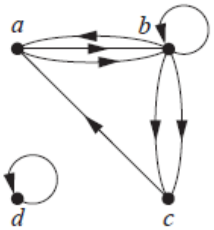
CS330 Homework 9[*]

Questions

1. Points = 4. What kind of graph can be used to model a highway system between major cities where
   a. There is an edge between the vertices representing cities if there is an interstate highway between them?
   b There is an edge between the vertices representing cities for each interstate highway between them?

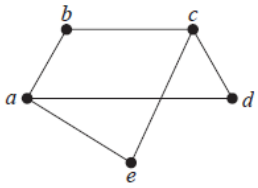
2. Points = 2. (a) Explain how graphs can be used to model electronic mail messages in a network. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed? (b) Describe a graph that models the electronic mail sent in a network in a particular week.

3. Points = 2. List the in-degrees and out-degrees of each vertex for the graph below. Give the sums of the in-degrees and the out-degrees (they should be equal and also equal to the number of edges in the graph).



4. Points = 2. In a Hollywood graph, actors are vertices; an edge between actors indicates they've been in one or more movies together. What does the degree of a vertex in the Hollywood graph represent? What does the neighborhood of a vertex represent? What do the isolated and pendant[†] vertices represent?

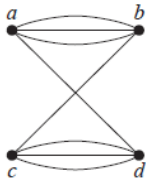
5. Points = 2. Is this graph bipartite? If it is, give a partition.



6. Points = 2. Is 6, 5, 4, 3, 2, 1 a possible degree sequence for a simple graph? If it is, draw the graph.

7. Points = 8.
   a. Draw a directed graph for this adjacency matrix:
   b. Give an adjacency matrix representation for this multigraph:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



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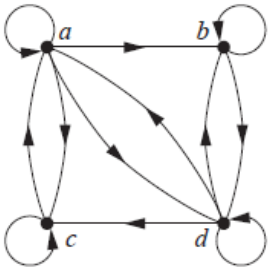
† A *pendant vertex* has a degree of 1.

c. Give an adjacency matrix for this graph

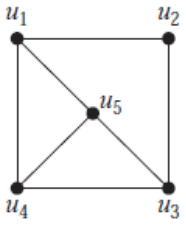


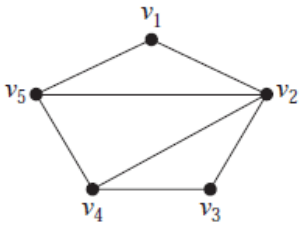
d. Draw a directed graph with the adjacency matrix below.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

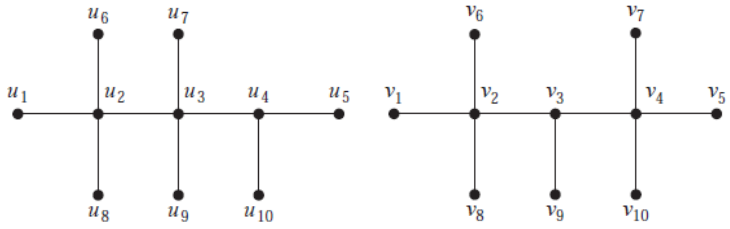
8. Points = 3. What is the sum of the entries in a row of the adjacency matrix for an undirected graph? For a directed graph?

9. Points = 3. Are these graphs isomorphic? If yes, give a correspondence of the vertices. If not, give a rigorous argument why not.

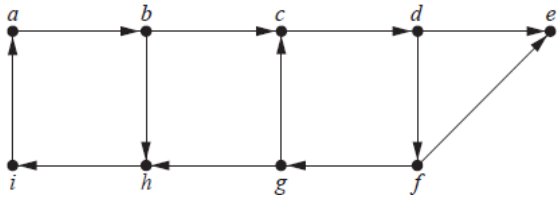




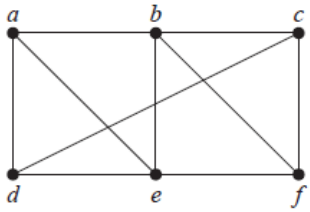
10. Points = 3. Repeat, for these two graphs.



11. Points = 3. In a directed graph, a *strongly-connected component* is a subgraph where for every pair of vertices v and w, there is a directed path from v to w and another from w to v. The subgraph should be *maximal* (you can't add another vertex without losing strong connectedness. Find the strongly-connected components of the following graph.



12. Points = 6. Consider this graph.



   a. Give its adjacency matrix representation, call it A.
   b. Calculate and give A^2. What is the value of $A^2[c, d]$; what does it value represent?
   c. Repeat for A^3 and $A^3[c, d]$.