CS330 Lecture Homework 1 — Due Fri 1/27 (at end of Recitation)*

1/23: p.2

Written Problems [40 points total]

1. Points = 6. Let p and q be the propositions p: I bought a lottery ticket this week; q: I won the million-dollar jackpot. Express each of these propositions as an English sentence.
   a. $\neg p$ d. $p \wedge q$ g. $\neg p \wedge \neg q$
   b. $p \vee q$ e. $p \leftrightarrow q$ h. $\neg p \vee (p \wedge q)$
   c. $p \rightarrow q$ f. $\neg p \rightarrow \neg q$

2. Points = 4. Write each of these statements in the form "if p, then q" in English. [Hint: Refer to the list of common ways to express conditional statements provided in the text.]
   a. It is necessary to wash the boss's car to get promoted.
   b. Winds from the south imply a spring thaw
   c. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
   d. Willy gets caught whenever he cheats.

3. Points = 2. Are these system specifications consistent?
   "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

4. Points = 3. Without using truth tables, show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent.

5. Points = 3. Determine if the following argument is correct or not and show in detail why.

   "If I play baseball, then I am sore. I use the swimming pool if I am sore. I did not use the swimming pool. Therefore, I did play baseball".

6. Points = 5. Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement "x has a dog", and $F(x)$ be the statement "x has a ferret"†. Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
   a. A student in your class has a cat, a dog, and a ferret.
   b. All students in your class have a cat, a dog, and a ferret.
   c. Some student in your class has a cat and a ferret, but not a dog.
   d. No student in your class has a cat, a dog, or a ferret.
   e. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

7. Points = 4. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
   a. $\exists x\,(x^2 = 2)$ b. $\exists x\,(x^2 = -1)$ c. $\forall x\,(x^2 + 2 \geq 1)$ d. $\forall x\,(x^2 \neq x)$

8. Points = 5. Let $Q(x, y)$ be the statement "student x has been a contestant on quiz show y." Express each of these sentences in terms of $Q(x, y)$, quantifiers, and logical connectives where the domain for x consists of all students at your school and the domain y consists of all quiz shows on television.
   a. There is a student at your school who has been a contestant on a television quiz show.
   b. No student at your school has ever been a contestant on a television quiz show.
   c. There is a student at your school who has been a contestant on *Jeopardy* and on *Wheel of Fortune*.
   d. Every television quiz show has had a student from your school as a contestant.
   e. At least two students from your school have been a contestant on *Jeopardy*.

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† By the way, a great name for a ferret is "Bueller".

9. Points = 8. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.[‡]

   a. "If I play hockey, then I am sore the next day."
   " I use the whirlpool if I am sore."
   " I did not use the whirlpool."

   b. " If I work, it is either sunny or partly sunny."
   " I worked last Monday or I worked last Friday."
   " It was not sunny on Tuesday."
   " It was not partly sunny on Friday."

   c. "All insects have six legs [1/23]."
   "Dragonflies are insects."
   "Spiders do not have six legs.
   "Spiders eat dragonflies."

   d. " Every student has an Internet account."
   " Homer does not have an Internet account."
   " Maggie has an Internet account."

[‡] On the topic of finding relevant conclusions, there's a joke about certain research papers having the form "*X* is a good language. Object-oriented languages are good. therefore, *X* is object-oriented." [This reasoning is unsound, by the way — try "*X* is a mammal; dogs are mammals; therefore *X* is a dog."]