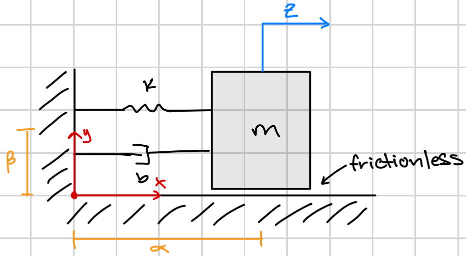
Carson Wynn
ME 431-Controls
HW #2

P.2a) Find KE of system

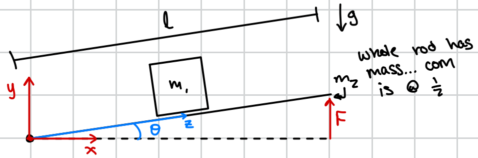


$q = [z]$

$$P = \begin{bmatrix} \alpha + z \\ \beta \\ 0 \end{bmatrix} \xrightarrow{\frac{d}{dt}} \begin{bmatrix} \dot z \\ 0 \\ 0 \end{bmatrix}$$

$$KE = \frac{1}{2} m \|V\| = \frac{1}{2} m \begin{bmatrix} \dot z & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot z \\ 0 \\ 0 \end{bmatrix} = \boxed{\frac{1}{2} m \dot z^2 = KE}$$

E.2a) Find KE of system



$$q = \begin{bmatrix} \theta \\ z \end{bmatrix}$$

$$P_1 = \begin{bmatrix} z\sin(\theta) \\ z\cos(\theta) \\ 0 \end{bmatrix} \xrightarrow{\frac{d}{dt}} \begin{bmatrix} \dot z \sin(\theta) + z\cos(\theta)\dot\theta \\ \dot z\cos(\theta) - z\sin(\theta)\dot\theta \\ 0 \end{bmatrix}$$

$$\begin{aligned} \|v_1\| &= \dot z^2 \sin(\theta)^2 + \cancel{2\dot z z \sin(\theta)\cos(\theta)} + z^2\cos(\theta)^2\dot\theta^2 \\ &+ \dot z^2 \cos(\theta)^2 - \cancel{2\dot z z \sin(\theta)\cos(\theta)} + z^2 \sin(\theta)^2 \dot\theta^2 \\ &= \dot z^2 + z^2\dot\theta^2 \end{aligned}$$

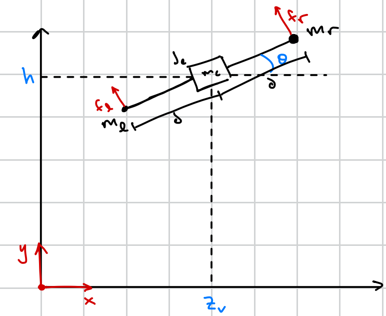
$$P_2 = \begin{bmatrix} \frac{1}{2} L \sin(\theta) \\ \frac{1}{2} L \cos(\theta) \\ 0 \end{bmatrix} \xrightarrow{\frac{d}{dt}} \begin{bmatrix} \frac{1}{2} L\cos(\theta)\dot\theta \\ -\frac{1}{2} L \sin(\theta)\dot\theta \\ 0 \end{bmatrix}$$

$$\|v_2\| = \frac{1}{4} L^2 \cos(\theta)^2 \dot\theta^2 + \frac{1}{4} L^2 \sin(\theta)^2 \dot\theta^2 = \frac{1}{4} L^2 \dot\theta^2$$

$$KE = \frac{1}{2} m_1 \|v_1\| + \frac{1}{2} m_2 \|v_2\| = \boxed{\frac{1}{2} m_1 \left(\dot z^2 + z^2 \dot\theta^2\right) + \frac{1}{8} m_2 L^2 \dot\theta^2 = KE}$$

F.2a on next page

F.2a) Find KE of system



$$m_r = m_l = m_s$$

$$q = \begin{bmatrix} h \\ z_v \\ \theta \end{bmatrix}$$

$$P_l = \begin{bmatrix} z_v + \alpha - \delta\cos(\theta) \\ h + \beta - \delta\sin(\theta) \\ 0 \end{bmatrix} \xrightarrow{\frac{\partial}{\partial t}} \begin{bmatrix} \dot{z}_v + \delta\sin(\theta)\dot{\theta} \\ \dot{h} - \delta\cos(\theta)\dot{\theta} \\ 0 \end{bmatrix}$$

$$\|v_l\| = \dot{z}_v^2 + 2\dot{z}_v\delta\sin(\theta)\dot{\theta} + \delta^2\sin(\theta)^2\dot{\theta}^2 + \dot{h}^2 - 2\dot{h}\delta\cos(\theta)\dot{\theta} + \delta^2\cos(\theta)^2\dot{\theta}^2$$
$$= \dot{z}_v^2 + \dot{h}^2 + \delta^2\dot{\theta}^2 + 2\delta\dot{\theta}\left(\dot{z}_v\sin(\theta) - \dot{h}\cos(\theta)\right)$$

$$P_c = \begin{bmatrix} z_v + \alpha \\ h + \beta \\ 0 \end{bmatrix} \; ? \xrightarrow{\frac{\partial}{\partial t}} \begin{bmatrix} \dot{z}_v \\ \dot{h} \\ 0 \end{bmatrix}$$

$$\|v_c\| = \dot{z}_v^2 + \dot{h}^2$$

$$P_r = \begin{bmatrix} z_v + \alpha + \delta\cos(\theta) \\ h + \beta + \delta\sin(\theta) \\ 0 \end{bmatrix} \xrightarrow{\frac{\partial}{\partial t}} \begin{bmatrix} \dot{z}_v - \delta\sin(\theta)\dot{\theta} \\ \dot{h} + \delta\cos(\theta)\dot{\theta} \\ 0 \end{bmatrix}$$

$$\|v_l\| = \dot{z}_v^2 - 2\dot{z}_v\delta\sin(\theta)\dot{\theta} + \delta^2\sin(\theta)^2\dot{\theta}^2 + \dot{h}^2 + 2\dot{h}\delta\cos(\theta)\dot{\theta} + \delta^2\cos(\theta)^2\dot{\theta}^2$$
$$= \dot{z}_v^2 + \dot{h}^2 + \delta^2\dot{\theta}^2 - 2\delta\dot{\theta}\left(\dot{z}_v\sin(\theta) - \dot{h}\cos(\theta)\right)$$

$$KE = \frac{1}{2}m_l\left(\dot{z}_v^2 + \dot{h}^2 + \delta^2\dot{\theta}^2 + 2\delta\dot{\theta}\left(\dot{z}_v\sin(\theta) - \dot{h}\cos(\theta)\right)\right) + \frac{1}{2}m_c\left(\dot{z}_v^2 + \dot{h}^2\right) + \frac{1}{2}m_r\left(\dot{z}_v^2 + \dot{h}^2 + \delta^2\dot{\theta}^2 - 2\delta\dot{\theta}\left(\dot{z}_v\sin(\theta) - \dot{h}\cos(\theta)\right)\right) + \frac{1}{2}J_c\dot{\theta}^2$$

$$\boxed{KE = \underbrace{m_s\left(\dot{z}_v^2 + \dot{h}^2 + \delta^2\dot{\theta}^2\right) + \frac{1}{2}m_c\left(\dot{z}_v^2 + \dot{h}^2\right)}_{\text{Translational Energy \& } m_r = m_l \text{ simplify}} + \underbrace{\frac{1}{2}J_c\dot{\theta}^2}_{\text{Rotational energy}}}$$