CS330 Lecture Homework 1 — Answer Key*

1. Below, $p \equiv$ I bought a lottery ticket this week and $q \equiv$ I won the million-dollar jackpot
   a. $\neg p$ — I didn't buy a lottery ticket this week.
   b. $p \vee q$ — I bought a lottery ticket this week or I won the million-dollar jackpot.
   c. $p \to q$ — If I bought a lottery ticket this week, then I won the million-dollar jackpot.
   d. $p \wedge q$ — I bought a lottery ticket this week and I won the million-dollar jackpot.
   e. $p \leftrightarrow q$ — I bought a lottery ticket this week if and only if I won the million-dollar jackpot.
   f. $\neg p \to \neg q$ — If I didn't buy a lottery ticket this week, then I didn't win the million-dollar jackpot.
   g. $\neg p \wedge \neg q$ — I didn't buy a lottery ticket this week and I didn't win the million-dollar jackpot.
   h. $\neg p \vee (p \wedge q)$ — I didn't buy a lottery ticket this week, or, I bought a lottery ticket this week and won the million-dollar jackpot.

2. (Statements rewritten in if-then format)
   a. It is necessary to wash the boss's car to get promoted.
      If you got promoted, you must have washed the boss's car
   b. Winds from the south imply a spring thaw.
      If there is a wind from the south, then there is a spring thaw.
   c. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
      If you bought the computer less than a year ago, then the warranty is good.
   d. Willy gets caught whenever he cheats.
      If Willy cheats, then he gets caught.

3. Define the following formal versions of our statements:

   $U \to \neg A$ — Whenever the system software is being upgraded, users cannot access the file system
   $A \to S$ — If users can access the file system, then they can save new files
   $\neg S \to \neg U$ — If users cannot save new files, then the system software is not being upgraded

   These are all satisfied if $U = \neg A = S =$ true, so they are consistent

4. One way to show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent is to show that they are both equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$:

   $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ — conditional identity (a.k.a. definition of $\leftrightarrow$)

   $(p \to q) \wedge (q \to p)$
   $\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ — conditional identity (a.k.a. definition of $\to$)
   $\equiv (\neg p \wedge (\neg q \vee p)) \vee (q \wedge (\neg q \vee p))$ — distribute $\vee$ over $\wedge$ (twice)
   $\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p)$ — distribute $\wedge$ over $\vee$ (twice)
   $\equiv (\neg p \wedge \neg q) \vee F \vee F \vee (q \wedge p)$ — complement (a.k.a. excluded middle) (twice)
   $\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$ — identity (twice)

5. Formalize our statements as follows

   $B \to S$ — If I play baseball, then I am sore
   $S \to U$ — I use the swimming pool if I am sore
   $\neg U$ — I did not use the swimming pool
   B — I played baseball

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We can show that $\neg B$ follows from the first three statements, so claiming B is inconsistent:

1. $\neg U$ Premise
2. $S \to U$ Premise
3. $\neg S$ Modus tollens, 2, 1
4. $B \to S$ Premise
5. $\neg B$ Modus tollens, 4, 3

6. Below, x ranges over students in our class
   a. $\exists x\,(C(x) \wedge D(x) \wedge F(x))$ A student in your class has a cat, a dog, and a ferret
   b. $\forall x\,(C(x) \wedge D(x) \wedge F(x))$ All students in your class have a cat, a dog, and a ferret
   c. $\exists x\,(C(x) \wedge F(x) \wedge \neg D(x))$ Some student in your class has a cat and a ferret, but not a dog
   d. $\neg \exists x\,(C(x) \vee D(x) \vee F(x))$ No student in your class has a cat, a dog, or a ferret
   e. $(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$ For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet

7. Since variables range over the reals,
   a. $\exists x\,(x^2 = 2)$ is correct (x can be the positive or negative square root of 2)
   b. $\exists x\,(x^2 = -1)$ is incorrect (over the reals; if the domain included complex numbers, the statement would be true)
   c. $\forall x\,(x^2 + 2 \geq 1)$ is correct ($x^2 + 2 \geq 0 + 2 \geq 1$)
   d. $\forall x\,(x^2 \neq x)$ is incorrect ($x = 1$ is a counterexample)

8. Here are formal versions of our statements; x and y range over students at our school
   a. $\exists x\, \exists y\, Q(x, y)$ There is a student at your school who has been a contestant on a television quiz show
   b. $\neg \exists x\, \exists y\, Q(x, y)$ No student at your school has ever been a contestant on a television quiz show
   c. $\exists x\,(Q(x, \mathit{Jeopardy}) \wedge Q(x, \mathit{Wheel\ of\ Fortune}))$
   There is a student at your school who has been a contestant on *Jeopardy* and on *Wheel of Fortune*
   d. $\forall y\, \exists x\, Q(x, y)$ Every television quiz show has had a student from your school as a contestant
   e. $\exists x_1\, \exists x_2\,(x_1 \neq x_2 \wedge Q(x_1, \mathit{Jeopardy}) \wedge Q(x_2, \mathit{Jeopardy}))$
   At least two students from your school have been a contestant on *Jeopardy*

9a. Formalize our premises as:

$\mathit{Hockey} \to \mathit{Sore}$ If I play hockey, then I am sore the next day.
$\mathit{Sore} \to \mathit{Whirlpool}$ I use the whirlpool if I am sore
$\neg \mathit{Whirlpool}$ I did not use the whirlpool

We can show $\neg \mathit{Hockey}$:

1. $\neg \mathit{Whirlpool}$ Premise
2. $\mathit{Sore} \to \mathit{Whirlpool}$ Premise
3. $\neg \mathit{Sore}$ Modus tollens, 2, 1
4. $\mathit{Hockey} \to \mathit{Sore}$ Premise
5. $\neg \mathit{Hockey}$ Modus tollens, 4, 3

9b. Define statements to formalize our premises as below, where d ranges over the days of the week

$\forall d\,(W(d) \to S(d) \vee P(d))$ If I work, it is either sunny or partly sunny
$W(\mathit{Mon}) \vee W(\mathit{Fri})$ I worked last Monday or I worked last Friday
$\neg S(\text{Tue})$ It was not sunny on Tuesday.
$\neg P(\mathit{Fri})$ It was not partly sunny on Friday

We can show $W(Fri) \to S(Fri)$:

1.	$\forall\, d\, (W(d) \to S(d) \vee P(d))$	Premise
2.	$W(Fri) \to S(Fri) \vee P(Fri)$	Instantiation, 1
3.	$\neg P(Fri)$	Premise
4.	$W(Fri) \to S(Fri) \vee F$	Substitution 2, 3
5.	$W(Fri) \to S(Fri)$	Identity, 4

Also, we can show $W(\text{Tue}) \to P(\text{Tue})$:

6.	$W(\text{Tue}) \to S(\text{Tue}) \vee P(\text{Tue})$	Instantiation, 1
7.	$\neg S(\text{Tue})$	Premise
8.	$W(\text{Tue}) \to F \vee P(\text{Tue})$	Substitution, 6, 7
9.	$W(\text{Tue}) \to P(\text{Tue})$	Identity, 4

Note we can't say anything definitive about whether it was sunny or Friday or partly sunny on Tuesday; we can only conclude these conditionally.

9c. Formal versions of our premisses are as follows. For the domain for x, we'll use insects, spiders, and dragonflies.

$\forall\, x\, (In(x) \to L6(x))$	All insects have six legs
$\forall\, x\, (Df(x) \to In(x))$	Dragonflies are insects
$\forall\, x\, (Sp(x) \to \neg L6(x))$	Spiders do not have six legs
$\forall\, x\, (Sp(x) \to Ed(x))$	Spiders eat dragonflies

We can show that all dragonflies are not spiders:

1.	$\forall\, x\, (Df(x) \to In(x))$	Premise
2.	$\forall\, x\, (In(x) \to L6(x))$	Premise
3.	$\forall x\, (Sp(x) \to \neg L6(x))$	Premise
4.	y is an arbitrary element	Hypothesis
5.	$Df(y) \to In(y)$	Universal instantiation 1, 4 (see table 1.12.1)
6.	$In(y) \to L6(y)$	Universal instantiation 2, 4
7.	$Sp(y) \to \neg L6(y)$	Universal instantiation 3, 4
8.	$Df(y) \to L6(y)$	Hypothetical syllogism 5, 6 (see table 1.11.1)
9.	$\neg Df(y) \vee L6(y)$	Conditional identity 8 (see table 1.5.1)
10.	$\neg Sp(y) \vee \neg L6(y)$	Conditional identity 7
11.	$\neg Df(y) \vee \neg Sp(y)$	Resolution 9, 10
12.	$Df(y) \to \neg Sp(y)$	Conditional identity 11
13.	$\forall\, y\, (Df(y) \to \neg Sp(y))$	Universal generalization 12

We know spiders eat dragonflies but not its inverse (not spider → doesn't eat dragonflies), so we can't say anything about dragonflies eating other dragonflies.

9d. Formal versions of our premisses, where x ranges all people. (Or at least, all Simpson characters.)

$\forall\, x\, (S(x) \to A(x))$	Every student has an Internet account
$\neg A(Homer)$	Homer does not have an Internet account
$A(Maggie)$	Maggie has an Internet account

We can show *Homer* is not a student:

1.	$\forall\, x\, (S(x) \to A(x))$	Premise
2.	$S(Homer) \to A(Homer)$	Universal instantiation, 1
3.	$\neg A(Homer)$	Premise
4.	$\neg S(Homer)$	Modus tollens, 2, 3

We know that students have accounts, but we don't know its converse (having an account makes you a student). So we know Maggie has an account but don't know her student status.