\frac{L}{C}} v_e^2} & \cancelto{0}{-2\sqrt{\frac{L}{C}}\dot{v}_e v_e} \quad 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{V}} \\ \tilde{V} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \sqrt{\frac{L}{C}} & 1 \\ 1 & 0 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{B}$$

$$Y = \left(\left(Y_r - Y G_4\right) G_2 + \left(Y_r - Y G_4\right) G_3\right) G_1$$

$$Y = Y_r G_1 G_2 - Y G_1 G_2 G_4 + Y_r G_1 G_3 - Y G_1 G_3 G_4$$

$$Y\left(1 + G_1 G_2 G_4 + G_1 G_3 G_4\right) = Y_r\left(G_1 G_2 + G_1 G_3\right)$$

$$\frac{Y}{Y_r} = \frac{G_1\left(G_2 + G_3\right)}{1 + G_1 G_4\left(G_2 + G_3\right)}$$

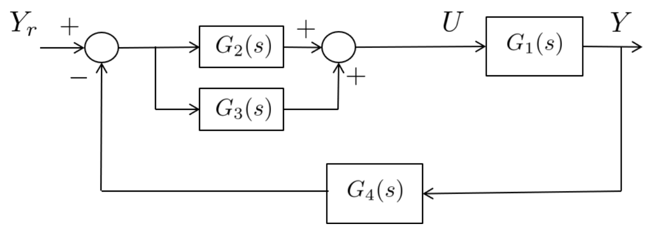


Figure 4: Block Diagram with 4 Different Transfer Functions

11. (5 pts) Given the block diagram in Figure 4, find the resulting transfer function $\frac{Y}{Y_r}$ in terms of the given transfer functions (G).

(a)
$$\frac{G_1(G_2 + G_3)}{1 + G_1G_4(G_2 + G_3)}$$

(b)
$$\frac{G_4G_1(G_2 + G_3)}{1 + G_1G_4(G_2 + G_3)}$$

(c)
$$\frac{G_4G_1(G_2 + G_3)}{G_1G_4(G_2 + G_3)}$$

(d)
$$\frac{G_1(G_2 + G_3)}{1 + G_1(G_2 + G_3)}$$

(e) None of the above.

12. (5 pts) For a problem where y and ϕ are outputs, and f is the input, we have obtained the following differential equations:

$$\begin{aligned} \ddot{y} + 1.1\dot{y} &= -0.5y + 0.5f \\ 0.1\dot{\phi} + \phi &= f \end{aligned}$$

$$Y(s^2 + 1.1s + 0.5) = 0.5\,F \qquad \frac{Y}{F} = \frac{0.5}{s^2 + 1.1s + 0.5}$$

$$\Phi(0.1s + 1) = 1\,F \qquad \frac{\Phi}{F} = \frac{1}{0.1s + 1}$$

Find the transfer function for $\frac{Y}{\Phi}$.

$$\frac{Y}{\Phi} = \frac{\frac{Y}{F}}{\frac{\Phi}{F}} = \frac{0.5}{(s^2 + 1.1s + 0.5)} \cdot \frac{(0.1s + 1)}{1}$$

$$= \frac{0.05s + 0.5}{s^2 + 1.1s + 0.5}$$

(a)
$$\frac{0.5}{(0.1s + 1)(s^2 + 1.1s + 0.5)}$$

(b)
$$\frac{(0.1s + 1)(s^2 + 1.1s + 0.5)}{0.5}$$

(c)
$$\frac{0.05s + 0.5}{s^2 + 1.1s + 0.5}$$

(d)
$$\frac{s^2 + 1.1s + 0.5}{0.05s + 0.5}$$

(e) None of the above.

13. (5 pts) The following linearized equations describe a state space model for a dynamic system that we want to control using PID methods:

$$\dot{\tilde{x}} = \begin{pmatrix} 0.5 & 0.1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{\tilde{x}}_1 \\ \tilde{x}_1 \end{pmatrix} + \begin{pmatrix} 0.1 \\ 0 \end{pmatrix} \tilde{\tau}$$

Find the transfer function $\frac{\tilde{X}_1(s)}{\tilde{\tau}(s)}$.

$\ddot{x} = 0.5\dot{x} + 0.1x$

$X(s^2 - 0.5s - 0.1) = 0.1\,\tau$

$* \ \frac{X}{\tau} = \frac{0.1}{s^2 - 0.5s - 0.1}$

(a) $\begin{bmatrix} \frac{0.1}{s^2-0.5s-0.1} \\ \\ \frac{0.1s}{s^2-0.5s-0.1} \end{bmatrix}$

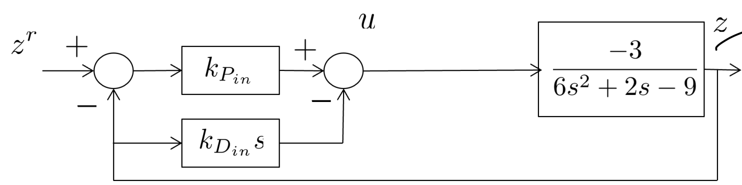
(b) $\left[\frac{0.1(s+1)}{s^2-0.5s-0.1}\right]$

(c) $\left[\frac{0.1s}{s^2-0.5s-0.1}\right]$

(d) $\left[\frac{0.1}{s^2-0.5s-0.1}\right]$

(e) None of the above.

14. (5 pts) Figure 5 shows a proportional-derivative (PD) control for the open-loop system $P(s) = \frac{-3}{6s^2+2s-9}$.



P

$$\frac{-\frac{1}{2}}{s^2 + \frac{1}{3}s - \frac{3}{2}}$$

Δ_{cl}

$$\frac{-\frac{1}{2}k_P}{s^2 + \left(\frac{1}{3} - \frac{1}{2}k_D\right)s - \left(\frac{3}{2} + \frac{1}{2}k_P\right)}$$

Figure 5: System under PD control. To be used in problems 14–19.

What is the closed-loop characteristic equation?

(a) $3s^2 + 6s + -9$

(b) $s^2 + (\frac{1}{3} - \frac{1}{2}k_{D_{in}})s + (-\frac{1}{2}k_{P_{in}} - \frac{3}{2})$

(c) $s^2 + (2 - 3k_{D_{in}})s + (-3k_{P_{in}} - 9)$

(d) $s^2 + (2 + 3k_{D_{in}})s + (3k_{P_{in}} - 9)$

(e) None of the above.

$$s^2 + \left(\frac{1}{3} - \frac{1}{2}k_D\right)s - \left(\frac{3}{2} + \frac{1}{2}k_P\right)$$

$$2\sigma = \left(\frac{1}{3} - \frac{1}{2}k_D\right)$$

$$2\sigma = 2\left(\frac{1}{6} - \frac{1}{4}k_D\right)$$

15. (5 pts) Referring to the system in Figure 5, which best describes the stability of the open-loop system?

(a) Stable.

(b) Unstable.

(c) Marginally Stable.

(d) Cannot be determined without $k_{P_{in}}$ and $k_{D_{in}}$.

(e) None of the above.

$$2\sigma = \left(\frac{1}{3} - \frac{1}{2}k_D\right)$$

$$2\sigma = 2\left(\frac{1}{6} - \frac{1}{4}k_D\right)$$

$$\sigma = \frac{1}{6} - \frac{1}{4}k_D \longrightarrow \text{if } \frac{1}{4}k_D > \frac{1}{6} \text{ then Stable}$$

else unstable

16. (5 pts) Referring to the system in Figure 5, what is the desired characteristic equation that results in a rise time of $t_r = 1.5$ seconds, and a damping ratio of $\zeta = 0.95$ (not $\zeta = 0.707$)?

(a) $s^2 + 1.3s + 0.46$

(b) $s^2 + 0.96s + 0.46$

(c) $s^2 + 6.4s + 11.2$

(d) $s^2 + 0.48s + 0.46$

(e) None of the above.

$$t_r = \frac{1}{2} \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \Rightarrow \omega_n = \frac{1}{2}\frac{\pi}{t_r\sqrt{1-\xi^2}} = \frac{\pi}{2}\left(\frac{1}{1.5\sqrt{1-0.95^2}}\right)$$

$\omega_n = 3.353$

$\omega_n^2 = 11.247$

$2\xi\omega_n = 6.372$

$$\frac{12k_P}{s^2+(6+12k_D)s+(-9+12k_P)}$$

C.L.

$\hookrightarrow k_D = 0$

$-9+12k_P = 6$

$12k_P = 15$

$k_P = \frac{15}{12} = \frac{5}{4}$

17. (5 pts) Referring to the system in Figure 5, if we replace the open-loop transfer function with $\frac{12}{s^2+6s-9}$, what are the proportional and derivative gains that result in the desired characteristic equation of $\Delta_{cl}^d(s) = s^2 + 6s + 6 = 0$?

   (a) ~~$k_{P_{in}} = 1/4$, $k_{D_{in}} = -1/4$.~~
   (b) $k_{P_{in}} = -1/4$, $k_{D_{in}} = 0$.
   (c) ~~$k_{P_{in}} = -5/4$, $k_{D_{in}} = 5/4$.~~
   (d) $k_{P_{in}} = 5/4$, $k_{D_{in}} = 0$.
   (e) None of the above.

18. (5 pts) Referring to the system in Figure 5 (including the original open-loop transfer function), if $k_{P_{in}} = -15$ and $k_{D_{in}} = -9\frac{1}{3}$, what is the DC-gain of the inner loop system?

   $\hookrightarrow -\frac{28}{3}$

   (a) DC-gain = 5/4
   (b) DC-gain = -5/4
   (c) DC-gain = 1
   (d) DC-gain = -1
   (e) None of the above.

$s^2 + \left(\frac{1}{3} - \frac{1}{2}k_D\right)s - \left(\frac{3}{2} + \frac{1}{2}k_P\right)$

$$DC = \lim_{s\to 0} \frac{\frac{28}{6}}{s^2+(5)s+(6)} = \frac{75}{6} = \frac{5}{4}$$

19. (5 pts) Again referring to the system in Figure 5 (including the original open-loop transfer function), if $k_{P_{in}} = -15$ and $k_{D_{in}} = -9\frac{1}{3}$, what are the poles of the closed-loop system?

(a) $s = -2, \cancel{-3}$

(b) $s = -1.4, 1.1$

(c) $s = -15, -9.3$

(d) $s = 2.4, 1.0$

(e) None of the above.

$s^2 + (\ 5\ )s + (\ 6\ )$

$\frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2} = -3, -2$

$$\frac{6}{S^2+7S}$$

$$\frac{6}{S^2+(7+6K_D)s+(6K_D)}$$

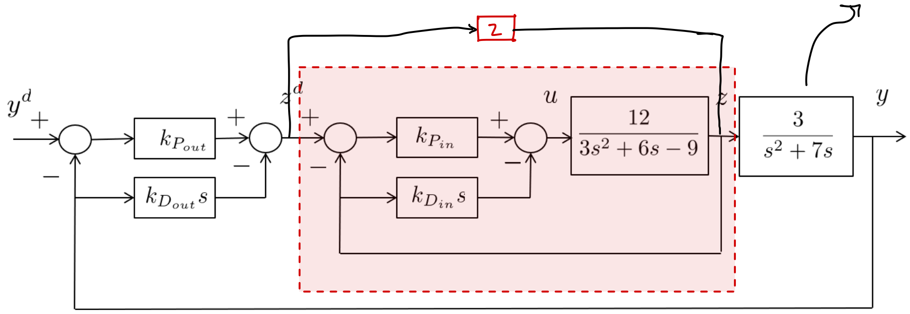


Figure 6: Inner-loop, outer-loop design. To be used in problem 20.

20. (5 pts) Given the system shown in Figure 6, if the gains for the inner loop are chosen such that it has a DC-gain of 2, what is the closed-loop characteristic polynomial of the outer loop?

(a) $s^2 + (7 + 36k_{D_{out}})s + 36k_{P_{out}}$

(b) $s^2 + (7 + 6k_{D_{out}})s + 6k_{P_{out}}$

(c) $s^2 + (7 + 1.5k_{D_{out}})s + 1.5k_{P_{out}}$

(d) $s^2 + (7 + 3k_{D_{out}})s + 3k_{P_{out}}$

(e) None of the above.