

Homework 2

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CS 330 - Discrete Structures

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February 4, 2017

Prove the following statements. Show and explain **ALL** your work. Unless otherwise specified, give direct proofs.

Problem 1. The sum of two even integers is always even.

Let a and b be integers.

Assume a and b are even.

Then by definition,

$a = 2m$ where m is an integer,

$b = 2n$ where n is an integer.

Then,

$$\begin{aligned} a + b &= (2m) + (2n) \\ &= 2(m + n) \end{aligned}$$

Therefore $a + b$ is even by definition.

Problem 2. State the contrapositive of (for all integers n , if n^2 is odd, then n is odd) and prove it.

Given : $\forall n \in \mathbb{Z} (n^2 \text{ odd} \rightarrow n \text{ odd})$

Contrapositive : $\forall n \in \mathbb{Z} (n \text{ even} \rightarrow n^2 \text{ even})$

Let n be an integer.

Assume n is even.

Then by definition, $n = 2k$ where $k \in \mathbb{Z}$.

So $n^2 = (2k)^2 = (4)(k \cdot k) = 2k \cdot 2k$.

And two even integers multiplied are always even.

Therefore n^2 is even.

Problem 3. Prove that if the sum of the digits of a 3-digit number n is divisible by 9, then n is divisible by 9.

TODO

Problem 4. Prove by contradiction that the product of two odd numbers is odd.

TODO

Problem 5. Prove that the product of two rational numbers is rational.

TODO