CS330 Homework 6 *

Questions

1. 4 points:
   a. Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0s.
   b. What are the initial conditions?
   c. How many bit strings of length seven contain three consecutive 0s?
2. 4 points: A bus driver pays all tolls using only nickels and dimes, by throwing one coin at a time into the mechanical toll collector.
   a. Find a recurrence relation for the number of different ways the bus driver can pay a toll of n cents (where the order in which the coins are used matters).
   b. In how many different ways can the driver pay a toll of 45 cents?
3. 4 points: Suppose that there are $n = 2^k$ teams in an elimination tournament, where there are $n/2$ games in the first round, with the $n/2 = 2^{k-1}$ winners playing in the second round, and so on. Develop a recurrence relation for the number of rounds in the tournament.
4. 4 points: Solve the recurrence relation in number 3.
5. 4 points: Suppose that each person in a group of n people votes for exactly two people from a slate of candidates to fill two positions on a committee. The top two finishers both win positions as long as each receives more than $n/2$ votes.
   a. Write out (in pseudocode or words) a divide-and-conquer algorithm that determines whether the two candidates who received the most votes each received at least $n/2$ votes and, if so, determine who these two candidates are.
   b. Use the master theorem to give an $O(\ldots)$ estimate for the number of **comparisons** needed by the algorithm you devised in part (a).

   Hint: Assume that n is even and split the sequence of votes into two sequences, each with $n/2$ elements. A candidate cannot receive a majority of votes without receiving a majority of votes in at least one of the two halves.
6. 8 points: Find the solution to each of these recurrence relations with the given initial conditions. Use an iterative approach.
   a. $a_n = -a_{n-1}$, $a_0 = 5$
   b. $a_n = a_{n-1} + 3$, $a_0 = 1$
   c. $a_n = a_{n-1} - n$, $a_0 = 4$
   d. $a_n = 2\,a_{n-1} - 3$, $a_0 = -1$
7. 9 points: Solve these recurrence relations together with the initial conditions given.
   a. $a_n = a_{n-1} + 6\,a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = 6$
   b. $a_n = 7\,a_{n-1} - 10\,a_{n-2}$ for $n \geq 2$, $a_0 = 2$, $a_1 = 1$
   c. $a_{n-1} = 6\,a_{n-1} - 8\,a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 10$
8. 3 points: A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.
   a. Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.
   b. Find L_n if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2.

---

* Some material from MATH230 Dr. Robert Ellis IIT-AMAT, zyBooks Disc. Math., and Disc. Math. and its Appl., Rosen (7th Ed.) is included under the academic/nonprofit Fair Use exception of US Copyright Law. This document must NOT be distributed beyond participants in IIT's CS330.