

ECEn 483 / MeEn 431  
Introduction to Feedback Control

## Winter 2024, Midterm 2

April 2 – April 6, 2024  
Professor: Marc Killpack

Time Limit: 3 hours.

Explanations are  
in **GREEN**

Name: CARSON Wynn  
Start Time: 14:42  
End Time: 16:09

Look @  
# 2  
# 14  
# 17

### Instructions:

- Work all problems and make sure to submit all written work.
- Test is closed book, closed notes, except...
- You are allowed two 8.5×11 sheets of notes (front and back).
- Calculator is allowed (but shouldn't be needed). You can use Python or MATLAB (or similar software), but only as a calculator for matrix math. You **MUST NOT** look at documentation at all or use MATLAB or Python for any feedback control functions in the “control” module. You may use Sympy.

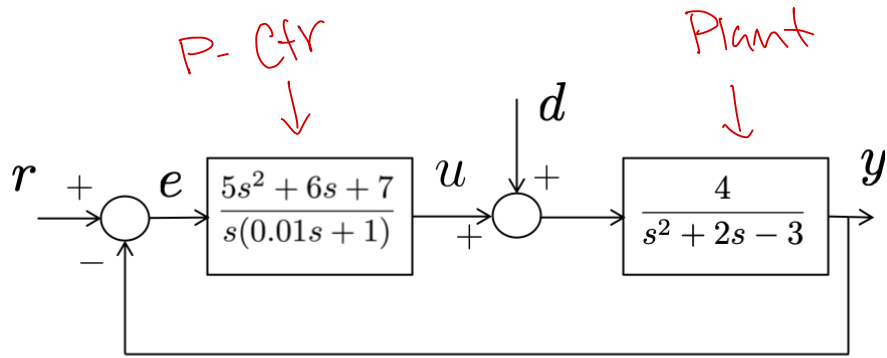


Figure 1: System used for problems 1–4.

1. (5 points) Consider the system shown in Figure 1. What is the system type with respect to the reference  $r$ ?

$$\frac{E}{Z} = \frac{1}{1+CP} = \frac{1}{1 + \left( \frac{5s^2 + 6s + 7}{s(0.01s + 1)} \right) \left( \frac{4}{s^2 + 2s - 3} \right)}$$

$$= \frac{s(0.01s + 1)(s^2 + 2s - 3)}{\dots}$$

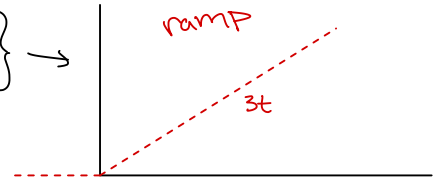
- (a) Type 0  
 (b) Type 1  
 (c) Type 2  
 (d) Type 3  
 (e) None of the above.

2. (5 points) Consider the system shown in Figure 1. What is the steady-state error when  $d(t) = 0$  and

$$\lim_{s \rightarrow 0} s \left( \frac{s(0.01s + 1)(s^2 + 2s - 3)}{s(0.01s + 1)(s^2 + 2s - 3) + (5s^2 + 6s + 7)(4)} \right) \left( \frac{1}{s^2} \right)$$

$$= \frac{(1)(-3)}{(7)(4)} = -\frac{3}{28}$$

$$r(t) = \begin{cases} 3t, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad ?$$



- (a) 0  
 (b)  $\infty$   
 (c)  $-\frac{3}{28}$   
 (d)  $-\frac{9}{28}$   
 (e) None of the above.

Mostly Correct. Work  
 Shown, but I forgot  
 to multiply my ramp  
 input by the constant 3

3. (5 points) Consider the system shown in Figure 1. What is the system type with respect to the **disturbance**  $d$ ?

$$\frac{E}{D} = \frac{P}{1+CP} = \frac{\frac{4}{s^2+2s-3}}{1 + \left( \frac{s^2+6s+7}{s(s+1)} \right) \left( \frac{4}{s^2+2s-3} \right)}$$

$$= \frac{s(s+1)(4)}{s(s+1)(s^2+2s-3) + (s^2+6s+7)(4)}$$

- (a) Type 0  
 (b) Type 1  
 (c) Type 2  
 (d) Type 3  
 (e) None of the above.

4. (5 points) Consider the system shown in Figure 1. What is the steady-state error when  $r(t) = 0$  and

$$d(t) = \begin{cases} 2u_s(t), & t \geq 0 \\ 0 & t < 0 \end{cases}$$

where  $u_s(t)$  is a unit step function?



(a) 0

(b)  $\infty$

(c)  $-\frac{2}{7}$

(d)  $\frac{2}{7}$

(e) None of the above.

5. (5 points) Consider the closed loop system shown in Figure 2. What is the closed-loop transfer function from  $R$  to  $Y$ , where  $R$  and  $Y$  are the Laplace transforms of  $r$  and  $y$ ?

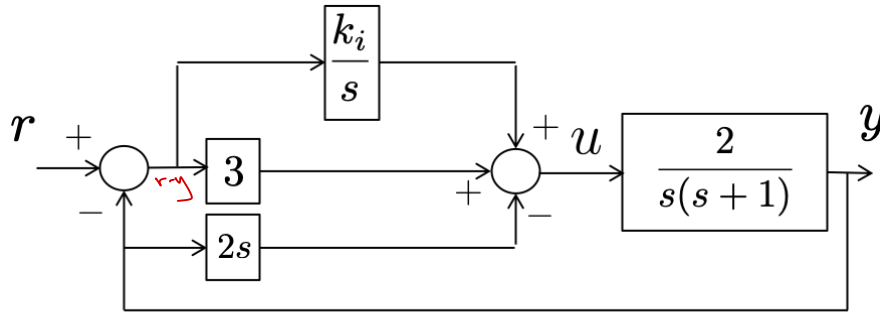


Figure 2: System used for problem 5.

$$u \left( \frac{2}{s(s+1)} \right) = y$$

(a)  $\frac{6s + 2k_i}{s^3 + 5s^2 + 6s + 2k_i}$

(b)  $\frac{2s^2 + 6s + 2k_i}{s^3 + 5s^2 + 6s + 2k_i}$

(c)  $\frac{4s^2 + 6s + 2k_i}{s^3 + s^2}$

(d)  $\frac{3s + k_i}{s^3 + 3s^2 + 3s + k_i}$

(e) None of the above.

$$3(r-y) + \frac{k_i}{s}(r-y) - 2sy = u$$

$$(r-y) \left( 3 + \frac{k_i}{s} \right) - 2sy = u$$

$$\left( 3 + \frac{k_i}{s} \right) r - \left( 3 + \frac{k_i}{s} + 2s \right) y = u$$

$$\left[ \left( 3 + \frac{k_i}{s} \right) r - \left( 3 + \frac{k_i}{s} + 2s \right) y \right] \left( \frac{2}{s(s+1)} \right) = y$$

$$\frac{2 \left( 3 + \frac{k_i}{s} \right)}{s(s+1)} r = \left( \frac{2 \left( 3 + \frac{k_i}{s} + 2s \right)}{s(s+1)} + 1 \right) y$$

$$\frac{Y}{R} = \frac{\cancel{s^3 + s^2}}{6s + 2k_i + 4s^2 + \cancel{s^3 + s^2}} \left( \frac{6s + 2k_i}{\cancel{s^3 + s^2}} \right)$$

$$= \frac{6s + 2k_i}{s^3 + 5s^2 + 6s + 2k_i}$$

6. (5 points) Suppose that the model of a system to be controlled is given in state-space control canonical form as

$$\dot{x} = \begin{pmatrix} 1 & 2 & -4 & -8 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u$$

$$y = (0 \quad 0 \quad -1 \quad 2) x.$$

$s^4 - s^3 - 2s^2 + 4s + 8$

What is the open-loop characteristic equation for the transfer function from the input  $u$  to the output  $y$ ?

- (a)  $\Delta_{ol}(s) = -s^3 - 2s^2 + 4s + 8$
- (b)  $\Delta_{ol}(s) = s^4 + 8s^3 + 4s^2 - 2s - 1$
- (c)  $\Delta_{ol}(s) = s^4 + s^3 + 2s^2 + 4s + 8$
- (d)  $\Delta_{ol}(s) = s^4 + s^3 + 2s^2 - 4s - 8$
- ☒ (e) None of the above.

7. (5 points) If the transfer function for a system is given by

$$H(s) = \frac{s^2 - 1}{s^3 + 3s^2 - 6},$$

what are the state space equations in controller canonic form?

$$A_c = \begin{bmatrix} -3 & 0 & 6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

(a)  $\dot{x} = \begin{pmatrix} -3 & 0 & 6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$

$$y = (1 \ 0 \ -1) x$$

(b)  $\dot{x} = \begin{pmatrix} 1 & -3 & 0 & 6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u$

$$y = (0 \ 1 \ 0 \ -1) x$$

(c)  $\dot{x} = \begin{pmatrix} -3 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} u$

$$y = (1 \ 0 \ 0) x$$

(d)  $\dot{x} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} u$

$$y = (1 \ 0 \ 0) x$$

(e) None of the above.

8. (5 points) Suppose that the controllability matrix of a system is given by

$$C_{A,B} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

$$\det(C_{A,B}) = 2 + 2 - 2 - 2 - 2 = 0$$

which of the following is true?

→ Same ... Bad!

- (a) The system is not controllable.  
 (b) The system is controllable.  
 (c) Not enough information to decide.

Questions 9–11 refer to the following system:

$$\dot{x} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u \quad (1)$$

$$y = \begin{pmatrix} 0 & 3 \end{pmatrix} x. \quad (2)$$

9. (5 points) Suppose that the system to be controlled is given in state-space form by Equations (1)–(2). What is the controllability matrix?

(a)  $\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$

(b)  $\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$

☒ (c)  $\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$

(d)  $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$

(e) None of the above.

$$C_{AB} = \begin{bmatrix} B & AB \end{bmatrix} \\ = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

10. (5 points) Suppose that the system to be controlled is given in state-space form by Equations (1)–(2). If the desired closed loop poles are at  $-2 \pm 3j$ , what is  $\alpha$ ?

(a)  $\alpha = (1 \ 4 \ 13)$

(b)  $\alpha = (-4 \ 13)$

(c)  $\alpha = (-4 \ -13)$

☒ (d)  $\alpha = (4 \ 13)$

(e) None of the above.

$$\begin{aligned} P_1 &= (-2 + 3j) \\ P_2 &= (-2 - 3j) \end{aligned} \left. \vphantom{\begin{aligned} P_1 &= (-2 + 3j) \\ P_2 &= (-2 - 3j) \end{aligned}} \right\} = s^2 + 4s + 13 \\ \alpha &= \begin{bmatrix} 4 & 13 \end{bmatrix}$$

11. (5 points) Suppose that the system to be controlled is given in state-space form by Equations (1)–(2), what is  $\mathcal{A}_A$ ?

(a)  $\mathcal{A}_A = (1 \ 1 \ 0 \ 1)$

(b)  $\mathcal{A}_A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

☒ (c)  $\mathcal{A}_A = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

(d)  $\mathcal{A}_A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

(e) None of the above.

$$\det(sI - A) = 0 \\ \det \begin{pmatrix} s-1 & 2 \\ 0 & s-1 \end{pmatrix} = 0 \\ s^2 - 2s + 1 = 0$$

$$A_A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

12. (5 points) Given the system:

$$(s+1)(s) - 4 \rightarrow s^2 + s - 4 \quad (3)$$

$$\dot{x} = \begin{pmatrix} -1 & 4 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & -1 \end{pmatrix} x. \quad (4)$$

What are the gains  $K$  that place closed-loop poles at  $-1 \pm 2j$ ?  $\rightarrow s^2 + 2s + 5$

(a)  $K = \begin{pmatrix} -1 & -9 \end{pmatrix}$

(b)  $K = \begin{pmatrix} 1 & 9 \end{pmatrix}$

(c)  $K = \begin{pmatrix} -3 & -1 \end{pmatrix}$

(d)  $K = \begin{pmatrix} 3 & 1 \end{pmatrix}$

(e) None of the above.

$$K = K_c = \alpha_A - \alpha_A = \begin{pmatrix} 1 & 9 \end{pmatrix}$$

13. (5 points) Consider the system given by Equations (3) and (4). If the feedback gains have been selected as

$$K = \begin{pmatrix} 2 & 6 \end{pmatrix},$$

what are the poles of the closed-loop system?

(a)  $s = \begin{pmatrix} 1 & 2 \end{pmatrix}$

(b)  $s = \begin{pmatrix} -2 & -1 \end{pmatrix}$

(c)  $s = \begin{pmatrix} -2.56 & 1.56 \end{pmatrix}$

(d)  $s = \begin{pmatrix} -2.56 & -1.56 \end{pmatrix}$

(e) None of the above.

$$C_{AB} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$K = K_c \overbrace{A_A^{-1} C_{AB}^{-1}}^I \quad \text{so} \quad K = K_c$$

$$\text{eig}(A - BK) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$



14. (5 points) Consider the system given by Equations (3) and (4). If the feedback gains have been selected as

$$K = \begin{pmatrix} 2 & 5 \end{pmatrix},$$

and the controller is given by

$$u = -Kx + k_r r,$$

what is the reference gain  $k_r$  that makes the DC-gain from  $r$  to  $y$  equal to one?

$$k_r = \frac{-1}{C(A-BK)^{-1}B}$$

$$k_r = 0.5$$

- (a)  $k_r = -1/3$
- ☒ (b)  $k_r = 1$
- (c)  $k_r = -1$
- (d)  $k_r = 1/2$
- (e) None of the above.

Work in the Test2.m File. I forgot to multiply by the -1 on top sign error

15. (5 points) Given the Equations (3) and (4), which of the following is the observability matrix?

☒ (a)  $\begin{pmatrix} 1 & -1 \\ -2 & 4 \end{pmatrix}$

(b)  $\begin{pmatrix} -1 & 2 \\ 1 & -4 \end{pmatrix}$

(c)  $\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$

(d)  $\begin{pmatrix} -1 & 4 \\ 1 & 0 \end{pmatrix}$

- (e) None of the above.

$$\begin{aligned} \mathcal{O}_{AC} &= \begin{bmatrix} C \\ CA \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} \end{aligned}$$

16. (5 points) In addition to using it in the Ackermann formula to find the observer gain matrix "L", what is the significance of the observability matrix?

- (a) ~~It is the transpose of the controllability matrix.~~
- (b) ~~It tells us if we can move all of the the eigenvalues or poles for our closed-loop system.~~
- ☒ (c) It tells us if we can estimate all of the states with our given measurements. ✓
- (d) ~~It tells us the system type which tells us about the steady state error.~~
- (e) None of the above.

→ C, A, B

$$\det(sI - A) = s + 5$$

$$A = [-5] \quad B = [1]$$

$$C = [1]$$

17. (5 points) Consider the system shown in Figure 3. The control law is given by

$$u = -Ky - k_i \int (r - y) dt.$$

Find  $K$  and  $k_i$  to place the closed loop pole at  $-1$  and the integrator pole at  $-2$ .

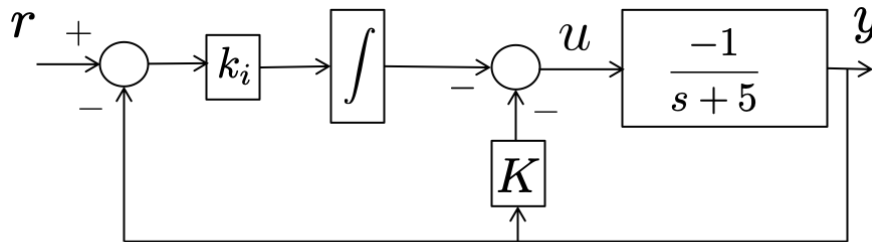


Figure 3: System used for problems 15.

$$\dot{X} = (A - BK)x$$

$$A' = \begin{bmatrix} -5 & 0 \\ 1 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k \\ k_i \end{bmatrix}$$

$$\begin{bmatrix} -5 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} k & k_i \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -5-k & -k_i \\ 1 & 0 \end{bmatrix}$$

$$\det(sI - (A' - B'K)) = (s+1)(s+2)$$

$$= s^2 + 3s + 2$$

$$\alpha = [3 \quad 2]$$

$$(s + (5+k))(s) + k_i = s^2 + 3s + 2$$

$$s^2 + (5+k)s + k_i = s^2 + 3s + 2$$

$$k = -2$$

$$k_i = 2$$

- (a)  $K = 2, k_i = 2$
- (b)  $K = 2, k_i = -2$
- (c)  $K = -2, k_i = 2$
- (d)  $K = -2, k_i = -2$
- (e) None of the above.

$$-(r-y) \frac{k_i}{s} - Ky = u$$

$$u \left( \frac{-1}{s+5} \right) = y$$

Work in the Test2.m File. I set  $B = [1]$  instead of  $B = [-1]$ , sign error

18. (5 points) What is the purpose of integrator anti-windup?

- (a) ~~To ensure~~ that the integrator and differentiator do not cancel the effect of each other.
- (b) ~~To ensure~~ that the root locus is in the left-half plane.
- (c) To avoid integrating when the system actuators are saturated. ✓
- (d) ~~To avoid pushing~~ the poles of the closed-loop system into the right-half plane.
- (e) ~~None of the above.~~

19. (5 points) Using the Tustin approximation (or transform), find the discrete-time equation for the following transfer function (where “n” denotes a discrete time step):

$$U_I(s) = \frac{\sigma}{s} E(s) \quad S = \frac{z}{T_s} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right)$$

Which of the following is the correct discrete-time implementation?

- (a)  $u[n] = 2u[n+1] + \frac{\sigma T_s}{2}(e[n] - e[n+1])$
- (b)  $u[n] = u[n-1] + \frac{\sigma T_s}{2}(e[n] + e[n-1])$
- (c)  $u[n] = \frac{u[n-1]}{2} + \frac{\sigma T_s}{2}(e[n] + e[n-1])$
- (d)  $u[n] = \frac{\sigma T_s}{2}(e[n] + e[n-1])$
- (e) None of the above.

$$U(s) = \frac{\sigma T_s (1 + z^{-1})}{2 (1 - z^{-1})} E(s)$$

$$U(s) - U(s) z^{-1} = \frac{\sigma T_s}{2} (E(s) + E(s) z^{-1})$$

$$u[n] = u[n-1] + \frac{\sigma T_s}{2} (e[n] + e[n-1])$$

20. (5 points) Which of the following is true when comparing full-state feedback control to PID control (without successive loop closure)?

- (a) ~~Integral control~~ is possible with PID, but not with full-state feedback. *False*
- (b) ~~Full-state~~ feedback allows us to change the eigenvalues, which is not possible with PID control.
- (c) We cannot use the dirty derivative to find any of our states for full-state feedback.
- ☒ (d) Full-state feedback makes it easier to set system pole values directly compared to PID control.
- (e) None of the above.