ECEn 483 / MeEn 431
Introduction to Feedback Control

Winter 2024, Midterm 2

April 2 – April 6, 2024
Professor: Marc Killpack

Time Limit: 3 hours.

Name: Carson Wynn
Start Time: 14:42
End Time: 16:09

Instructions:

- Work all problems and make sure to submit all written work.
- Test is closed book, closed notes, except...
- You are allowed two 8.5×11 sheets of notes (front and back).
- Calculator is allowed (but shouldn't be needed). You can use Python or MATLAB (or similar software), but only as a calculator for matrix math. You **MUST NOT** look at documentation at all or use MATLAB or Python for any feedback control functions in the "control" module. You may use Sympy.

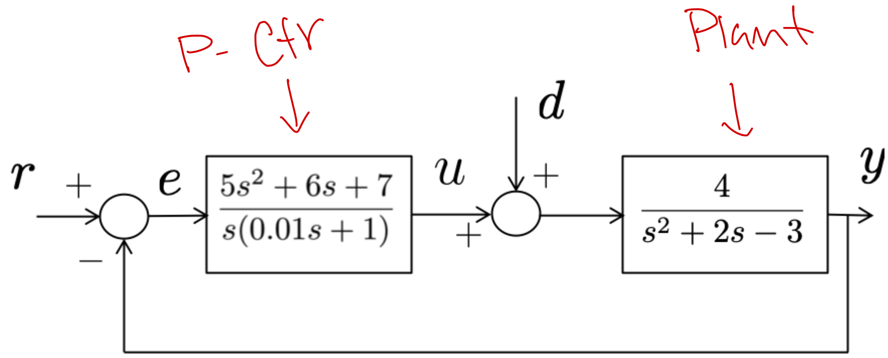


Figure 1: System used for problems 1–4.

1. (5 points) Consider the system shown in Figure 1. What is the system type with respect to the reference r?

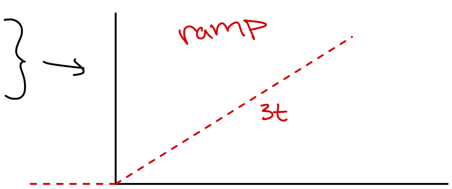
$$\frac{E}{Z} = \frac{1}{1+CP} = \frac{1}{1+\left(\frac{5s^2+6s+7}{s(0.01s+1)}\right)\left(\frac{4}{s^2+2s-3}\right)}$$

$$= \frac{s(0.01s+1)(s^2+2s-3)}{\cdots\cdots}$$

   (a) Type 0
   (b) Type 1
   (c) Type 2
   (d) Type 3
   (e) None of the above.

2. (5 points) Consider the system shown in Figure 1. What is the steady-state error when $d(t) = 0$ and

$$r(t) = \begin{cases} 3t, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad ?$$



$$\lim_{s\to 0} \cancel{s}\left(\frac{\cancel{s}\,\cancel{(0.01s+1)}\,(\cancel{s^2+2s}-3)}{\cancel{s(0.01s+1)(s^2+2s-3)} + (5s^2+6s+7)(4)}\right)\left(\frac{1}{\cancel{s^2}}\right)$$

$$= \frac{(1)(-3)}{(7)(4)} = \frac{-3}{28}$$

   (a) ~~0~~
   (b) ~~∞~~
   (c) $-\frac{3}{28}$
   (d) $-\frac{9}{28}$
   (e) None of the above.

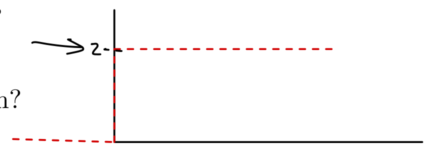
3. (5 points) Consider the system shown in Figure 1. What is the system type with respect to the **disturbance** d?

$$\frac{E}{D} = \frac{P}{1+CP} = \frac{\frac{4}{s^2+2s-3}}{1+\left(\frac{5s^2+6s+7}{s(0.01s+1)}\right)\left(\frac{4}{s^2+2s-3}\right)}$$

$$= \frac{s(0.01s+1)(4)}{s(0.01s+1)(s^2+2s-3)+(5s^2+6s+7)(4)}$$

(a) Type 0
(b) Type 1
(c) Type 2
(d) Type 3
(e) None of the above.

4. (5 points) Consider the system shown in Figure 1. What is the steady-state error when $r(t) = 0$ and $d(t) = \begin{cases} 2u_s(t), & t \geq 0 \\ 0 & t < 0 \end{cases}$ where $u_s(t)$ is a unit step function?



(a) 0
(b) ∞
(c) $-\dfrac{2}{7}$
(d) $\dfrac{2}{7}$
(e) None of the above.

5. (5 points) Consider the closed loop system system shown in Figure 2. What is the closed-loop transfer function from R to Y, where R and Y are the Laplace transforms of r and y?

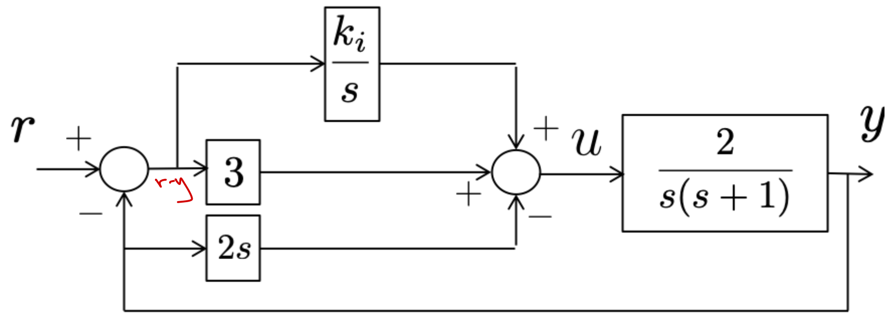


Figure 2: System used for problem 5.

$$u\left(\frac{2}{s(s+1)}\right) = y$$

(a) $\dfrac{6s+2k_i}{s^3+5s^2+6s+2k_i}$

(b) $\dfrac{2s^2+6s+2k_i}{s^3+5s^2+6s+2k_i}$

(c) $\dfrac{4s^2+6s+2k_i}{s^3+s^2}$

(d) $\dfrac{3s+k_i}{s^3+3s^2+3s+k_i}$

(e) None of the above.

$$3(r-y) + \frac{k_i}{s}(r-y) - 2s(y) = u$$

$$(r-y)\left(3+\frac{k_i}{s}\right) - 2sy = u$$

$$\left(3+\frac{k_i}{s}\right)r - \left(3+\frac{k_i}{s}+2s\right)y = u$$

$$\left[\left(3+\frac{k_i}{s}\right)r - \left(3+\frac{k_i}{s}+2s\right)y\right]\left(\frac{2}{s(s+1)}\right) = y$$

$$\frac{2\left(3+\frac{k_i}{s}\right)}{s(s+1)}r = \left(\frac{2\left(3+\frac{k_i}{s}+2s\right)}{s(s+1)} + 1\right)y$$

$$\frac{Y}{R} = \frac{\cancel{s^3+s^2}}{6s+2k_i+4s^2+s^3+s^2}\left(\frac{6s+2k_i}{\cancel{s^3+s^2}}\right)$$

$$= \frac{6s+2k_i}{s^3+5s^2+6s+2k_i}$$

6. (5 points) Suppose that the model of a system to be controlled is given in state-space control canonical form as

$$\dot{x} = \begin{pmatrix} 1 & 2 & -4 & -8 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & -1 & 2 \end{pmatrix} x.$$

$s^4 - s^3 - 2s^2 + 4s + 8$

What is the open-loop characteristic equation for the transfer function from the input u to the output y?

(a) $\Delta_{ol}(s) = -s^3 - 2s^2 + 4s + 8$

(b) $\Delta_{ol}(s) = s^4 + 8s^3 + 4s^2 - 2s - 1$

(c) $\Delta_{ol}(s) = s^4 + s^3 + 2s^2 + 4s + 8$

(d) $\Delta_{ol}(s) = s^4 + s^3 + 2s^2 - 4s - 8$

(e) None of the above.

7. (5 points) If the transfer function for a system is given by

$$H(s) = \frac{s^2 - 1}{s^3 + 3s^2 - 6},$$

what are the state space equations in controller canonic form?

$$A_c = \begin{bmatrix} -3 & 0 & 6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

(a) $\dot{x} = \begin{pmatrix} -3 & 0 & 6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$

$y = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} x$

(b) $\dot{x} = \begin{pmatrix} 1 & -3 & 0 & 6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u$

$y = \begin{pmatrix} 0 & 1 & 0 & -1 \end{pmatrix} x$

(c) $\dot{x} = \begin{pmatrix} -3 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} u$

$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x$

(d) $\dot{x} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} u$

$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x$

(e) None of the above.

8. (5 points) Suppose that the controllability matrix of a system is given by

$$\mathcal{C}_{\mathcal{A},\mathcal{B}} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 1 \end{pmatrix}.$$

$\det(\mathcal{C}_{AB}) = 2 + 2 + 2 - 2 - 2 - 2 = 0$

Same ... Bad!

which of the following is true?

(a) The system is not controllable.

(b) The system is controllable.

(c) Not enough information to decide.

Questions 9–11 refer to the following system:

$$\dot{x} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u \quad (1)$$

$$y = \begin{pmatrix} 0 & 3 \end{pmatrix} x. \quad (2)$$

9. (5 points) Suppose that the system to be controlled is given in state-space form by Equations (1)–(2). What is the controllability matrix?

(a) $\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$

(e) None of the above.

$$C_{AB} = \begin{bmatrix} B & AB \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

10. (5 points) Suppose that the system to be controlled is given in state-space form by Equations (1)–(2). If the desired closed loop poles are at $-2 \pm 3j$, what is $\boldsymbol{\alpha}$?

(a) $\boldsymbol{\alpha} = \begin{pmatrix} 1 & 4 & 13 \end{pmatrix}$

(b) $\boldsymbol{\alpha} = \begin{pmatrix} -4 & 13 \end{pmatrix}$

(c) $\boldsymbol{\alpha} = \begin{pmatrix} -4 & -13 \end{pmatrix}$

(d) $\boldsymbol{\alpha} = \begin{pmatrix} 4 & 13 \end{pmatrix}$

(e) None of the above.

$$\left.\begin{array}{l} P_1 = (-2+3j) \\ P_2 = (-2-3j) \end{array}\right\} = s^2 + 4s + 13$$

$$\alpha = \begin{bmatrix} 4 & 13 \end{bmatrix}$$

11. (5 points) Suppose that the system to be controlled is given in state-space form by Equations (1)–(2), what is $\mathcal{A}_A$?

(a) $\mathcal{A}_A = \begin{pmatrix} 1 & 1 & 0 & 1 \end{pmatrix}$

(b) $\mathcal{A}_A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

(c) $\mathcal{A}_A = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

(d) $\mathcal{A}_A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

(e) None of the above.

$$\det\underbrace{(sI - A)} = 0$$

$$\det\left(\begin{bmatrix} s-1 & 2 \\ 0 & s-1 \end{bmatrix}\right) = 0$$

$$s^2 - 2s + 1 = 0$$

$$A_A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

12. (5 points) Given the system:

$(s+1)(s)-4 \rightarrow s^2+s-4$

$a_A = [1 \quad -4]$

$$\dot{x} = \begin{pmatrix} -1 & 4 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u \tag{3}$$

$$y = \begin{pmatrix} 1 & -1 \end{pmatrix} x. \tag{4}$$

What are the gains K that place closed-loop poles at $-1 \pm 2j$? $\} \rightarrow s^2+2s+5$

$\alpha_A = [2 \quad 5]$

(a) $K = \begin{pmatrix} -1 & -9 \end{pmatrix}$
(b) $K = \begin{pmatrix} 1 & 9 \end{pmatrix}$
(c) $K = \begin{pmatrix} -3 & -1 \end{pmatrix}$
(d) $K = \begin{pmatrix} 3 & 1 \end{pmatrix}$
(e) None of the above.

$K = K_C = \alpha_A - a_A = [1 \quad 9]$

13. (5 points) Consider the system given by Equations (3) and (4). If the feedback gains have been selected as

$$K = \begin{pmatrix} 2 & 6 \end{pmatrix},$$

what are the poles of the closed-loop system?

(a) $s = \begin{pmatrix} 1 & 2 \end{pmatrix}$
(b) $s = \begin{pmatrix} -2 & -1 \end{pmatrix}$
(c) $s = \begin{pmatrix} -2.56 & 1.56 \end{pmatrix}$
(d) $s = \begin{pmatrix} -2.56 & -1.56 \end{pmatrix}$
(e) None of the above.

$C_{AB} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$A_A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$K = K_C \underbrace{A_A^{-1} C_{AB}^{-1}}_{I}$ so $K = K_C$

$\mathrm{eig}(A - BK) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

14. (5 points) Consider the system given by Equations (3) and (4). If the feedback gains have been selected as

$$K = \begin{pmatrix} 2 & 5 \end{pmatrix},$$

and the controller is given by

$$u = -Kx + k_r r,$$

what is the reference gain k_r that makes the DC-gain from r to y equal to one?

$K_r = \dfrac{-1}{C(A-BK)^{-1}B}$

$K_r = 0.5$

(a) $k_r = -1/3$
(b) $k_r = 1$
(c) $k_r = -1$
(d) $k_r = 1/2$
(e) None of the above.

15. (5 points) Given the Equations (3) and (4), which of the following is the observability matrix?

(a) $\begin{pmatrix} 1 & -1 \\ -2 & 4 \end{pmatrix}$
(b) $\begin{pmatrix} -1 & 2 \\ 1 & -4 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$
(d) $\begin{pmatrix} -1 & 4 \\ 1 & 0 \end{pmatrix}$
(e) None of the above.

$$O_{AC} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix}$$

16. (5 points) In addition to using it in the Ackermann formula to find the observer gain matrix "L", what is the significance of the observability matrix?

(a) ~~It is the~~ transpose of the controllability matrix.
(b) ~~It tells us if~~ we can move all of the the eigenvalues or poles for our closed-loop system. $C_{A,B}$
(c) It tells us if we can estimate all of the states with our given measurements. ✓
(d) ~~It tells us the~~ system type which tells us about the steady state error.
(e) None of the above.

$\det(sI - A) = s+5 \qquad A = [-5] \quad B = [1]$

$C_r = [1]$

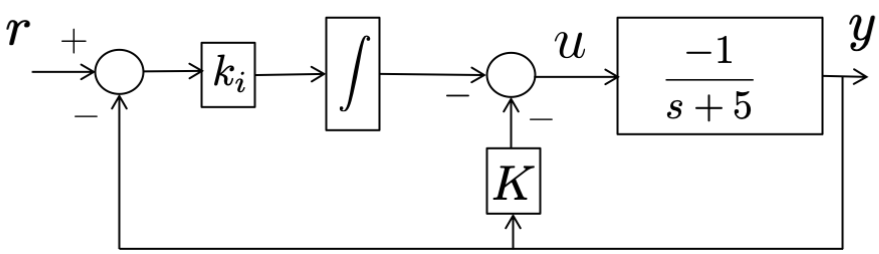
$P = \dfrac{-1}{s+5}$

$y(s+5) = u(-1)$

17. (5 points) Consider the system shown in Figure 3. The control law is given by

$$u = -Ky - k_i \int (r - y)\, dt.$$

Find K and k_i to place the closed loop pole at -1 and the integrator pole at -2.



CL.P. S.P.

$(s+1)(s+2)$

$\dot{x} = (A - BK)x$

Figure 3: System used for problems 15.

(a) $K = 2, k_i = 2$
(b) $K = 2, k_i = -2$
(c) $K = -2, k_i = 2$
(d) $K = -2, k_i = -2$
(e) None of the above.

$-(r-y)\dfrac{K_i}{s} - Ky = u$

$u\left(\dfrac{-1}{s+5}\right) = y$

$A' = \begin{bmatrix} -5 & 0 \\ 1 & 0 \end{bmatrix}$

$B' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} K & K_i \end{bmatrix}$

$\det(sI - (A' - B'K')) = (s+1)(s+2)$

$= s^2 + 3s + 2$

$a = [3 \quad 2]$

$\begin{bmatrix} -5 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} K & K_i \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -5-K & -K_i \\ 1 & 0 \end{bmatrix}$

$(s + (5+K))(s) + K_i = s^2 + 3s + 2$

$s^2 + (5+K)s + K_i = s^2 + 3s + 2$

$K = -2 \qquad K_i = 2$

18. (5 points) What is the purpose of integrator anti-windup?

   (a) ~~To ensure that~~ the integrator and differentiator do not cancel the effect of each other.
   (b) ~~To ensure~~ that the root locus is in the left-half plane.
   (c) To avoid integrating when the system actuators are saturated.
   (d) ~~To avoid pu~~shing the poles of the closed-loop system into the right-half plane.
   (e) ~~None of~~ the above.

19. (5 points) Using the Tustin approximation (or transform), find the discrete-time equation for the following transfer function (where "n" denotes a discrete time step):

$$U_I(s) = \frac{\sigma}{s} E(s)$$

$$S = \frac{2}{T_s}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

Which of the following is the correct discrete-time implementation?

$$U(s) = \frac{\sigma T_s (1 + z^{-1})}{2(1 - z^{-1})} E(s)$$

   (a) $u[n] = 2\,u[n+1] + \dfrac{\sigma T_s}{2}(e[n] - e[n+1])$
   (b) $u[n] = u[n-1] + \dfrac{\sigma T_s}{2}(e[n] + e[n-1])$
   (c) $u[n] = \dfrac{u[n-1]}{2} + \dfrac{\sigma T_s}{2}(e[n] + e[n-1])$
   (d) $u[n] = \dfrac{\sigma T_s}{2}(e[n] + e[n-1])$
   (e) None of the above.

$$U(s) - u(s) z^{-1} = \frac{\sigma T_s}{2}\left(E(s) + E(s) z^{-1}\right)$$

$$u[n] = u[n-1] + \frac{\sigma T_s}{2}\left(e[n] + e[n-1]\right)$$

20. (5 points) Which of the following is true when comparing full-state feedback control to PID control (without successive loop closure)?

(a) ~~Integral con~~trol is possible with PID, but not with full-state feedback. Poles

(b) ~~Full-state f~~eedback allows us to change the eigenvalues, which is not possible with PID control.

(c) We cannot use the dirty derivative to find any of our states for full-state feedback.

(d) Full-state feedback makes it easier to set system pole values directly compared to PID control.

(e) None of the above.