Controls Test #1 Notes

EL: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = F$

$$L = KE - PE = \underbrace{\left[\sum_{i=1}^{n} \frac{1}{2} m v^T v + \frac{1}{2}\omega^T J \omega\right]}_{KE} + \underbrace{\left[P_{gravity} + P_{springs}\right]}_{PE}$$

Getting KE

1) Find generalized coords
2) Define inertial coord frame
3) Find position vectors of each body
4) Take time der. to get velocity
5) Add rotational terms
6) Combine all bodies & linear & rot

Eq. Forms

State Variable: $\dot{x} = f(x, u)$

$y = g(x, u)$

State Space: $\dot{x} = Ax + Bu$

$y = Cx + Du$

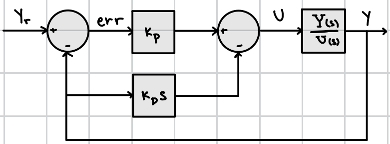
Jacobian Linearization

$$A = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}\right|_{\substack{x=x_0 \\ u=u_0}} \qquad B = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_n} \end{bmatrix}\right|_{\substack{x=x_0 \\ u=u_0}}$$

Feedback Linearization

$$\dot{x} = f(x, u_{Fl} + u_{ctrl}) \Rightarrow Ax + Bu_{ctrl}$$

Modified PD controller



$$Y(s) = \frac{b_0 K_P}{s^2 + (a_1 + b_0 K_D)s + (a_0 + b_0 K_P)} Y_r(s) \quad \text{real}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \quad \text{desired}$$

Desired Closed Loop

$t_r = \frac{2.2}{\omega_n}$ if $\zeta = 0.707$

$t_r = \frac{1}{2} \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Code

```python
import sympy as sp
from sympy import eye, sin, cos, diff, Matric, symbols, simplify
from sympy.physics.vector import dynamicsymbols
from sympy.physics.vector.printing import vlatex
from IPython.display import Math, display

t, m = symbols('t, m')
z = dynamicsymbols('z')
V = P.diff(t)
LHS = simplify((L.diff(q)).diff(t) - L.diff(q))
display(Math(vlatex(LHS)))

A = svf.jacobian(states)
B = svf.jacobian(inputs)

TF = simplify(C @ (s*I - A).inv() @ B + D)
```

$$s \mapsto \frac{2}{T_s}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \text{O.L.}$$

$$\left.\begin{aligned} \dot{x} &= (A - BK)x + Br \\ y &= Cx \end{aligned}\right\} \text{Full-State Feedback}$$

$\Delta_{OL} = \det(sI - A) = 0$

poles: eig(A)

$$Y(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} U(s)$$

$$A_c \triangleq \begin{pmatrix} -a_{n-1} & -a_{n-2} & \cdots & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \tag{11.14}$$

$$B_c \triangleq \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{11.15}$$

$$C_c \triangleq \begin{cases} \begin{pmatrix} 0 & \cdots & 0 & b_m & \cdots & b_1 & b_0 \end{pmatrix} & m < n \\ \begin{pmatrix} b_{n-1} - b_n a_{n-1}, & \cdots & b_1 - b_n a_1, & b_0 - b_n a_0 \end{pmatrix} & m = n \end{cases} \tag{11.16}$$

$$D \triangleq \begin{cases} 0 & m < n \\ b_n & m = n \end{cases}. \tag{11.17}$$

$$A_o = A_c^\top \qquad B_o = C_c^\top \qquad C_o = B_c^\top.$$

$$\dot{\hat{x}} = \underbrace{A\hat{x} + Bu}_{\text{predictor}} + \underbrace{L\,(y - C\hat{x})}_{\text{corrector}}.$$

$$L = \mathcal{O}_{A,C}^{-1} \mathcal{A}_A^{-\top} (\boldsymbol{\beta} - \mathbf{a}_A)^\top,$$

$$\mathcal{O}_{A,C} \triangleq \mathcal{C}_{A^\top,C^\top}^\top = \begin{pmatrix} C^\top & A^\top C^\top & (A^\top)^2 C^\top & \cdots & (A^\top)^{n-1} C^\top \end{pmatrix}^\top = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix},$$

$$\mathbf{a}_A = (a_{n-1}, a_{n-2}, \ldots, a_1, a_0) \text{ - open-loop}$$

$$\boldsymbol{\alpha} = (\alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_1, \alpha_0) \text{ - closed-loop}$$

$$K_c = (k_1, k_2, \ldots, k_{n-1}, k_n), \qquad K_c = \alpha - a_A$$

$$\mathcal{A}_A = \begin{pmatrix} 1 & a_{n-1} & a_{n-2} & \cdots & a_2 & a_1 \\ 0 & 1 & a_{n-1} & \vdots & a_3 & a_2 \\ 0 & 0 & 1 & \vdots & a_4 & a_3 \\ \vdots & & & \ddots & \vdots & \\ 0 & 0 & 0 & \cdots & 1 & a_{n-1} \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

$$\mathcal{C}_{A,B} = \begin{pmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{pmatrix},$$

$$P = \mathcal{C}_{A,B} \mathcal{A}_A.$$

$$K \triangleq (\boldsymbol{\alpha} - \mathbf{a}_A) \mathcal{A}_A^{-1} \mathcal{C}_{A,B}^{-1}.$$

Therefore, the DC-gain is equal to one if

$$k_r = -\frac{1}{C(A - BK)^{-1}B}.$$

$$\begin{aligned} u &= -(\boldsymbol{\alpha} - \mathbf{a}_A) P^{-1} x + \nu \\ &= -(\boldsymbol{\alpha} - \mathbf{a}_A) \mathcal{A}_A^{-1} \mathcal{C}_{A,B}^{-1} x + \nu \\ &= -Kx + \nu, \end{aligned}$$

$$\begin{pmatrix} \dot{x} \\ \dot{x}_I \end{pmatrix} = \begin{pmatrix} A & \mathbf{0} \\ -C_r & \mathbf{0} \end{pmatrix} \begin{pmatrix} x \\ x_I \end{pmatrix} + \begin{pmatrix} B \\ \mathbf{0} \end{pmatrix} u + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r,$$

$$\Delta_{obs}^d(s) = (s - q_1)(s - q_2) \cdots (s - q_n) = s^n + \beta_{n-1}s^{n-1} + \cdots + \beta_1 s + \beta_0,$$

and construct the row vector

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_{n-1}, & \beta_{n-2}, & \ldots, & \beta_1, & \beta_0 \end{pmatrix}.$$