CS 330 - DISCRETE STRUCTURES
HOMEWORK #4

1. Points=2

Let P(n) be the statement that $1^3 + 2^3 + \ldots + n^3 = (n(n+1)/2)^2$, for positive integer n. Prove by induction that $\forall n \in :\mathbb{N}$ (P(n).

2. Points=2

Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well defined, find a formula for $f(n)$ when n is a nonnegative integer and prove that your formula is valid.

a) $f(0) = 1$, $f(n) = -f(n-1)$ for $n \geq 1$

b) $f(0) = 1$, $f(1) = 0$, $f(2) = 2$, $f(n) = 2f(n-3)$ for $n \geq 3$

c) $f(0) = 0$, $f(1) = 1$, $f(n) = 2f(n+1)$ for $n \geq 2$

d) $f(0) = 0$, $f(1) = 1$, $f(n) = 2f(n-1)$ for $n \geq 1$

3. Points=2

Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

a) $a_n = 4n - 2$.

b) $a_n = 1 + (-1)^n$.

c) $a_n = n(n+1)$.

d) $a_n = n^2$.

4. Points=2

Give a recursive definition of

a) the set of odd positive integers.

5. Points=3

Give a recursive algorithm for finding the maximum of a finite set of integers, making use of the fact that the maximum of n integers is the larger of the last integer in the list and the maximum of the first $n-1$ integers in the list.

6. Points=3

Devise a recursive algorithm to find a^{2^n}, where a is a real number and n is a positive integer.

7. Points=3

Give a recursive algorithm for computing n*a ("n times a") using addition, where "n" is a positive integer and "a" is a real number.

8. Points=3

Write the algorithm and the loop invariant (for the outer loop) for the iterative version of Bubble Sort and prove all three cases (Initialization, Maintenance, Termination).