Auditing Course Material

Part 25 of 61 (Chapters 2401-2500)

17. Illustrations

A firm has EBITA of Rs 30 lakh and a 40% tax rate. It is able to borrow at an interest rate of 14%, whereas its equity capitalization rate in the absence of borrowing is 18%. The earnings of the company are not expected to grow, and all earnings are paid out to shareholders in the form of dividends. In the presence of corporate but no personal taxes:

(i) what is the value of the firm in an M&M world with no financial leverage?

(ii) what is the value of the firm With Rs 40 lakh in debt?

Solution:

No Leverage

EBITA	30,00,000
INTEREST	0
EBT	30,00,000
Taxes	12,00,000
EAT	18,00,000

$$\text{VALUE OF FIRM} = \frac{EAT}{Ke}$$

$$= \frac{18,00,000}{0.18} = 100,00,000$$

Rs 40 Lakh Debt

$$V_L = V_U + \text{Tax Shield}$$

$$= 100,00,000 + 0.40 \times 40,00,000$$

$$= 116,00,000$$

17. Illustrations

A firm has Rs 1,000 in debt at 10 percent interest. The expected annual net operating income (NOI or EBIT) figure is Rs 1,000, and that the overall capitalization rate (WACC) is 15%. Compute the required return on equity and the value of firm? Suppose that the firm increases the amount of debt from Rs 1,000 to Rs 3,000 and uses the proceeds of the debt issue to repurchase common stock. What will be its impact on the required return on equity and the value of firm ? Consider Net Operating Income approach for your analysis.

Solution:

EBIT = Rs 1000 WACC = 15%

VALUE OF FIRM = $\frac{1000}{0.15}$ = Rs 6667

WHEN DEBT IS 1000

Debt = 10% of 1000 = Rs 100

EAI = 1000 - 100 = Rs 900

EQUITY VALUE = 6667 - 1000 = Rs 5667

$Ke = \frac{900}{5667} = 15.88\%$

WHEN DEBT IS 3000

Debt = 10% of 3000 = Rs 300

EAI = 1000 - 300 = Rs 700

EQUITY VALUE = 6667 - 3000 = Rs 3667

$Ke = \frac{700}{3667} = 19.09\%$

Ke Increases

17. Illustrations

The XYZ firm's EBIT Rs 5,00,000. The company has 10%, RS 20 lakh debentures. The equity capitalization rate is 16%. Compute the followings:

(i) Market value of equity and value of firm

(ii) Overall cost of capital

Solution:

EBIT = Rs. 5,00,000

Interest on debentures = 10% of 20,00,000 = Rs. 2,00,000

EAI = 5,00,000 − 2,00,000 = Rs. 3,00,000 ← Earning for Equity holders

Ke = 16%

Value of Equity $= \frac{3,00,000}{0.16} = 18,75,000$

Value of Debt = 20,00,000

Value of Firm = 18,75,000 + 20,00,000 = 38,75,000

WACC $= \frac{5,00,000}{38,75,000} = 12.90\%$

17. Illustrations

Firm ABC has an EBIT of Rs 1,00,000. The company makes use of both the debt and equity capital. The firm has 10% debentures of Rs 5,00,000 and the firm’s equity capitalization rate is 15%. Compute the followings:
(i) Total value of the firm
(ii) Overall cost of capital

Solution:

EBIT = 1,00,000 , Ke = 15%

Interest on Debentures = 10% of 5,00,000 = Rs. 50,000

EAI = 1,00,000 − 50,000 = Rs. 50,000

Value of Equity = $\frac{50,000}{0.15}$ = Rs. 3,33,333

Value of Debt = Rs. 5,00,000

Value of Firm = 5,00,000 + 3,33,333 = Rs. 8,33,333

WACC = $\frac{100,000}{8,33,333}$ = 12.00%

17. Illustrations

A company has an expected EBIT of Rs 57,000 in perpetuity and a tax rate of 35%. The firm has Rs 90,000 in outstanding debt at an interest rate of 8%, and its unlevered cost of capital is 15 percent. What is the value of the firm according to MM Proposition I with taxes? Should the company change its debt–equity ratio if the goal is to maximize the value of the firm? Explain.

Solution:

$$V_L = \underbrace{V_U}_{\frac{EBIT(1-t)}{K_o}} + \underbrace{\text{DEBT} \times \text{TAX RATE}}_{0.35 \times 90,000}$$

$$= \frac{57000(1-0.35)}{0.15} + 0.35 \times 90,000$$

$$= 2,47,000 + 31,50,00 = \boxed{2,78,500}$$

Applying M&M Proposition I with taxes, the firm has increased its value by issuing debt. As long as M&M Proposition I holds, that is, there are no bankruptcy costs and so forth, then the company should continue to increase its debt/equity ratio to maximize the value of the firm.

17. Illustrations

A company expects an EBIT of Rs 19,750 every year forever. The company currently has no debt, and its cost of equity is 15 percent.

(i) What is the current value of the company?

(ii) Suppose the company can borrow at 10 percent. If the corporate tax rate is 35 percent, what will the value of the firm be if the company takes on debt equal to 50 percent of its unlevered value?

(iii) What will the value of the firm be if the company takes on debt equal to 50 percent of its levered value?

Solution:

$$V_u = \frac{EBIT(1-t)}{K_o} = \frac{19750(1-0.35)}{0.15}$$

$$= 85,583.33$$

$$V_L = V_u + t \times Debt$$

$$= 85,583.33 + 0.35 \times 50\% \text{ of } V_u$$

$$= 85,583.33 + 0.35 \times 0.50 \times 85,583.33$$

$$= 1,00,562.42$$

$$V_L = V_u + t \times Debt$$

$$= 85,583.33 + 0.35 \times 50\% \text{ of } V_L$$

$$= 85,583.33 + 0.35 \times 0.50 \times V_L$$

$$\Rightarrow V_L = 1,31,666.67$$

17. Illustrations

ABC Ltd, an all equity financed company is considering the repurchase of Rs 275 lakhs equity shares and to replace it with 15% debentures of the same amount. Current market value of the company is Rs 1,750 lakhs with its cost of capital of 20%. The company's Earnings before Interest and Taxes (EBIT) are expected to remain constant in future years. The company also has a policy of distributing its entire earnings as dividend.
Assuming the corporate tax rate as 30%, you are required to Compute the impact on the following on account of the change in the capital structure as per Modigliani and Miller (MM) Approach:
(i) Market value of the company
(ii) Overall Cost of capital
(iii) Cost of equity

Solution:

$EBIT = x$

$Taxes = 0.30x$

$EAT = x - 0.30x$
$= 0.70x$

$$\frac{EAT}{MV \text{ of firm}} = K_o$$

$$\frac{0.70x}{1750} = 0.20$$

$\implies x = $ Rs 500 lakh

$V_L = V_u + t \times Debt$

$= 1750 + 0.30 \times 275$

$= 1832.5$

$V_u = 1750$ $\quad$ $V_L = 1832.5$

$EBIT = 500$

Interest $= 15\%$ of 275
$= 41.25$

$EBT = 458.75$

Taxes @ 30% $= 137.63$

$EAT = 321.12$

MARKET VALUE OF FIRM = MARKET VALUE OF DEBT + MARKET VALUE OF EQUITY

$1832.50 = 275 + x$

$x = 1557.50$

$Ke = \frac{321.12}{1557.50} = 0.2062$ **20.62%**

$kd = 15(1-0.30)$
$= 0.1050$ 10.50%

$WACC = \frac{1557.50}{1832.50} \times 20.62 + \frac{275}{1832.50} \times 10.50 =$ **19.11%**

ANOTHER APPROACH OF FINDING Ke

$Ke = Ko + \frac{D}{E}(Ko - Kd)$

$= 0.20 + \frac{275}{1557.50}(0.20 - 0.1050)$

$= 0.20 + 0.016 = 0.216$ **21.6%**

ANOTHER APPROACH OF WACC

$WACC = \frac{EBITA(1-t)}{Value\ of\ Firm} = \frac{500(1-0.30)}{1832.50} =$ **19.09%**

17. Illustrations

Locomotive Corporation is planning to repurchase part of its common stock by issuing corporate debt. As a result, the firm's debt–equity ratio is expected to rise from 35 percent to 50 percent. The firm currently has Rs 36 lakh worth of debt outstanding. The cost of this debt is 8 percent per year. Locomotive expects to have an EBIT of Rs 13.5 lakh per year in perpetuity. Locomotive pays no taxes.
(i) What is the market value of Locomotive Corporation before and after the repurchase announcement?
(ii) what is the expected return on the firm's equity before the announcement of the stock repurchase plan?
(iii) What is the expected return on the equity of an otherwise identical all-equity firm?
(iv) What is the expected return on the firm's equity after the announcement of the stock repurchase plan?

Solution:

(i) What is the market value of Locomotive Corporation before and after the repurchase announcement?

$$\text{DEBT EQUITY RATIO} = \frac{D}{E}$$

$$0.35 = \frac{36,00,000}{E}$$

$$E = 102,85,714$$

MARKET VALUE OF FIRM

$$= D + E$$
$$= 36,00,000 + 102,85,714$$
$$= 138,85,714$$

According to MM Proposition I without taxes, changes in a firm's capital structure have no effect on the overall value of the firm. Therefore, the value of the firm will not change after the announcement of the stock repurchase plan.

(ii) what is the expected return on the firm's equity before the announcement of the stock repurchase plan?

EBITA	13,50,000	
INTEREST @ 8%	2,88,000	(8% of 36,00,000)
EAI	10,62,000	

$$Ke = \frac{10,62,000}{102,85,714} = 0.1033 \quad 10.33\%$$

(iii) What is the expected return on the equity of an otherwise identical all-equity firm?

$$K_e = K_o + \frac{D}{E}(K_o - K_d)$$

$$0.1033 = K_o + 0.35(K_o - 0.08)$$

$$\Rightarrow K_o = 0.0972 \quad 9.72\%$$

(iv) What is the expected return on the firm's equity after the announcement of the stock repurchase plan?

$$K_e = K_o + \frac{D}{E}(K_o - K_d)$$

$$= 0.1033 + 0.50(0.1033 - 0.08)$$

$$= 0.1058 \quad 10.58\%$$

17. Illustrations

A company has equity with a market value of Rs 230 lakh and debt with a market value of Rs 70 lakh. Treasury bills that mature in one year yield 5% percent per year, and the expected return on the market portfolio is 12 percent. The beta of company's equity is 1.15. The firm pays no taxes.

(a) What is company's debt–equity ratio?

(b) What is the firm's weighted average cost of capital?

(c) What is the cost of capital for an otherwise identical all-equity firm?

Solution:

$$\text{DEBT EQUITY RATIO} = \frac{\text{MV of Debt}}{\text{MV of Equity}}$$

$$= \frac{70,00,000}{230,00,000} = \frac{7}{23} = 0.30$$

USING CAPM for Ke

$$r_s(ke) = r_f + \beta(r_m - r_f)$$

$$= 0.05 + 1.15(0.12 - 0.05) = 0.1305$$

$$13.05\%$$

An assumption of the Modigliani-Miller theorem is that the company debt is risk-free, so we can use the Treasury bill rate as the cost of debt for the company.

Cost of Debt = Risk free return

$$Kd = r_f = 0.05$$

$$WACC = \frac{70,00,000}{70,00,000 + 230,00,000} \times 0.05 + \frac{230,00,000}{70,00,000 + 230,00,000} \times 0.1305$$

$$= 0.1117 \quad 11.17\%$$

$$k_e = k_o + \frac{D}{E}(k_o - k_d)$$

$$0.1305 = k_o + 0.30(k_o - 0.05)$$

$$\Rightarrow k_o = 0.1117 \quad 11.17\%$$

Above result is consistent with Modigliani-Miller's proposition that, in the absence of taxes, the cost of capital for an all-equity firm is equal to the weighted average cost of capital of an otherwise identical levered firm.

17. Illustrations

Company A and Company B are identical except for capital structures. Company A has 50 percent debt and 50 percent equity financing, whereas Company B has 20 percent debt and 80 percent equity financing. The borrowing rate for both companies is 13 percent in a no-tax world, and capital markets are assumed to be perfect. The earnings of both companies are not expected to grow, and all earnings are paid out to shareholders in the form of dividends.

(a) If you own 2 percent of the common stock of Company A, what is your return if the company has net operating income of Rs 3,60,000 and the overall capitalization rate of the company is 18 percent? What is the implied equity capitalization rate?

(b) Company B has the same net operating income as Company A. What is the implied equity capitalization rate of Company B? Why does it differ from Company A?

Solution:

FIRM A

$$\text{VALUE OF FIRM} = \frac{\text{EBITA}}{\text{WACC}} = \frac{3,60,000}{0.18} = 20,00,000$$

Since 50% Debt, Value of Debt = 10,00,000

Value of Equity = 10,00,000

EBITA	3,60,000
Interest	1,30,000 (13% of 10,00,000)
EAI	2,30,000

SINCE YOU OWN 2%

$$\text{YOUR EARNINGS} = \frac{2}{100} \times 2,30,000 = 4,600$$

$$Ke = \frac{2,30,000}{10,00,000} = 23\%$$

FIRM B

$$\text{VALUE OF FIRM} = \frac{\text{EBITA}}{\text{WACC}} = \frac{3,60,000}{0.18} = 20,00,000$$

Since Debt is 20%, Value of Debt = 20% of 20,00,000 = 4,00,000

Value of Equity = 20,00,000 − 4,00,000 = 16,00,000

EBITA	3,60,000
Interest	52,000 (13% of 4,00,000)
EAI	3,08,000

$$Ke = \frac{3,08,000}{16,00,000} = 19.25\%$$

We see that Ke decreases with decrease in $\frac{D}{E}$ from $\frac{50}{50}$ to $\frac{20}{80}$.

17. Illustrations

ABC Co. and XYZ Co. are identical firms in all respects except for their capital structure. ABC is all equity financed with Rs 7,50,000 in stock. XYZ uses both stock and perpetual debt; its stock is worth Rs 3,75,000 and the interest rate on its debt is 8 percent. Both firms expect EBIT to be Rs 86,000. Ignore taxes.
Rohan owns Rs 30,000 worth of XYZ’s stock. Show how Richard could generate exactly the same cash flows and rate of return by investing in ABC and using homemade leverage.

Solution:

XYZ FIRM

EBITA 86,000

INTEREST @ 8% 30,000

EAI 56,000

$$Ke = \frac{56,000}{3,75,000} = 14.93\%$$

TOTAL EARNINGS RECEIVED FROM XYZ = 14.93% of 30,000
= Rs 4480

ABC FIRM

EBITA 86,000

$$Ko = \frac{86,000}{7,50,000} = 11.47\%$$

Since Debt : Equity of XYZ is 50:50

→ Borrow additional 30,000

Interest on this 30,000 $= \frac{8}{100} \times 30,000$

$= 2400$

Total to be invested in ABC = 30 000 + 30 000

= 60,000

EARNINGS FROM 60,000 = 11.47% of 60,000

= 6882

INTEREST TO BE PAID = 2400

NET EARNINGS = 6882 - 2400

= 4482 $\cong$ 4480

THUS NET EARNINGS ARE SAME WITH HOMEMADE LEVERAGE

17. Illustrations

Following data is available in respect of two companies having same business risk:
Capital employed = Rs 2,00,000, EBIT = Rs 30,000 and Ke = 12.5%

	Levered Firm	Unlevered Firm
Debt@10%	1,00,000	Nil
Equity	1,00,000	2,00,000

An investor is holding 15% shares in levered company.
Explain the process of homemade leverage (arbitrage) and show the increase in annual earnings of investor if he switches his holding from Levered firm to Unlevered firm.

Solution:

	Levered Firm	Unlevered Firm
Debt@10%	1,00,000	Nil
Equity	1,00,000	2,00,000
EBIT	30,000	30,000
Interest at 10%	− 10,000	−
EAI	20,000	30,000
Value of Equity $\left(\frac{EAI}{Ke}\right)$	1,60,000	2,40,000
Value of Debt	1,00,000	−
Value of Firm	2,60,000	2,40,000

Ke = 12.5%

SINCE 2,60,000 > 2,40,000
Sell in Levered → Buy in Unlevered.

Sell 15% in levered for, 15% of 1,60,000 = Rs 24000
Create homemade leverage by borrowing 15% of 1,00,000
= Rs 15,000

TOTAL TO BE INVESTED IN UNLEVERED FIRM: 24000 + 15000 = 39000

Income from Unlevered firm = 12.5% of 39,000

= Rs 4875

Interest on borrowed money = 10% of 15,000

= Rs 1500

Net Income from Unlevered = 4875 - 1500 = Rs 3375

Net Income from Levered = 12.5% of 24000 = Rs 3000

ADDITIONAL INCOME = 3375 - 3000

= Rs 375

17. Illustrations

Following data is available in respect of two companies having same business risk:

Capital employed = Rs 2,00,000

EBIT = Rs 30,000

Cost of Equity Capital for Unlevered is 12.5% and for Levered is 20%.

	Levered Firm	Unlevered Firm
Debt@10%	1,00,000	Nil
Equity	1,00,000	2,00,000

An investor is holding 15% shares in Unlevered company.

Explain the process of homemade leverage (arbitrage) and show the increase in annual earnings of investor if he switches his holding from Unlevered firm to Levered firm.

Solution:

	Levered Firm	Unlevered Firm
Debt@10%	1,00,000	Nil
Equity	1,00,000	2,00,000
EBIT	30,000	30,000
Debt Interest	− 10,000	−
EAI	20,000	30,000
ke	20%	12.5%
Value of Equity $\left(\frac{EAI}{ke}\right)$	1,00,000	2,40,000
Value of Debt	1,00,000	−
Value of Firm	2,00,000	2,40,000

$2,00,000 < 2,40,000 \Rightarrow$ Sell Unlevered

→ Buy levered

Sell 15% in Unlevered for , 15% of 2,40,000 = Rs 36,000

Create home made leverage by Lending 50% of 36,000 = Rs 18,000

Buy shares of levered company for 36000 − 18000
= Rs 18,000

Income from Levered = 20% of 18,000 = 3600

Interest received on money = 10% of 18,000 = Rs 1800

Total Income from levered = 3600 + 1800 = 5400

Income from Unlevered = 12.5% of 36,000 = Rs 4500

ADDITIONAL INCOME = 5400 − 4500 = Rs 900

17. Illustrations

A company has no debt but can borrow at 8 percent. The firm's WACC is currently 11 percent, and the tax rate is 35 percent.
(a) What is company's cost of equity?
(b) If the firm converts to 25 percent debt, what will its cost of equity be?
(c) What is company's WACC in part (b)?

Solution:

(a) What is company's cost of equity?

SINCE ZERO DEBT

$K_0 = WACC = 11\%$

(b) If the firm converts to 25 percent debt, what will its cost of equity be?

25% Debt $\Rightarrow \frac{D}{E} = \frac{0.25}{0.75}$

$$Ke = K_0 + \frac{D}{E}(K_0 - Kd)(1-t)$$

$$= 0.11 + \frac{0.25}{0.75}(0.11 - 0.08)(1 - 0.35)$$

$$= 0.1165 \quad 11.65\%$$

(c) What is company's WACC in part (b)?

WACC with 25% Debt

$$WACC = 0.75 \times 0.1165 + 0.25 \times 0.08 \times (1 - 0.35)$$

$$= 0.1004 \quad 10.04\%$$

18. Signaling

Signaling refers to how investors interpret and react to news of a company's debt levels.

When a company **increases its debt**, it may signal to the market that the company is confident in its ability to generate future cash flows and repay its obligations. This confidence can lead investors to perceive the company as more valuable, potentially resulting in a higher share price.

Conversely, if a company **decreases its debt**, it may signal financial distress or an inability to pay interest, causing investors to perceive the company as less valuable. This can lead to a decrease in the company's share price.

In essence, managers signal information to the market through changes in leverage, with debt levels serving as a signal of the firm's value. This signaling effect influences investor perceptions and can impact the company's stock price.

Companies frequently adjust their debt levels through **exchange offers**, which come in two forms.

The first type permits shareholders to swap some of their stock for debt, thus boosting leverage. Conversely, the second type allows bondholders to exchange a portion of their debt for stock, thereby reducing leverage. Notably, stock prices typically surge significantly upon the announcement of an exchange offer that increases leverage. Conversely, a substantial decline in stock price accompanies the announcement of an offer that decreases leverage.

At times, managers may attempt to deceive investors by increasing debt levels, thereby inflating share prices, even if the company's existing debt level is already optimal.

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19. Trading on Equity

Trading on equity refers to using fixed-cost modes of financing, such as debt or preferred stock, to increase the earnings of stockholders.

Trading on Thin Equity

This term refers to a situation where a company has a relatively small amount of equity capital compared to its total capital structure. In other words, the equity portion of the company's capital is thin or minimal.

When a company trades on thin equity, it means that it relies heavily on debt or other fixed-cost financing methods to fund its operations or investments. This amplifies the effect of financial leverage.

The potential benefit of trading on thin equity is that it can magnify returns to equity shareholders when the return on assets (ROA) exceeds the cost of debt or preferred stock. However, it also increases the company's financial risk because the higher the leverage, the higher the financial risk.

Trading on Thick Equity

Conversely, trading on thick equity refers to a situation where a company has a relatively large amount of equity capital compared to its total capital structure.

When a company trades on thick equity, it means that it relies less on debt and more on equity financing to fund its operations or investments.

While trading on thick equity may reduce financial risk because of lower leverage, it may also limit the potential returns to equity shareholders compared to trading on thin equity. This is because there is less magnification of returns through leverage.

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20. FRICTO framework

In analyzing an appropriate capital structure, the FRICTO framework encompasses six key factors that businesses consider when determining their optimal mix of debt and equity financing:

1. Flexibility

Flexibility refers to the ability of a company to adjust its capital structure over time in response to changing market conditions, business needs, or strategic objectives. A capital structure that offers flexibility allows the company to adapt to unforeseen events, seize growth opportunities, or manage financial distress more effectively. For example, having access to various financing options such as debt facilities, equity issuance, or convertible securities provides flexibility in raising capital when needed.

2. Risk

Risk considerations are paramount in determining capital structure. A company must assess its risk tolerance and the impact of different financing alternatives on its overall risk profile. While debt financing offers tax benefits and can amplify returns, it also increases financial leverage and exposes the company to higher levels of financial risk, including bankruptcy risk and liquidity risk. Equity financing, on the other hand, dilutes ownership but can enhance financial flexibility and reduce financial distress risk. Balancing risk and return is crucial in designing an optimal capital structure.

3. Income

The cost of capital plays a significant role in capital structure decisions. A company evaluates the cost of debt, equity, and other financing options to minimize its overall cost of capital and maximize shareholder value. Debt financing typically offers lower interest rates than equity financing due to its priority claim on company assets and tax-deductible interest payments. However, excessive reliance on debt may increase financial risk and raise the cost of capital through higher borrowing costs or credit rating downgrades. Striking a balance between debt and equity to minimize the weighted average cost of capital (WACC) is essential for maximizing income or profitability.

4. Control

Control considerations involve assessing the impact of capital structure decisions on ownership and decision-making rights within the company. Debt financing does not dilute ownership but entails contractual obligations, such as interest payments and debt covenants, which may restrict managerial autonomy and shareholder control. Equity financing, on the other hand, dilutes ownership but preserves decision-making authority and flexibility in dividend payments and corporate governance. Companies must weigh the trade-offs between maintaining control and accessing external capital markets when structuring their capital.

5. Timing

Timing considerations focus on the timing and sequencing of capital structure decisions in alignment with the company's growth trajectory, investment opportunities, and market conditions. Companies may strategically time equity issuance or debt refinancing to capitalize on favorable market conditions, investor sentiment, or changes in interest rates. Additionally, companies may consider the maturity and amortization schedules of debt instruments to match cash flows from investment projects or asset disposals. Effective timing of capital structure adjustments enhances financial flexibility and minimizes financing costs.

6. Others

The "others" category encompasses various qualitative and quantitative factors that influence capital structure decisions but may not fit neatly into the other categories. These factors include regulatory constraints, market perceptions, industry norms, corporate culture, and stakeholder preferences. For example, regulatory requirements may limit the amount of debt a company can issue relative to its equity capital, while market perceptions of financial stability or growth prospects may affect investor demand for different types of securities. Understanding and addressing these miscellaneous factors are essential for crafting a holistic and effective capital structure strategy.

21. Summary

Factors Influencing Capital Structure Choices are listed below

1. Corporate Taxes

- Corporate taxes play a significant role in capital structure decisions due to the tax deductibility of interest payments on debt. Firms with higher corporate tax rates may benefit more from the tax shield provided by debt, making debt financing more attractive.
- The tax shield refers to the reduction in taxable income resulting from deductible interest expenses. Therefore, firms often seek to maximize the tax shield by leveraging debt to reduce their overall tax burden.

2. Financial Distress Costs

- Financial distress costs refer to the potential costs incurred by a firm when facing financial difficulties, such as bankruptcy or restructuring expenses. These costs can include legal fees, lost business opportunities, damage to reputation, and potential loss of value for shareholders and creditors.
- Firms must weigh the benefits of debt financing against the potential costs of financial distress. Too much debt can increase the likelihood of financial distress, leading to higher bankruptcy costs and ultimately impacting the firm's value.

3. Uncertainty of Operating Incomes

- The level of uncertainty surrounding a firm's operating income influences its capital structure decisions. High-tech industries often have unpredictable cash flows and future earnings, leading them to prefer low or no debt to avoid the risk of financial distress.
- Conversely, capital-intensive industries such as airlines, refineries, and hotels may have more stable cash flows and higher tangible assets, making them better candidates for debt financing.
- Start-ups may initially rely on equity financing due to the uncertainty surrounding their future cash flows and profitability.

4. Industry Average of Debt/Equity Ratios

- Firms may benchmark their capital structure decisions against industry averages to ensure they remain competitive and align with market norms. However, it's essential to consider the specific circumstances and risk profiles of individual firms within an industry.
- Deviations from industry averages may occur due to differences in business models, risk tolerance, and financial strategies. Firms should carefully evaluate their own financial position and objectives when determining their optimal capital structure.

5. Personal Taxes

- Personal taxes can influence capital structure decisions, particularly for closely held corporations or firms where shareholders are taxed at different rates on dividends and capital gains.
- If dividends are taxed at a lower rate compared to corporate taxes, shareholders may prefer dividend payouts over debt-financed interest payments to maximize after-tax returns. This preference may influence the firm's financing choices and overall capital structure.

1. Introduction

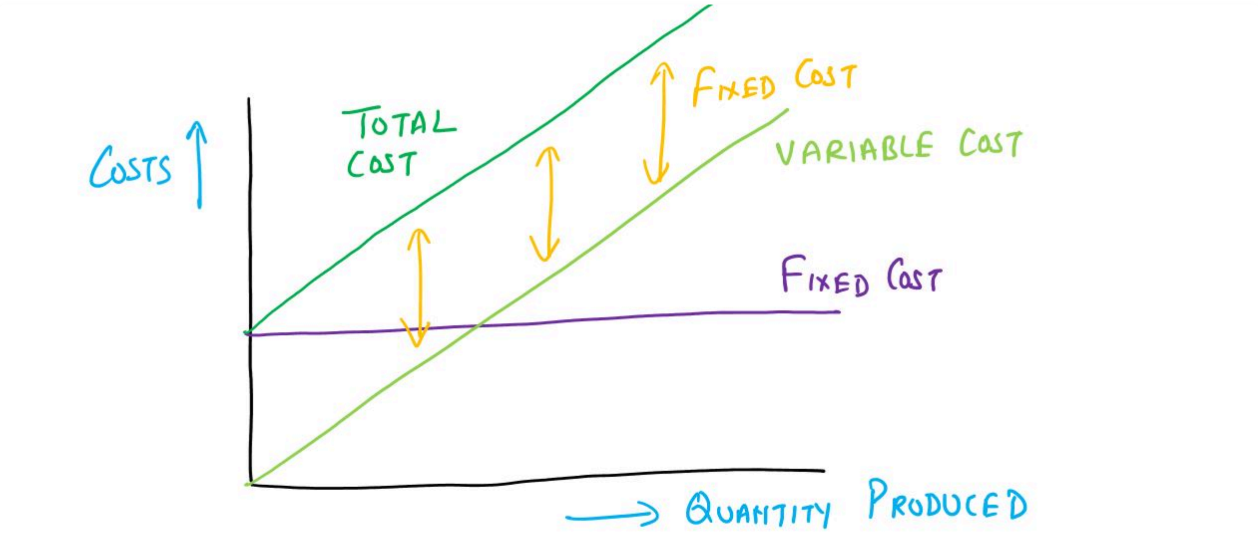
When a lever is employed effectively, a force applied at one point is translated or amplified into a larger force or motion at another point. While this concept is commonly associated with mechanical leverage, such as using a crowbar, it also applies to business contexts. In business, leverage refers to the utilization of fixed costs to potentially increase profitability, a practice known as "levering up."

Operating leverage and financial leverage are the two primary forms of leverage, in finance.

Operating leverage stems from fixed operating costs linked to the production of goods or services, while financial leverage arises from fixed financing costs, notably interest on debt. Both types of leverage influence the level and variability of a firm's earnings, consequently impacting the firm's overall risk and return profile.

2. Fixed and Variable Costs

In any production process, costs can typically be categorized into two main types: fixed costs and variable costs.



Fixed costs remain constant regardless of the level of production, at least within a certain range, while variable costs change in direct proportion to the level of production.

Examples of **fixed costs** include:

- Rent or lease payments for production facilities or office space
- Salaries or wages for permanent staff
- Insurance premiums for business operations
- Depreciation expenses for equipment or machinery
- Property taxes on facilities

On the other hand, examples of **variable costs** include:

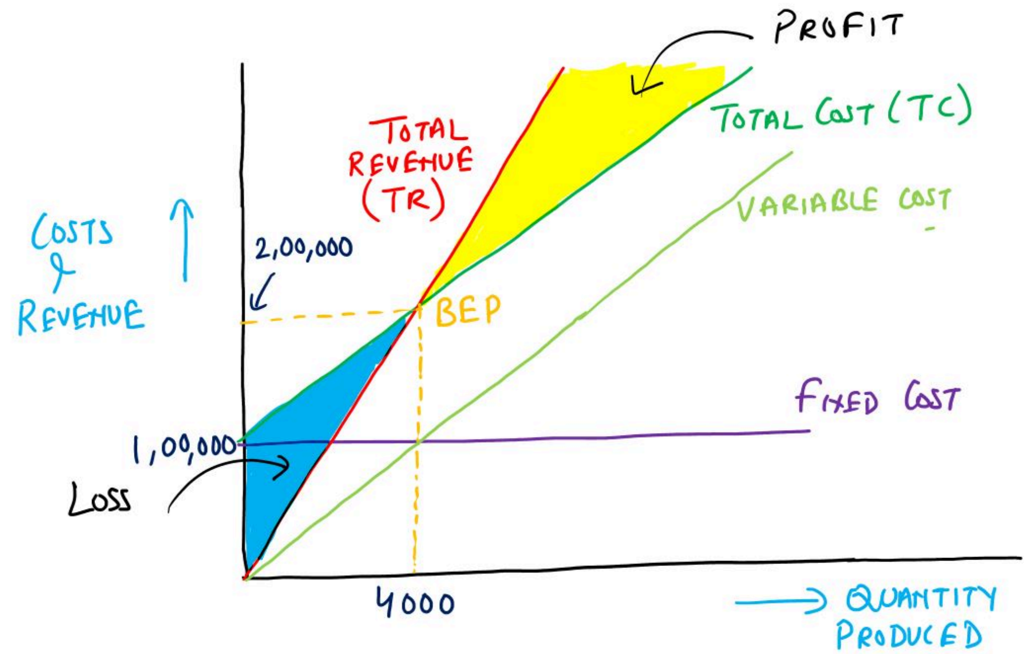
- Raw materials and components used in the production process
- Direct labor costs for workers directly involved in production
- Utilities such as electricity, water, and gas, which may vary based on production volume
- Packaging materials
- Shipping and transportation costs

In the short run, fixed costs are considered unavoidable that do not change with fluctuations in production levels. Variable costs, on the other hand, are more directly linked to the quantity of goods or services produced, increasing as production levels rise and decreasing as production levels fall.

3. Break Even Analysis

It is important to study break even analysis to understand operating leverage.

Consider a firm that produces shirts, which are sold at Rs 50 per shirt. The company has annual fixed operating costs of Rs 1,00,000, and variable operating costs are Rs 25 per shirt.



We wish to study the relationship between total operating costs and total revenues. One means for doing so is with the break-even chart in figure, which shows the relationship among total revenues (TR), total operating costs (TC), and profits for various levels of production and sales. As we are concerned only with operating costs at this point, we define profits here to mean operating profits before taxes.

The intersection of the total costs (TC) line with the total revenues (TR) line determines the break-even point (BEP). The BEP is the sales volume required for total revenues to equal total operating costs or for operating profit to equal zero. In our figure, this BEP is 4,000 shirts of output (or Rs 2,00,000 in sales since each shirt sells for Rs 50).

Mathematically, the computation of BEP (in units) is given below:

$$\text{TOTAL REVENUE} = \text{TOTAL COST}$$

$$P \times Q = P \times VC + FC$$

$$Q = \frac{FC}{P-VC}$$

$$\text{BREAK EVEN POINT} = \frac{FC}{P-VC} = \frac{FC}{\text{CONTRIBUTION PER UNIT}}$$

P = PRICE PER UNIT

Q = QUANTITY

VC = VARIABLE COST PER UNIT

P - VC = CONTRIBUTION PER UNIT

Contribution per unit refers to the revenue generated from the sale of each shirt, which is available to cover fixed costs. Any amount remaining after covering fixed costs contributes to operating profits.

The Break-Even Point (BEP) is when a business covers all its fixed costs but makes no profit. It's like reaching the point where you just break even, neither making money nor losing any.

Once a business sells more than the break-even quantity, it starts making a profit. But if it sells less than that, it starts losing money because it can't cover all its fixed costs.

We have calculated the Break-Even Point in terms of how many units need to be sold.

Another way to look at it is in terms of total sales volume. To find this, we just multiply the Break-Even Point (in units) by the price of each unit sold.

$$\text{BREAK EVEN POINT (SALES)} = \text{BREAK EVEN POINT (UNITS)} \times \text{SELLING PRICE}$$

In our example, the Break-Even Point (units) and Break-Even Point (sales) are computed below:

$$\text{BREAK EVEN POINT (UNITS)} = \frac{FC}{P-VC} = \frac{1{,}00{,}000}{50-25} = 4000 \text{ UNITS}$$

$$\text{BREAK EVEN POINT (SALES)} = 4000 \times 50 = \text{Rs } 2{,}00{,}000$$

Another formula to compute the Break-Even Point (sales) is given by:

$$\text{BREAK EVEN POINT (SALES)} = \frac{FC}{1 - \frac{VC}{SALES}}$$

$\frac{VC}{SALES}$ will be constant for any level of production

In our example, the Break-Even Point (sales) can be computed, above formula:

$$\text{BREAK EVEN POINT (SALE)} = \frac{FC}{1 - \frac{VC}{SALES}} = \frac{1{,}00{,}000}{1 - \frac{25}{50}} = \text{Rs } 2{,}00{,}000$$

Similarly, if we have to calculate the sales volume required to produce a “target” operating income (EBIT), we can add our target or minimum desired operating income figure to fixed costs (FC) in any of above equations. The resulting answers will be our target sales volume needed to produce the target operating income figure.

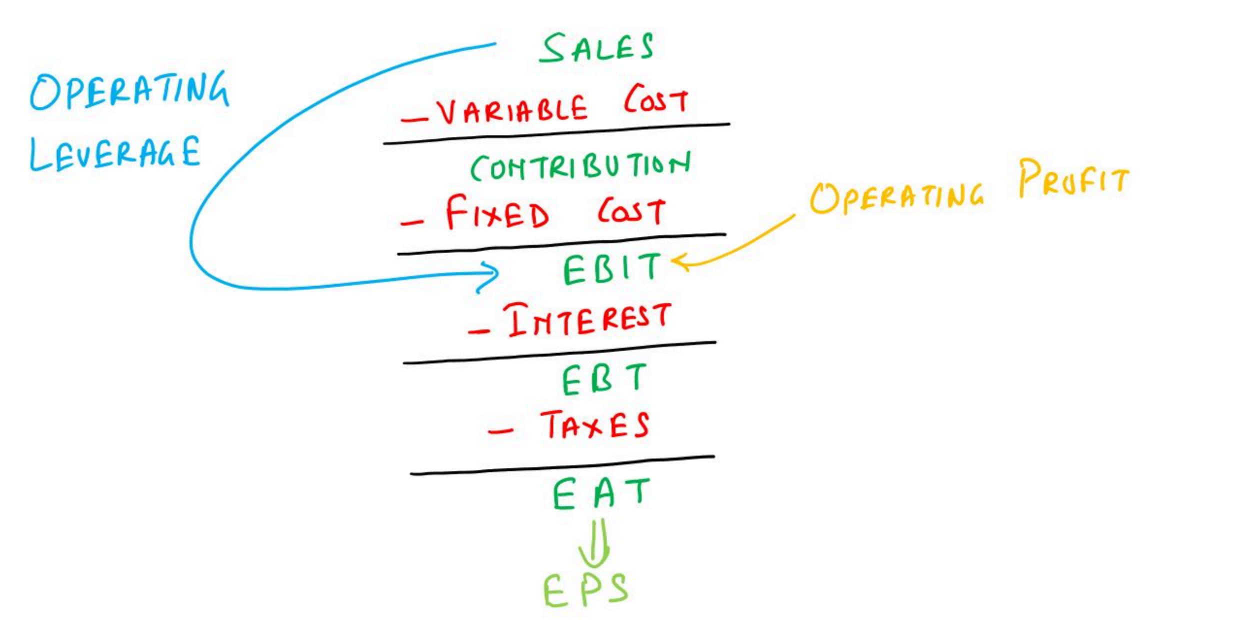
SALES VOLUME TO PRODUCE x EBIT

$$\text{In Units} = \frac{FC + x}{P - VC} = \frac{FC + x}{\text{CONTRIBUTION PER UNIT}}$$

$$\text{In Sale Volume} = \frac{FC + x}{1 - \frac{VC}{\text{SALES}}}$$

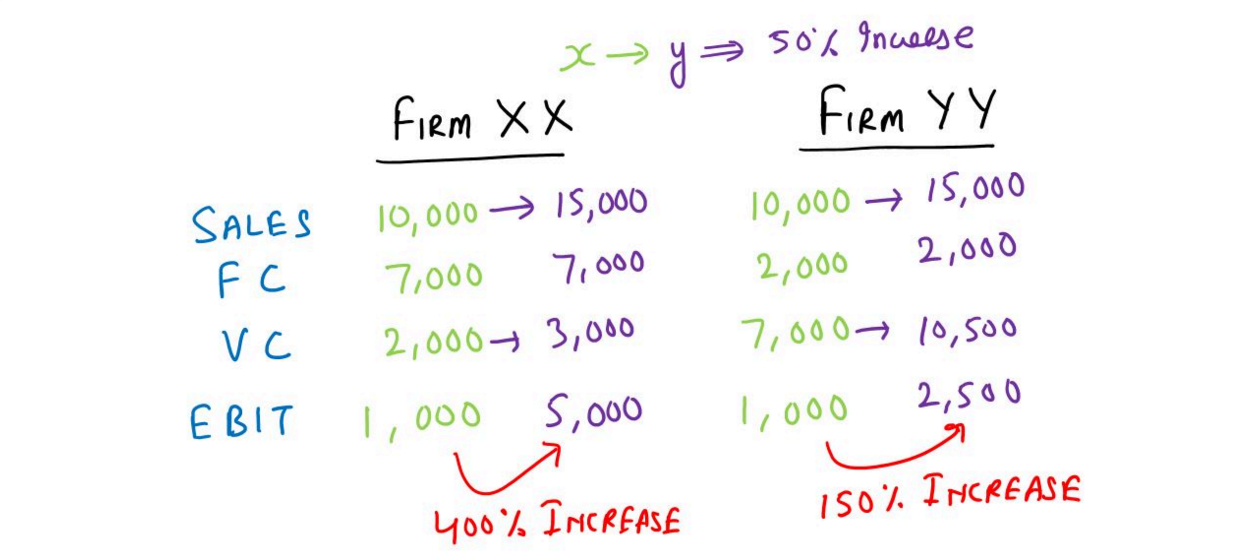
4. Operating Leverage

One interesting effect caused by the presence of fixed operating costs is that a change in the volume of sales results in a more than proportional change in operating profit (or loss).



Thus, like a lever used to magnify a force applied at one point into a larger force at some other point, the presence of fixed operating costs causes a percentage change in sales volume to produce a magnified percentage change in operating profit (or loss). Thus Fixed Costs is called Operating Leverage.

Consider two hypothetical firms, XX and YY, operating in the same industry. Firm XX has a higher proportion of fixed costs compared to Firm YY. To illustrate, let's assume that Firm XX has fixed costs of Rs 7,000 and variable costs of Rs 2,000, while Firm YY has fixed costs of Rs 2,000 and variable costs of Rs 7,000. This implies that Firm XX has higher fixed costs relative to Firm YY. Further, we suppose that both firms have a sales revenue of Rs 10,000.



If the sales of both firms increase by 50%, resulting in a new sales revenue of Rs 15,000, the variable costs of both firms will also increase by 50% due to the higher sales volume. However, the fixed costs of both firms remain the same, as they do not vary with the level of production or sales.

Upon analyzing the impact of this change in sales revenue on the earnings before interest and taxes (EBIT), we observe interesting outcomes. For Firm XX, the 50% increase in sales revenue leads to a substantial 400% increase in EBIT. On the other hand, Firm YY experiences only a 150% increase in EBIT for the same 50% change in sales revenue.

This disparity in the percentage change in EBIT relative to the change in sales revenue can be attributed to the concept of operating leverage. Operating leverage refers to the extent to which a firm's cost structure is comprised of fixed costs versus variable costs. In the case of Firm XX, with its higher fixed costs, a relatively small increase in sales revenue results in a disproportionately larger increase in EBIT. This is akin to a lever amplifying the force applied at one point to generate a larger force at another point. Consequently, Firm XX exhibits higher operating leverage compared to Firm YY.

5. Degree of Operating Leverage

We have learnt that the sensitivity of the percentage change in earnings before interest and taxes (EBIT) relative to the change in sales revenue is influenced by the level of fixed costs in a firm's cost structure.

This sensitivity is quantified by a metric known as the **Degree of Operating Leverage** (DOL). The sensitivity of the firm to a change in sales as measured by DOL will be different at each level of output (or sales). Therefore, we always need to indicate the level of output (or sales) at which DOL is measured – as in DOL at Q units.

The Degree of Operating Leverage (DOL) is calculated using the following formula:

DEGREE OF OPERATING LEVERAGE

$$DOL = \frac{\%\ \text{Change in EBIT}}{\%\ \text{change in SALES}}$$

This formula compares the percentage change in EBIT to the percentage change in sales revenue. A higher DOL indicates that a firm has a greater proportion of fixed costs in its cost structure, resulting in higher operating leverage. Conversely, a lower DOL suggests that a firm has fewer fixed costs relative to variable costs, resulting in lower operating leverage.

Above formula is good for defining and understanding DOL, but we would need a few simple alternative formulas for actually computing DOL values:

$$DOL \rightarrow \frac{Q}{Q - Q_{BE}} \quad \text{(when sales is given in units)}$$

$$DOL \rightarrow \frac{EBIT + FC}{EBIT} \quad \text{(when sales is given amount)}$$

Q = Sales Volume
Q_{BE} = BREAK EVEN SALES

We can compute DoL at 5,000 shirts in our example:

DOL at $Q = 5000$ units

$$DOL = \frac{Q}{Q - Q_{BE}} = \frac{5000}{5000 - 4000} = 5$$

"DoL of 5 at 5000 units" means that a 1% change in sales from the 5,000-unit sales position causes a 5% change in EBIT. A 3% decrease in sales causes a 15% decrease in EBIT, and a 4% increase in sales causes a 20% increase in EBIT.

5. Degree of Operating Leverage

A Company produces and sells 10,000 shirts. The selling price per shirt is Rs 500. Variable cost is Rs 200 per shirt and fixed operating cost is Rs 25,00,000.
(a) Calculate operating leverage.
(b) If sales are up by 70%, then what is impact on EBIT?

Solution:

SALES	$10{,}000 \times 500 =$	50 lakh
VC	$10{,}000 \times 200 =$	20 lakh
Contribution		30 lakh
FC		25 lakh
EBIT		5 lakh

$$DOL = \frac{EBIT + FC}{EBIT} = \frac{5+25}{5} = \boxed{6}$$

METHOD 1

BREAK EVEN QUANTITY

$$Q_{BE} = \frac{FC}{P-VC} = \frac{25 \text{ lakh}}{500-200} = \frac{25000}{3}$$

$$DOL = \frac{Q}{Q - Q_{BE}} = \frac{10000}{10000 - \frac{25000}{3}} = \boxed{6}$$

METHOD 2

SINCE DOL = 6

70% increase in Sales $\Rightarrow$ $70 \times 6 = 420\%$ increase in EBIT

5. Degree of Operating Leverage

A company has a DOL of 2 at its current production and sales level of 10,000 units. The resulting operating income figure is Rs 1,000.
(i) If sales are expected to increase by 20% from the current 10,000-unit sales position, what would be the resulting operating profit figure?
(ii) At the company's new sales position of 12,000 units, what is the firm's "new" DOL figure?

Solution:

(i) If sales are expected to increase by 20% from the current 10,000-unit sales position, what would be the resulting operating profit figure?

% change in EBIT (operating profit)

$$= \% \text{ change in sale} \times DOL$$

$$= 20\% \times 2 = 40\%$$

(ii) At the company's new sales position of 12,000 units, what is the firm's "new" DOL figure?

$$\text{DOL at } 10{,}000 = \frac{10{,}000}{10{,}000 - Q_{BE}} = 2$$

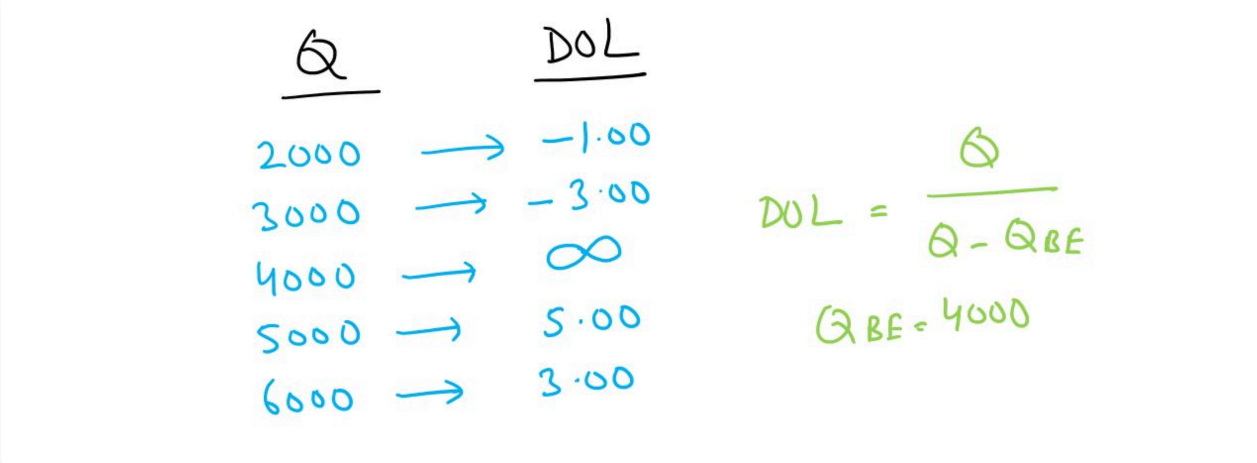
$$\Rightarrow Q_{BE} = 5000$$

$$\text{DOL at } 12{,}000 = \frac{12{,}000}{12000 - 5{,}000} = 1.7$$

6. DoL and Break Even Point

In previous section, we calculated DOL at 5000 shirts and it came out to be 5.

Similarly, we can compute DoL at 6,000 shirts, 7000 shirts, 4,000 shirts, 3000 shirts and 2000 shirts.

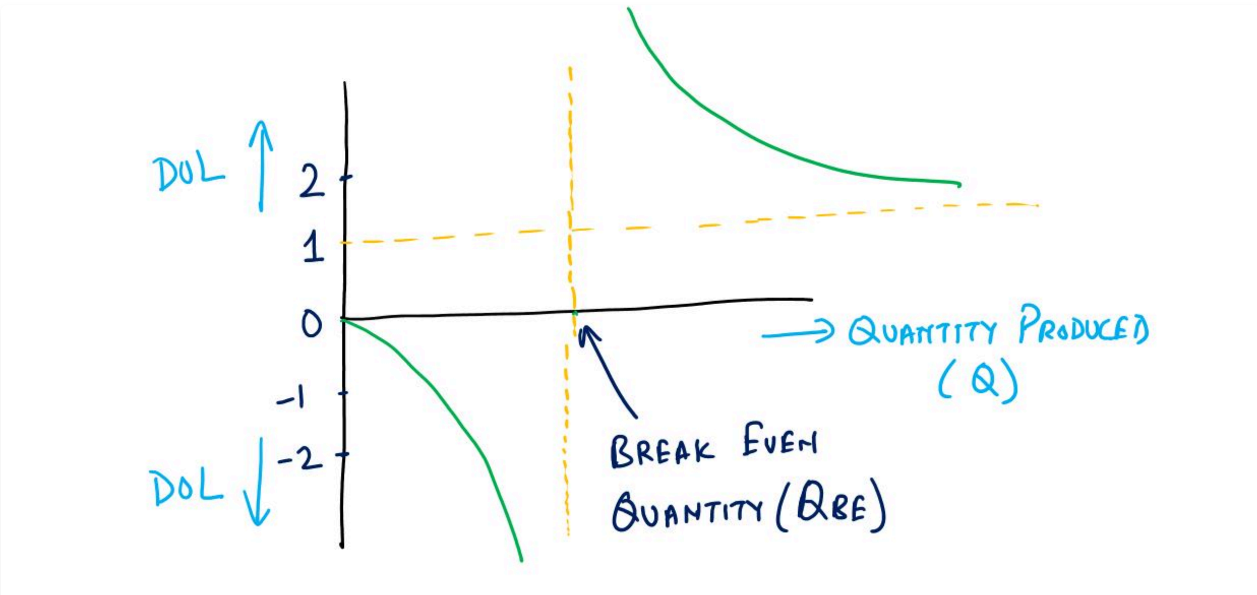


It may be noted that when output was increased from 5,000 to 6,000 shirts, the DoL decreased from a value of 5 to a value of 3. Thus, the further the level of output is from the break-even point (4000), the lower the degree of operating leverage.

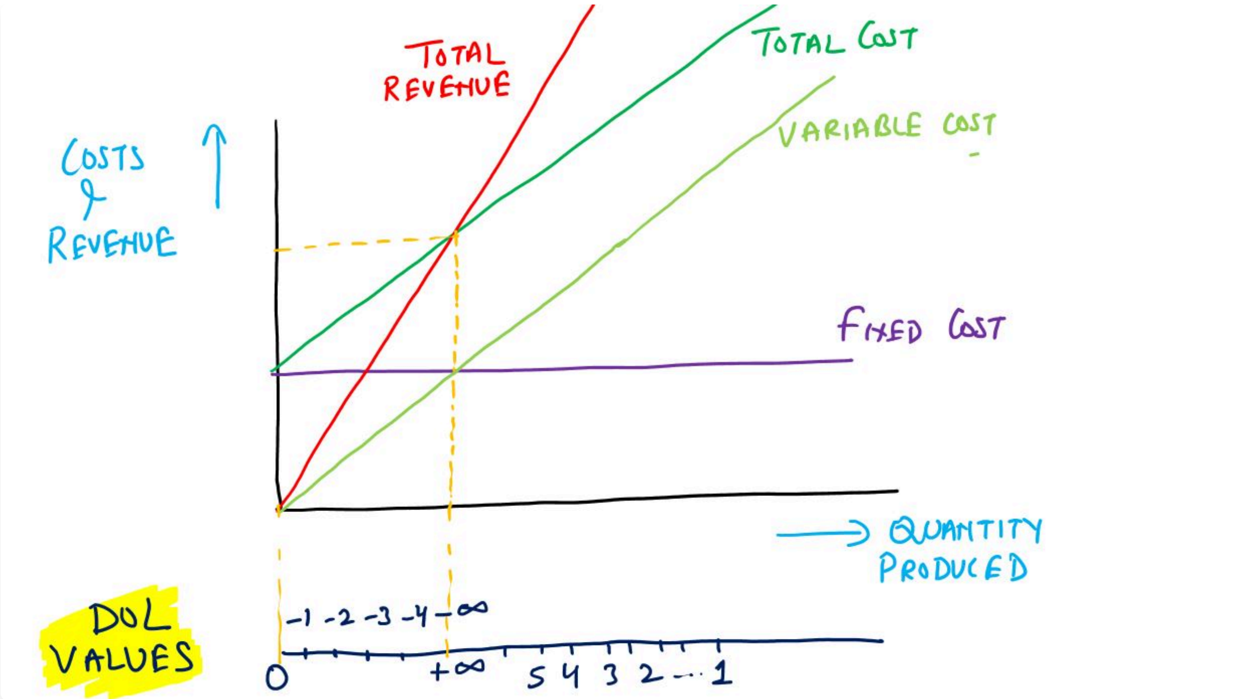
We see that the further we move from the firm's break-even point:
(i) the lower is the relative sensitivity of operating profit to changes in output (sales) as measured by DOL (we can see this in computed DoL values at various sale levels).
(ii) the greater is the absolute value of the firm's operating profit or loss (we can see this in break even graph)

How close a firm operates to its break-even point determines how sensitive its operating profits (EBIT) will be to a change in output or sales.

Now, we have understood, why we introduced break even analysis for understanding of Operating Leverage.



From figure, we can see that as DOL approaches positive (or negative) infinity as sales approach the break-even point from above (or below) that point. DOL approaches 1 as sales grow beyond the break-even point. This implies that the magnification effect on operating profits caused by the presence of fixed costs diminishes toward a simple 1-to-1 relationship as sales continue to grow beyond the break-even point.



Even firms with large fixed costs will have a low DOL if they operate well above their break-even point. By the same token, a firm with very low fixed costs will have an enormous DOL if it operates close to its breakeven point.

6. DoL and Break Even Point

A Company has fixed operating costs of Rs 30 lakhs a year. Variable operating costs are Rs 1.75 per pencil produced, and the average selling price is Rs 2 per pencil.

(i) What is the annual operating break-even point in units and in sales volume (Rupees)?

(ii) If variable operating costs decline to Rs 1.68 per pencil, what would happen to the operating break-even point in units?

(iii) If fixed costs increase to Rs 37.5 lakhs per year, what would be the effect on the operating break-even point in units?

(iv) Compute the degree of operating leverage (DOL) at the current sales level of 160 lakh pencils.

(v) If sales are expected to increase by 15 percent from the current sales position of 160 lakh pencils, what would be the resulting percentage change in operating profit (EBIT) from its current position?

Solution:

(i) What is the annual operating break-even point in units and in sales volume (Rupees)?

FC = 30 lakh   VC = Rs 1.75   P = Rs 2

$$\text{BREAK EVEN POINT (UNITS)} = \frac{FC}{P - VC} = \frac{30 \text{ lakh}}{2 - 1.75} = 120 \text{ lakh}$$

$$\text{BREAK EVEN POINT (SALE)} = \frac{FC}{1 - \frac{VC}{Sale}} = \frac{30 \text{ lakh}}{1 - \frac{1.75}{2}} = \text{Rs } 240 \text{ lakh}$$

(ii) If variable operating costs decline to Rs 1.68 per pencil, what would happen to the operating break-even point in units?

VC 1.75 → 1.68

$$\text{BREAK EVEN POINT (UNITS)} = \frac{30}{2 - 1.68} = 93.75 \text{ lakh}$$

$$\text{BREAK EVEN POINT (SALE)} = 93.75 \times 2 = \text{Rs } 187.50$$

(iii) If fixed costs increase to Rs 37.5 lakhs per year, what would be the effect on the operating break-even point in units?

FC 30 lakh → 37.5 lakh

$$\text{BREAK EVEN POINT (UNITS)} = \frac{37.5}{2 - 1.75} = 150 \text{ lakh}$$

$$\text{BREAK EVEN POINT (SALE)} = 150 \times 2 = \text{Rs } 300 \text{ lakh}$$

(iv) Compute the degree of operating leverage (DOL) at the current sales level of 160 lakh pencils.

$$DOL = \frac{Q}{Q - Q_{BE}} = \frac{160}{160 - 120} = \frac{160}{40} = 4$$

(v) If sales are expected to increase by 15 percent from the current sales position of 160 lakh pencils, what would be the resulting percentage change in operating profit (EBIT) from its current position?

Change in EBIT = $4 \times 15\% = 60\%$

7. How DOL Helps Financial Managers

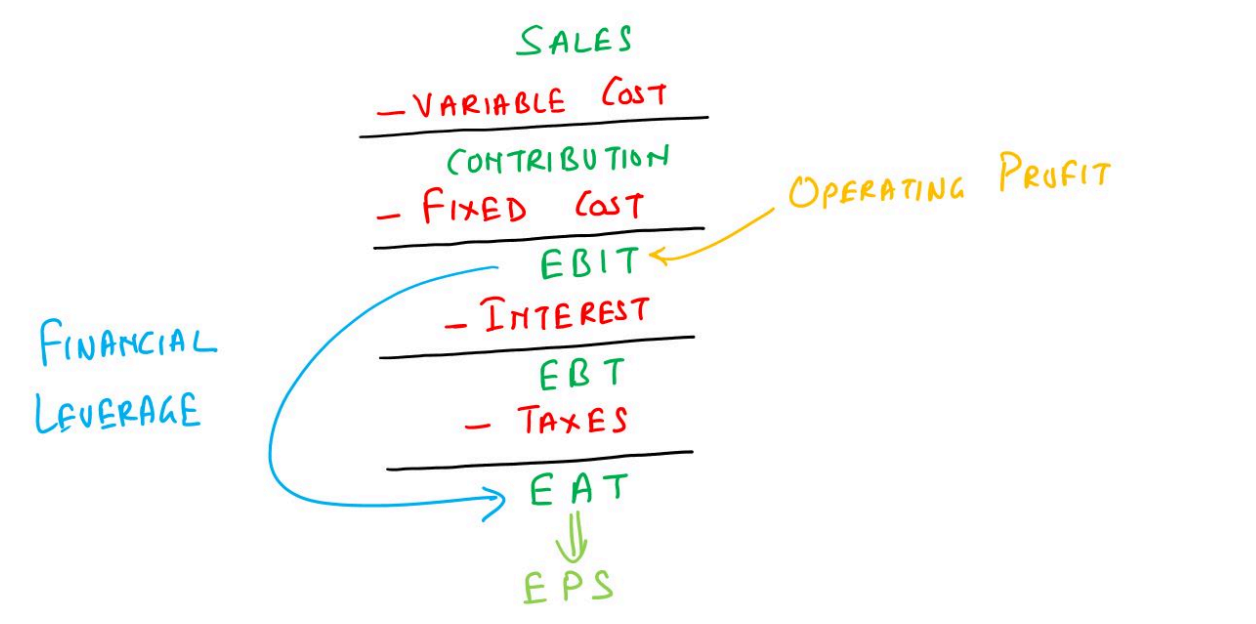
Knowing a firm's DOL helps managers predict how changes in sales will affect operating profits. Armed with this information, managers can make informed decisions about adjusting sales strategies or cost structures to mitigate risk. Generally, firms prefer lower DOL because high DOL means even a small drop in sales could lead to operating losses.

Understanding the degree of operating leverage (DOL) is important, but it's just one part of a firm's overall business risk. Other key factors contributing to business risk include the uncertainty of sales and production costs. DOL amplifies the impact of these factors on operating profits, but it isn't the direct cause of variability. For instance, a high DOL doesn't matter if a firm keeps sales and costs stable. It's a mistake to equate DOL with business risk because it only becomes significant when sales and costs vary.

8. Financial Leverage

Till now we have learnt that the Operating Leverage refers to the situation where a company's fixed costs (FC) are high relative to its variable costs (VC). While this can amplify earnings before interest and taxes (EBIT) when sales increase, it is not always possible to adjust FC to increase EBIT, especially in industries with inherently high FC like airlines, refineries, steel plants.

No worries, we have another type of leverage, called **Financial Leverage**.



Financial leverage involves using fixed-cost financing, such as debt or preferred stock, to potentially boost returns for common shareholders. This choice to employ financial leverage is usually well within the control of the company's management.

When financial leverage is used effectively, it can lead to favorable or positive leverage. This occurs when the returns generated by the company's operations exceed the fixed financing costs, allowing any remaining profits to benefit common shareholders, in the form of increased earnings per share (EPS).

Thus, the Financial leverage represents the second step in a two-step process aimed at increasing profitability. In the first step, operating leverage magnifies the impact of sales changes on EBIT. Then, in the second step, financial leverage further amplifies the effects of changes in EBIT on EPS. Financial Leverage is also called Capital Gearing.

To understand how EPS changes with changes in EBIT for different financing methods, financial managers conduct EBIT-EPS analysis. This analysis helps in making decisions about the best financing options for the firm.

9. EBIT-EPS Analysis

EBIT-EPS analysis helps businesses decide on the best way to fund their operations by comparing different financing choices (like common stock, debt, preferred stock). It looks at how changes in EBIT affect the company's earnings per share (EPS) under different financial choices.

By doing this analysis, companies can figure out which financing option will give them the highest EPS, for a givel level of EBIT.

Consider a Company with long-term financing of Rs 100 lakh, consisting entirely of common stock equity, wishes to raise another Rs 50 lakh for expansion through one of three possible financing plans.
(i) all common stock
(ii) all debt at 12% rate of interest
(iii) all preferred stock with 11% dividend.
Present annual earnings before interest and taxes (EBIT) are Rs 15 lakh but with new expansion are expected to rise to Rs 27 lakh.
The income tax rate is 40%
2,00,000 shares of common stock are now outstanding.
Common stock can be sold at Rs 50 per share under the first financing option. So we need to issue 1,00,000 shares.

Earnings per share (EPS) for any level of EBIT can be computed using below formula:

$$EPS = \frac{(EBIT - I)(1 - t) - PD}{NS}$$

I = INTEREST ON DEBT

PD = PREFERENCE DIVIDEND

t = CORPORATE TAX RATE

NS = SHARES OUTSTANDING

On the horizontal axis we plot EBIT, and on the vertical axis we plot EPS. For each of three financing alternative, we draw a straight line to reflect EPS for all possible levels of EBIT. Because two points determine a straight line, we need two data points for each financing alternative.

One point of EPS for each of three financing alternatives can be computed at EBIT level of Rs 27 lakh, which is shown below.

CALCULATION OF EPS at EBIT = 27 lakh (all data in lakh except EPS)

	COMMON STOCK	DEBT @ 12%	PREFERRED STOCK @ 11%
EBIT	27	27	27
INTEREST	–	6	–
EBT	27	21	27
TAXES @ 40%	10.8	8.4	10.8
PD	–	–	5.5
EARNINGS FOR SHAREHOLDERS	16.2	12.6	10.7
EPS	5.40	6.30	5.35

The second data point of each of three financing alternatives can be obtained, where EPS is zero. We can calculate corresponsing EBIT as shown below.

FOR COMMON STOCK

$$\frac{(EBIT-I)(1-t)-PD}{NS}=0 \Rightarrow (EBIT-0)(1-0.40)-0=0$$
$$\Rightarrow EBIT=0$$

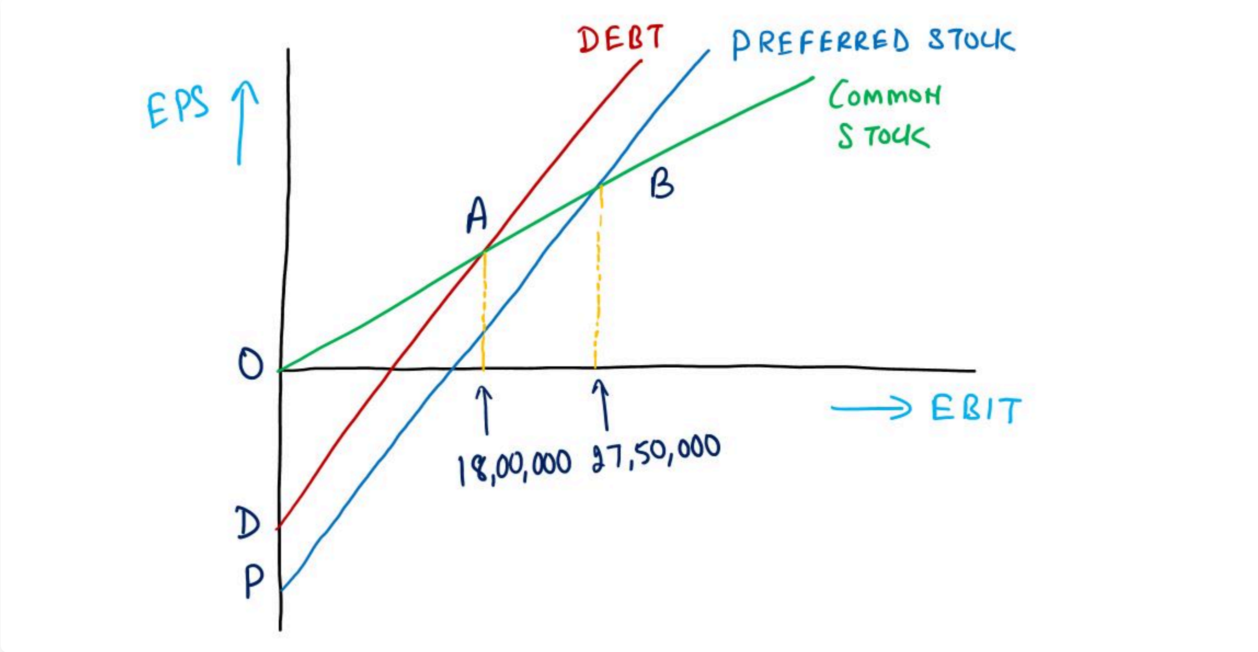
FOR DEBT

$$\frac{(EBIT-I)(1-t)-PD}{NS}=0 \Rightarrow (EBIT-6{,}00{,}000)(1-0.40)-0=0$$
$$\Rightarrow EBIT=6{,}00{,}000$$

FOR PREFERRED STOCK

$$\frac{(EBIT-I)(1-t)-PD}{NS}=0 \Rightarrow (EBIT-0)(1-0.40)-5{,}50{,}000=0$$
$$\Rightarrow EBIT=9{,}16{,}667$$

Now, that we have two points for each of three financing alternatives, we can draw our graph, which is shown below.



Indifference Points

We see from the figure that the EPS is same at point A for the debt and common stock. If EBIT is below that point, the common stock alternative will provide higher EPS. Above point A the debt alternative produces higher EPS. This point is called indifference point or break even point between two financing options. Since this is indifference point between the debt and common stock, it can be compted by equating EPS of two financing alternatives.

INDIFFERENCE POINT BETWEEN COMMON STOK AND DEBT

EPS of COMMON STOCK = EPS of DEBT

$$\frac{(EBIT-0)(1-0.40)-0}{3,00,000} = \frac{(EBIT-6,00,000)(1-0.40)-0}{2,00,000}$$

$$\Rightarrow EBIT = 18,00,000$$

If EBIT value is below Rs 18 lakh, the common stock alternative will provide higher EPS. If EBIT is above Rs 18 lakh, then the debt alternative produces higher EPS.

Similarly, the indifference point (Point B) between the preferred stock and the common stock alternative can be computed, as given below.

INDIFFERENCE POINT BETWEEN PREFERRED AND COMMON STOCKS

$$\text{EPS of Preferred Stock} = \text{EPS of Common Stock}$$

$$\frac{(EBIT-0)(1-0.40)-5{,}50{,}000}{2{,}00{,}000} = \frac{(EBIT-0)(1-0.40)-0}{3{,}00{,}000}$$

$$\Rightarrow EBIT = 27{,}50{,}000$$

Above EBIT of Rs 27.5 Lakh, the preferred stock alternative produces more favorable EPS. Below EBIT of Rs 27.5 lakh, the common stock alternative leads to higher EPS.

As seen from the graph, there is no indifference point between the debt and preferred stock alternatives. The debt alternative dominates for all levels of EBIT and by a constant amount of EPS.

Intercept with Y axis

The line for Common Stock (non-levered firm) starts from the origin (pount O), indicating that if EBIT is zero, EPS is also zero. As EBIT increases, EPS rises proportionally.

In contrast, the lines representing both debt and preferrence stock starts from the negative y-axis (Points D and P respectively). This signifies that if EBIT is zero, EPS will be negative due to fixed interest expenses (in case of debt) or preferred dividend (in case of preferred stock). Even with no earnings (EBIT), the firm must still pay these obligations.

Slope of lines

The slope of the line for the debt and preferred stock are higher than that of the common stock. This is because common stock has more shares outstanding. Thus, any increase in EBIT results in a more substantial rise in EPS for the debt and preferred stock compared to the common stock.

9. EBIT-EPS Analysis

ABC Ltd. is planning an expansion programme. It requires Rs 20 lakhs of external financing for which it is considering two alternatives. The first alternative calls for issuing 15,000 equity shares of Rs 100 each and 5,000 10% Preference Shares of Rs 100 each; the second alternative requires 10,000 equity shares of Rs 100 each, 2,000 10% Preference Shares of Rs 100 each and Rs 8,00,000 Debentures carrying 9% interest. The company is in the tax bracket of 50%. You are required to calculate the indifference point for the plans and verify your answer by calculating the EPS.

Solution:

	PLAN I	PLAN II
EQUITY	15,00,000	10,000
PREFERENCE SHARES @ 10%	5,00,000	2,00,000
DEBENTURE @ 9%	–	8,00,000
TOTAL	20,00,000	20,00,000
NS	15,000	10,00,000

INDIFFERENCE POINT

EPS under PLAN I = EPS under PLAN II

$$\frac{(EBIT-0)(1-0.50)-50,000}{15,000} = \frac{(EBIT-72,000)(1-0.50)-20,000}{10,000}$$

$$\Longrightarrow EBIT = 1,36,000$$

VERIFICATION

	PLAN I	PLAN II
EBIT	1,36,000	1,36,000
INTEREST	–	72,000
EBT	1,36,000	64,000
TAX	68,000	32,000
PD	50,000	20,000
EARNINGS AVAILABLE	18,000	12,000
EPS	1.20	1.20

9. EBIT-EPS Analysis

A company has no debt outstanding and a total market value of Rs 2,75,000. Earnings before interest and taxes, EBIT, are projected to be Rs 21,000 if economic conditions are normal. If there is strong expansion in the economy, then EBIT will be 25% higher. If there is a recession, then EBIT will be 40% lower.

The company is considering a Rs 99,000 debt issue with an interest rate of 8%. The proceeds will be used to repurchase shares of stock. There are currently 5,000 shares outstanding. The company has a tax rate of 35%.

(a) Calculate earnings per share, EPS, under each of the three economic scenarios before any debt is issued. Also calculate the percentage changes in EPS when the economy expands or enters a recession.

(b) What if the company goes through with recapitalization?

Solution:

BEFORE RECAPITALIZATION

	RECESSION	40%↓	NORMAL	25%↑	EXPANSION
EBIT	12,600		21,000		26,250
INTEREST	–		–		–
TAXES	4410		7350		9188
EAT	8190		13650		17063
EPS	$\frac{8190}{5000} = 1.64$		$\frac{13650}{5000} = 2.73$		$\frac{17063}{5000} = 3.41$

40%↓ 25%↑

RECAPITALIZATION

$$\text{Share Price} = \frac{2,75,000}{5,000} = \text{Rs } 55$$

$$\text{No of shares repurchased} = \frac{99000}{55} = 1800$$

$$\text{Remaining shares outstanding} = 5000 - 1800 = 3200$$

AFTER RECAPITALIZATION

	RECESSION		NORMAL		EXPANSION
		40% ↓		25% ↑	
EBIT	12,600		21,000		26,250
INTEREST	7,920		7,920		7,920
TAXES	1638		4578		6416
EAT	3042		8502		11915
EPS	$\frac{3042}{3200} = 0.95$		$\frac{8502}{3200} = 2.66$		$\frac{11915}{3200} = 3.72$
		64.22% ↓		40.14% ↑	

10. Degree of Financial Leverage

Just like the Degree of Operating Leverage (DOL) mesaures the sensitivity of change in EBIT relative to the change in sales, the Degree of Financial Leverage (DFL) mesaures the sensitivity of change in EBS relative to the change in EBIT.

The degree of financial leverage (DFL) at a particular level of EBIT (operating profit) is simply the percentage change in earnings per share (EPS) over the percentage change in operating profit (EBIT).

DEGREE OF FINANCIAL LEVERAGE

$$DFL = \frac{\%\text{ change in EPS}}{\%\text{ Change in EBIT}}$$

Whereas above equation is useful for defining and undertsanding DFL, a simple alternative derived formula is used for actually computing DFL values:

$$DFL = \frac{EBIT}{EBIT - I - \frac{PD}{(1-t)}}$$

For our example firm, using the debt-financing alternative at Rs 27 lakh in EBIT, we have:

$$DFL = \frac{EBIT}{EBIT - I - \frac{PD}{(1-t)}}$$

$$= \frac{27,00,000}{27,00,000 - 6,00,000 - 0} \quad \left(\begin{array}{l}\text{Debt financing} \\ \text{at EBIT} = 27,00,000\end{array}\right)$$

$$= 1.29$$

For the preferred stock financing alternative, the DFL is:

$$DFL = \frac{EBIT}{EBIT - I - \frac{PD}{(1-t)}}$$

$$= \frac{27{,}00{,}000}{27{,}00{,}000 - 0 - \frac{5{,}50{,}000}{(1-0.60)}}$$

(Prefered Stock financing at EBIT = 27,00,000)

$$= 1.51$$

We can see that the DFL is greater under the preferred stock option than under the debt option. This is because of the tax deductibility of interest and the non-deductibility of preferred dividends. It is often argued that preferred stock financing is of less risk than debt financing for the issuing firm. With regard to the risk of cash insolvency, this is probably true. But the DFL tells us that the relative variability of EPS will be greater under the preferred stock financing arrangement, everything else being equal.

10. Degree of Financial Leverage

A Company currently has Rs 30 lakh in debt outstanding, bearing an interest rate of 12 percent. It wishes to finance a Rs 40 lakh expansion program and is considering three alternatives:
Option 1: additional debt at 14% interest (option 1),
Option 2: preferred stock with a 12% dividend (option 2), and
Option 3: sale of common stock at Rs 16 per share
The company currently has 800,000 shares of common stock outstanding and is in a 40% tax bracket.

(i) If earnings before interest and taxes are currently Rs 15 lakh, what would be earnings per share for the three alternatives, assuming no immediate increase in operating profit?
(ii) Develop a break-even, or indifference, chart for these alternatives. What are the approximate indifference points? To check one of these points, mathematically determine the indifference point between the debt plan and the common stock plan. What are the horizontal axis intercepts?
(iii) Compute the degree of financial leverage (DFL) for each alternative at the expected EBIT level of Rs 15 lakh.
(iv) Which alternative do you prefer? How much would EBIT need to increase before the next alternative would be "better" (in terms of EPS)?

Solution:

(i) If earnings before interest and taxes are currently Rs 15 lakh, what would be earnings per share for the three alternatives, assuming no immediate increase in operating profit?

INTEREST ON EXISTING DEBT = 12% of 30 lakh
= 3.6 lakh

	OPTION 1 DEBT	OPTION 2 PREFERRED STOCK	OPTION 3 COMMON STOCK
EBIT	15,00,000	15,00,000	15,00,000
INTEREST	3,60,000 + 5,60,000	3,60,000	3,60,000
PBT	5,80,000	11,40,000	11,40,000
TAXES	2,32,000	4,56,000	4,56,000
PD	—	4,80,000	—
EARNINGS AVAILABLE	3,48,000	2,04,000	6,84,000
NS	8,00,000	8,00,000	10,50,000
EPS	0.435	0.255	0.651

(ii) Develop a break-even, or indifference, chart for these alternatives. What are the indifference points? What are the horizontal axis intercepts?

INDIFFERENCE POINT B/W DEBT AND COMMON STOCK

$$\frac{(EBIT - 9{,}20{,}000)(1-0.40) - 0}{8{,}00{,}000} = \frac{(EBIT - 3{,}60{,}000)(1-0.40) - 0}{10{,}50{,}000}$$

$\Longrightarrow$ EBIT = 27,12,000

INDIFFERENCE POINT B/W PREFERRED AND COMMON STOCK

$$\frac{(EBIT - 3{,}60{,}000)(1-0.40) - 4{,}80{,}000}{8{,}00{,}000} = \frac{(EBIT - 3{,}60{,}000)(1-0.40) - 0}{10{,}50{,}000}$$

$\Longrightarrow$ EBIT = 37,20,000

X-axis intercept of Debt

$(EBIT - 9{,}20{,}000)(1-0.40) - 0 = 0$

$\Longrightarrow$ EBIT = 9,20,000

X-axis intercept of Common stock

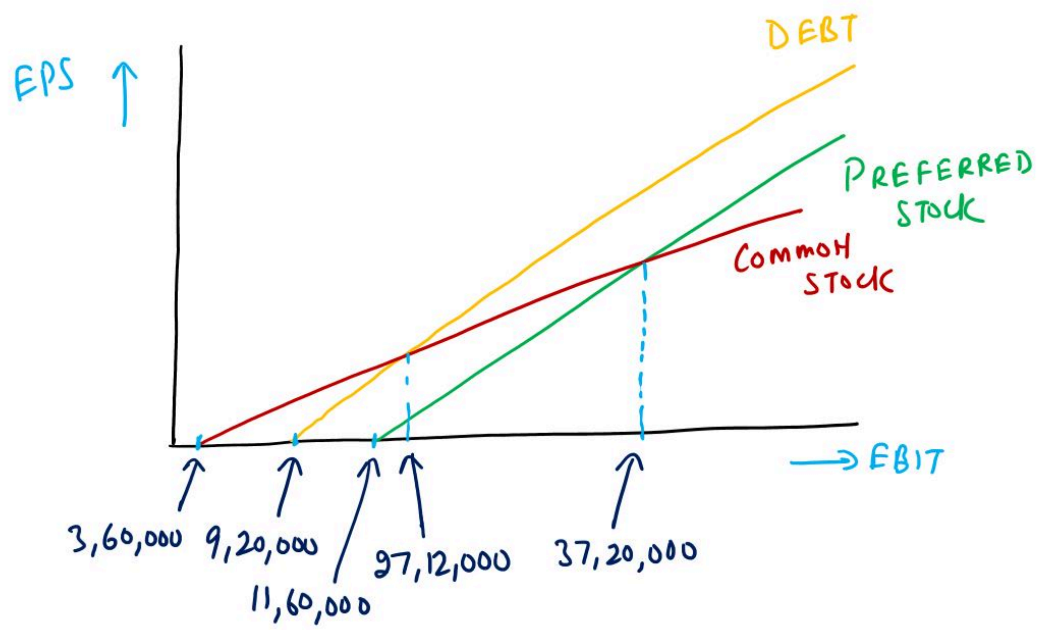
$(EBIT - 3{,}60{,}000)(1-0.40) - 0 = 0$

$\Longrightarrow$ EBIT = 3,60,000

X-axis intercept of Preferred stock

$(EBIT - 3{,}60{,}000)(1-0.40) - 4{,}80{,}000 = 0$

$\Longrightarrow$ EBIT = 11,60,000



(iii) Compute the degree of financial leverage (DFL) for each alternative at the expected EBIT level of Rs 15 lakh.

DFL at EBIT of 15,00,000

DEBT $\dfrac{15,00,000}{15,00,000 - 9,20,000} = 2.59$

PREFERRED STOCK $\dfrac{15,00,000}{15,00,000 - 9,20,000 - \dfrac{4,80,000}{1-0.40}} = 4.41$

COMMON STOCK $\dfrac{15,00,000}{15,00,000 - 3,60,000} = 1.32$

(iv) Which alternative do you prefer? How much would EBIT need to increase before the next alternative would be “better” (in terms of EPS)?

FOR EBIT of 15,00,000

$\Longrightarrow$ COMMON STOCK is PREFERABLE

(EPS IS HEIGHEST AT 0.651)

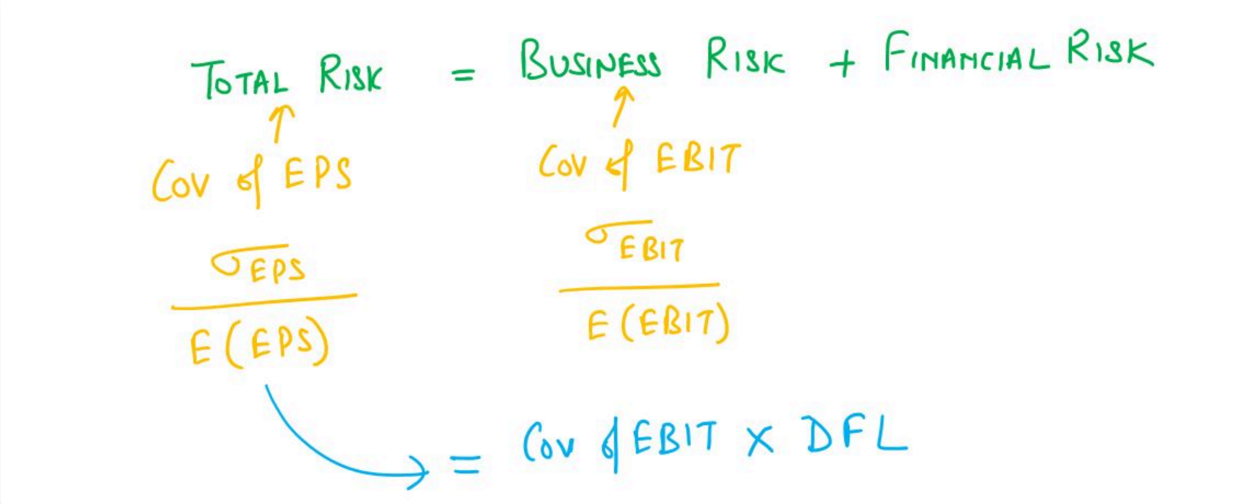
EBIT needs to increase by:

27,12,000 − 15,00,000 = Rs 12,12,000

For the present EBIT level, **common stock is clearly preferable**. EBIT would need to increase by Rs 12,12,000 before an indifference point with debt is reached.

11. DFL and Financial Risk

When we talk about a company's risk, we consider two main factors: business risk and financial risk. Business risk is how much the company's earnings can vary, while financial risk involves the possibility of running out of money and how borrowing money affects the company's earnings.



To measure the riskiness of a company, we can look at the variation in its earnings per share (EPS) compared to its expected earnings. This gives us the coefficient of variation, which tells us how much the earnings can differ relative to what's expected.

One way to figure out the business risk is by looking at the coefficient of variation of earnings before interest and taxes (EBIT), which is a measure of how much the company's operating earnings can vary.

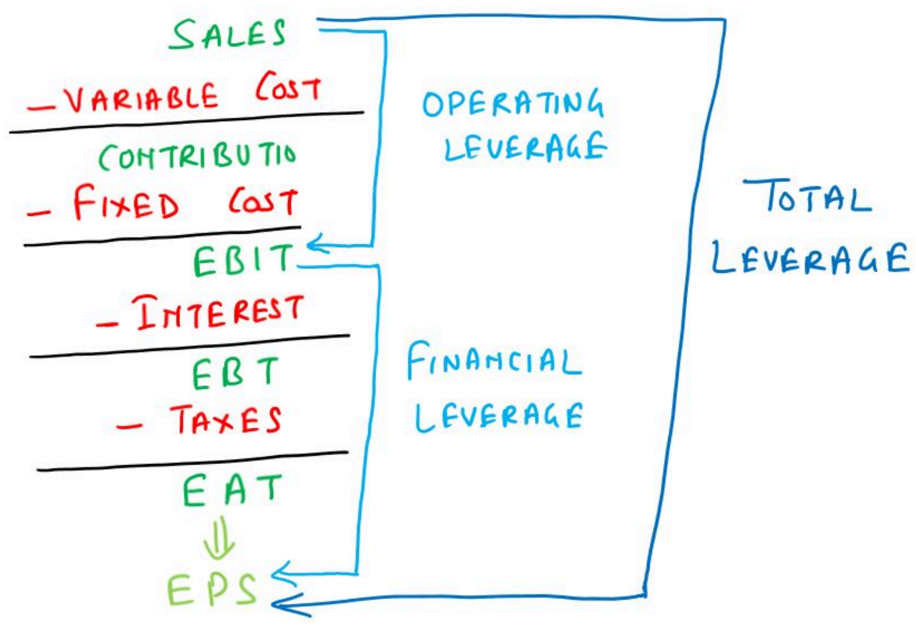
Financial risk, on the other hand, includes both the risk of going bankrupt and how borrowing money can make the earnings per share more unpredictable. When a company borrows money, it has to pay fixed amounts back, which can increase the chances of running out of cash.

We can calculate the coefficient of variation of EPS directly by dividing the standard deviation of EPS by the expected EPS. But, in some cases, we can also calculate it by multiplying the coefficient of variation of EBIT by the degree of financial leverage (DFL) at the expected EBIT level.

DFL shows how much the financial structure of the company amplifies the impact of business risk on the variability of EPS. So, while DFL isn't the same as financial risk, it does affect how much extra risk is caused by borrowing money. Companies with higher business risk may choose to use less financial leverage to limit the extra risk, and vice versa.

12. Total Leverage

Total leverage refers to the combined effect of both financial leverage and operating leverage on a company's earnings per share (EPS) in response to changes in sales volume. It is also called Combined Leverage.



Total leverage magnifies any change in sales into a larger relative change in EPS through a two-step process:
(i) Operating leverage magnifies the impact of changes in sales on operating income (EBIT).
(ii) Financial leverage further magnifies the impact of changes in operating income on earnings per share (EPS).

13. Degree of Total Leverage

The degree of total leverage (DTL) of a firm at a particular level of output (or sales) is equal to the percentage change in earnings per share (EPS) over the percentage change in output (or sales) that causes the change in EPS. It is also called Degree of Combined Leverage (DCL).

Thus DTL (or DCL) is given as:

DEGREE OF TOTAL LEVERAGE

$$DTL = DOL \times DFL$$

$$= \frac{\%\ \text{change in EPS}}{\%\ \text{change in SALES}}$$

We can make use of the fact that the DTL is simply the product of the DOL and the DFL.

We can use thefollowing derived formula to compute the DTL:

$$DTL = \frac{EBIT + FC}{EBIT - I - \frac{PD}{1-t}}$$

$EBIT + FC$ = TOTAL CONTRIBUTION

13. Degree of Total Leverage

A firm produces bicycle helmet that sells for Rs 50 a unit. The company has annual fixed operating costs of Rs 1,00,000, and variable operating costs are Rs 25 a unit. The firm has Rs 2,00,000 in debt at 8% interest. Assume that the tax rate is 40 percent. Determine the degree of total leverage at 8,000 units of production and sales.

Solution:

$$DOL = \frac{EBIT + FC}{EBIT} = \frac{2{,}00{,}000}{1{,}00{,}000} = 2$$

$$DFL = \frac{EBIT}{EBIT - I - \frac{PD}{1-t}} = \frac{1{,}00{,}000}{1{,}00{,}000 - 16000} = 1.19$$

$$DTL = DOL \times DFL = 2 \times 1.19 = 2.38$$

Sales	$50 \times 8000 =$	4,00,000
VC	$25 \times 8000 =$	2,00,000
Contribution		2,00,000
FC		1,00,000
EBIT		1,00,000
Interest @ 8%		16,000
EBT		84,000

$$DTL = \frac{2{,}00{,}000}{84{,}000} = 2.38$$

14. Level of Fixed Cost Financing

Determining an appropriate level of debt for a firm is crucial for its financial health and stability. Let us evaluate the firm's debt capacity, which essentially refers to the maximum amount of debt that the company can reasonably support given its projected cash flows and financial obligations.

One common metric used to assess a company's debt capacity is the **interest coverage ratio (ICR)**. This ratio measures the firm's ability to cover its interest payments with its operating income. It is calculated as below:

$$\text{INTEREST COVERAGE RATIO} = \frac{\text{EBIT}}{\text{INTEREST}}$$

While a higher interest coverage ratio suggests that the firm has sufficient earnings to comfortably meet its interest obligations, a ratio below 1 indicates that the company's earnings are insufficient to cover its interest expenses, which could raise concerns about its ability to service its debt. However, it is important to note that certain industries or business models may tolerate lower interest coverage ratios due to more stable or predictable cash flows.

However, the interest coverage ratio has limitations as it only considers interest payments and does not account for the repayment of principal debt amounts.

To address this, the **debt service coverage ratio (DSCR)** provides a more comprehensive analysis of a firm's ability to meet its debt obligations. The DSCR incorporates both interest payments and principal repayments into its calculation, adjusted for taxes. It is given by:

$$\text{DEBT SERVICE COVERAGE RATIO} = \frac{\text{EBIT}}{\text{INTEREST} + \frac{\text{PRINCIPAL}}{1-t}}$$

PRINCIPAL IS NOT TAX DEDUCTIBLE

By analyzing these metrics, firms can determine an appropriate level of debt that aligns with their financial capabilities and risk tolerance.

14. Level of Fixed Cost Financing

A company has Rs 74 lakh in long-term debt having the following payment schedule:

(a) 15% serial bonds of amount Rs 24,00,000, with Rs 1,00,000 payable annually in principal

(b) 13% first-mortgage bonds of amount Rs 30,00,000, with Rs 1,50,000 payable annually in principal

(c) 18% subordinated debentures of amount Rs 30,00,000, with interest only until maturity in 10 years

The Company’s common stock has a book value of Rs 83 lakh and a market value of Rs 60 lakh. The corporate tax rate is 50%. Its expected EBIT is Rs 20 lakh, with a standard deviation of Rs 15 lakh. The average debt-to-equity ratio of other companies in the industry is 0.47. Determine the interest coverage and the debt-service coverage ratios for the company.

Solution:

TOATAL ANNUAL INTEREST

15% of 24,00,000 = Rs 3,60,000

13% of 30,00,000 = Rs 3,90,000

18% of 20,00,000 = Rs 3,60,000

11,10,000

$$\text{INTEREST COVERAGE RATIO} = \frac{20,00,000}{11,10,000} = 1.80$$

PRINCIPAL PAID ANNUALLY = 1,00,000 + 1,50,000

= 2,50,000

DEBT SERVICE COVERAGE RATIO

$$= \frac{20,00,000}{11,10,000 + \frac{2,50,000}{1-0.50}} = 1.24$$

15. Margin of Safety (MOS)

Margin of Safety (MOS) is the difference between actual sales and break-even sales. In other words, all sales revenue that a company collects over and above its break-even point represents the margin of safety.

MARGIN OF SAFETY

$$MoS = \frac{\text{SALES} - \text{BREAK EVEN SALES}}{\text{SALES}} \times 100$$

$$= \frac{EBIT}{EBIT + FC}$$

which is inverse of DoL

$$MOS = \frac{1}{DOL}$$

Higher margin of safety indicates lower business risk and higher profit and vice versa.

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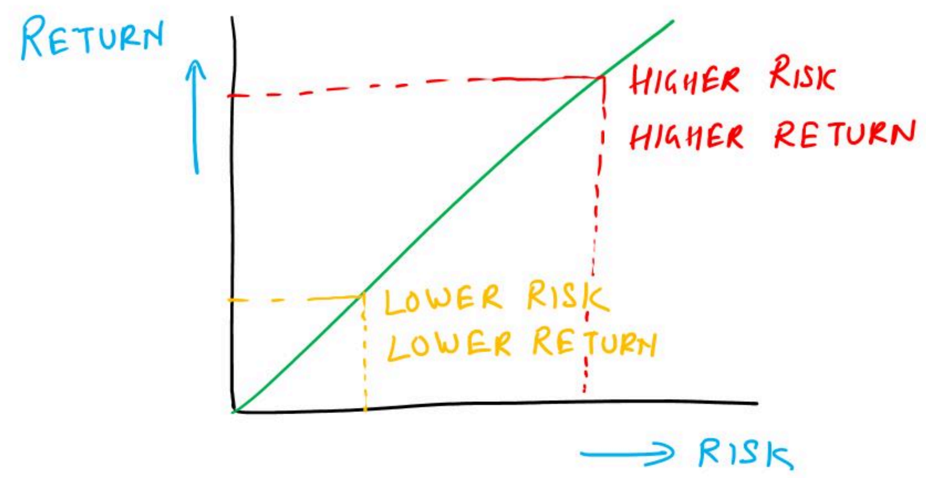
1. Introduction

Risk and return are two fundamental concepts in finance that are closely related and essential for investors to consider when making investment decisions.



Return refers to the gain or loss made on an investment over a specific period, typically expressed as a percentage of the initial investment. It is the reward or profit that investors receive for taking on investment risk. Returns can come in various forms, including capital gains (increase in the value of an asset) and income (such as interest or dividends).

Risk refers to the uncertainty or variability associated with the potential returns of an investment. It represents the possibility that the actual returns of an investment may differ from the expected returns. For example, investing in stocks carries risk. The value of a stock may fluctuate due to market conditions, potentially resulting in losses or gains.

Risk-return Tradeoff

The relationship between risk and return is summarized by the principle of "risk-return tradeoff." This principle suggests that higher returns are generally associated with higher levels of risk, and conversely, lower levels of risk are typically associated with lower potential returns.

This relationship implies that investors must be compensated for taking on additional risk. In other words, if an investment is riskier, investors will demand a higher expected return to justify that risk. Conversely, if an investment is less risky, investors will generally accept a lower expected return.

For instance, investing in a fixed deposit (FD) with a bank in India may yield around 7%, representing a low-risk option. Conversely, investing in a mutual fund might offer returns of around 12%, but with higher risk compared to a fixed deposit. Real estate investments may yield returns of approximately 15%, while investing in stocks of stable companies like Infosys may offer returns of around 18%. However, investing in stocks of new age companies like Paytm, Zomato could potentially yield returns of approximately 25%, but with the highest associated risk. These are illustrative numbers.

2. Measuring Risk

Now that we acknowledge the presence of risk inherent in returns, how can we quantify this risk?

The actual rate of return can be conceptualized as a random variable, subject to uncertainty and fluctuation. As such, it follows a probability distribution. Any probability distribution can be characterized by two fundamental variables: the expected value and the standard deviation.

Therefore, the variability in returns (risk), can be summarized by considering 2 key parameters:

1. Expected Return
This represents the average or mean return an investor can anticipate over a specified period. This is the return that an individual expects a security. Of course, because this is only an *expectation*, the actual return may be either higher or lower. It serves as a central measure around which actual returns are likely to fluctuate.

$$\text{EXPECTED RETURN, } \bar{r} = \sum_{i=1}^{n} r_i \cdot p_i$$

n = Number of possibilities
p_i = Probability of ith possibility
r_i = Return for ith possibility

Thus the expected return is nothing but a weighted average of the possible returns, with the weights being the probabilities of occurrence of returns.

2. Standard Deviation
The standard deviation quantifies the degree of dispersion or variability of returns around the expected return. It provides insight into the spread of potential returns and thus the level of risk associated with the investment.

$$\text{VARIANCE, } \sigma^2 = \sum (r_i - \bar{r})^2 \times p_i$$

$$\text{STANDARD DEVIATION, } \sigma = \sqrt{\sum (r_i - \bar{r})^2 \times p_i}$$

The greater the standard deviation of returns, the greater the variability of returns, and the greater the risk of the investment.

The square of standard deviation is the variance.

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3. Comparative Risk

The standard deviation alone will not accurately reflect the **comparative risk** or uncertainty between different investment options, particularly if they vary in size.

Let us consider two investment opportunities, A and B.

	OPTION A	OPTION B
EXPECTED RETURN	80	240
STANDARD DEVIATION	6	8
COEFFICIENT OF VARIATION	0.075	0.033

While it may seem intuitive to conclude that because the standard deviation of B is larger than that of A, it is inherently the riskier investment, this conclusion is not accurate.

In reality, relative to the size of the expected return, investment A exhibits greater variation.

To address this issue of scale, the standard deviation can be adjusted by dividing it by the expected return to compute the coefficient of variation (CV). The coefficient of variation serves as a measure of relative dispersion or risk, indicating the risk "per unit of expected return." A higher CV signifies a larger relative risk associated with the investment.

$$\text{COEFFICIENT OF VARIATION, } CV = \frac{\text{STANDARD DEVIATION}}{\text{EXPECTED RETURN}} = \frac{\sigma}{\bar{x}}$$

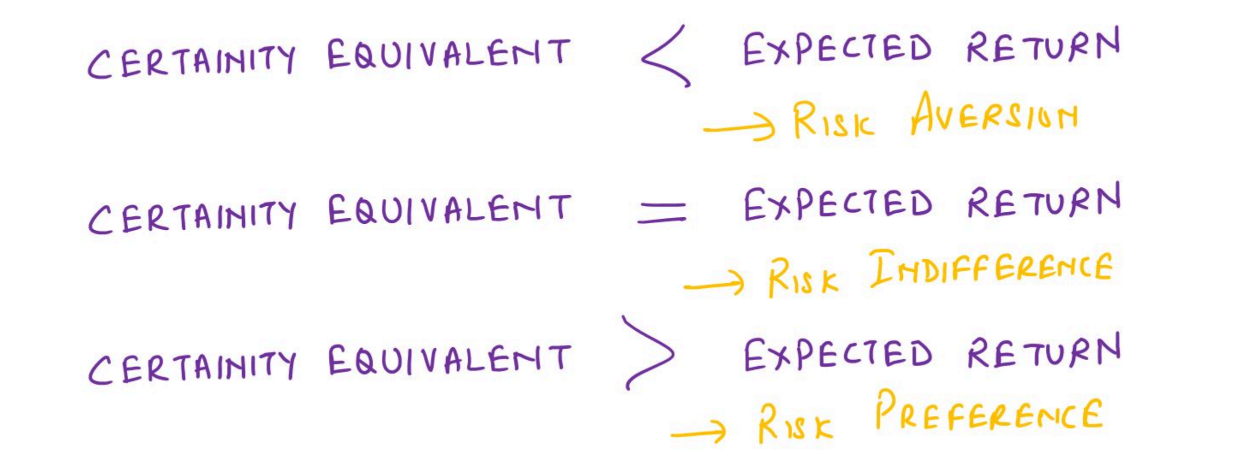
By using the coefficient of variation as our risk measure, we find that investment A, with a return distribution CV of 0.075, is perceived as riskier compared to investment B, which has a CV of only 0.033. This adjustment allows for a more accurate assessment of the risk inherent in each investment option relative to their respective expected returns.

4. Certainty Equivalent

Imagine you have two options:
(i) one where you know exactly what you'll get, and
(ii) another where you might get more or less (some element of probability).

The **certainty equivalent** is the amount of money you'd need to be guaranteed to feel just as happy as taking the riskier option.

Imagine you're offered a gamble where you have a 50% chance of winning Rs 200 and a 50% chance of winning nothing. The expected value (expected return) of this gamble is Rs 100.



Now, let's say you're given the option to take a certain amount of money instead of playing the gamble. Say, if you'd accept Rs 90 for sure instead of taking the gamble, then Rs 90 is your certainty equivalent for this risky opportunity. It means you value the certain Rs 90 as much as the uncertain gamble with an expected value of Rs 100.

There can be 3 scenarios:

1. If your certainty equivalent is less than the expected value, you're **risk-averse**. You prefer the sure thing over the risky option. For example certainty equivalent is Rs 80 and the expected return is Rs 100.

2. If your certainty equivalent equals the expected value, you're **risk-neutral**. You're indifferent between the sure thing and the risky option. For example certainty equivalent is Rs 100 and the expected return is Rs 100.

3. If your certainty equivalent is greater than the expected value, you have **risk-preference.** You prefer the risky option over the sure thing. For example certainty equivalent is Rs 130 and the expected return is Rs 100.

4. Certainty Equivalent

Suppose, you recently purchased a stock that is expected to earn 12 percent in a booming economy, 8 percent in a normal economy and lose 5 percent in a recessionary economy. There is a 15 percent probability of a boom, a 75 percent chance of a normal economy, and a 10 percent chance of a recession. What is your expected rate of return on this stock?

Solution:

STATE OF ECONOMY	RETURN (r_i)	PROBABILITY (p_i)	$r_i \times p_i$
BOOMING	0.12	0.15	0.018
NORMAL	0.08	0.75	0.060
RECESSION	-0.05	0.10	-0.005
			0.073
			7.3%

$$\sum_{i=1}^{n} r_i \times p_i$$

4. Certainty Equivalent

A stock's return has the following distribution:

Demand for the Company's Products	Probability of this demand occurring	Rate of return if this demand occurs (%)
Weak	0.1	-50
Below average	0.2	(5)
Average	0.4	16
Above average	0.2	25
Strong	0.1	60
Total	1.0	

Calculate the stock's expected return, standard deviation, and coefficient of variation.

Solution:

DEMAND	RETURN (r_i)	PROBABILITY (p_i)	$r_i \times p_i$
WEAK	−50	0.1	−5
BELOW AVERAGE	−5	0.2	−1
AVERAGE	16	0.4	6.4
ABOVE AVERAGE	25	0.2	5
STRONG	60	0.1	6
			11.4

EXPECTED RETURN → 11.4

$\bar{r} = \sum p_i \times r_i$

$\bar{r} = 11.4$

DEMAND	RETURN (r_i)	PROBABILITY (p_i)	$(r_i - \bar{r})^2$	$(r_i - \bar{r})^2 \times p_i$
WEAK	−50	0.1	3769.96	377
BELOW AVERAGE	−5	0.2	268.96	53.79
AVERAGE	16	0.4	21.16	8.464
ABOVE AVERAGE	25	0.2	184.96	36.99
STRONG	60	0.1	2361.96	236.20
				712.44

STANDARD DEVIATION, (σ) $\sqrt{(r_i-\bar{r})^2 \times p_i} = \sqrt{712.44}$

$= 26.69$

Coefficient of Variation (CV) $= \frac{\sigma}{\bar{x}} = \frac{26.69}{11.4} = 2.34$

4. Certainty Equivalent

Based on the following information, calculate the expected return and standard deviation for Stock Alpha (A) and Stock Beta (B).

State of Economy	Probability of Occurance	Rate of Return of Stock A	Rate of Return of Stock B
Recession	20%	6%	-20%
Normal	55%	7%	13%
Boom	25%	11%	33%

Solution:

STOCK ALPHA

EXPECTED RETURN $= w_1\bar{r}_1 + w_2\bar{r}_2 + w_3\bar{r}_3$

$= 0.20 \times 0.06 + 0.55 \times 0.07 + 0.25 \times 0.11$

$= 0.0780$ 7.80%

VARIANCE $= \sum (r_i - \bar{r})^2 \times p_i$

$= 0.20 \times (0.06 - 0.078)^2 + 0.55 \times (0.07 - 0.078)^2 + 0.25 \times (0.11 - 0.078)^2$

$= 0.00036$

STANDARD DEVIATION $= \sqrt{\text{VARIANCE}} = \sqrt{0.00036} = 0.0189$ 1.89%

STOCK BETA

EXPECTED RETURN $= 0.20 \times -0.20 + 0.55 \times 0.13 + 0.25 \times 0.33$

$= 0.1140$ 11.40%

VARIANCE $= 0.20 \times (-0.20 - 0.1140)^2 + 0.55 \times (0.13 - 0.1140)^2 + 0.25 \times (0.33 - 0.1140)^2$

$= 0.03152$

STANDARD DEVIATION $= \sqrt{0.03152} = 0.1775$ 17.75%

5. Geometric Average Return

Geometric average return is an alternative to arithmetic average return, providing insights into compound returns over a specific period.

While arithmetic average return indicates the average annual return over a period, geometric average return measures the average compound return per year during the same period.

The arithmetic average return answers the question, “What was your return in an average year over a particular period?”

$$\left[(1+r_1)(1+r_2)(1+r_3)\cdots\cdots \times (1+r_n)\right]^{\frac{1}{n}} - 1$$

$\Longrightarrow$ Geometric Average Return

$$\frac{r_1+r_2+r_3+\cdots r_n}{n}$$

$\Longrightarrow$ Arithemetic Average Return

As such, arithmetic returns do not account for the effects of compounding. Geometric returns do account for the effects of compounding. As an investor, the more important return of an asset is the geometric return.

The geometric average tells you what you actually earned per year on average, compounded annually. The arithmetic average tells you what you earned in a typical year and is an unbiased estimate of the true mean of the distribution. The geometric average is very useful in describing the actual historical investment experience. The arithmetic average is useful in making estimates of the future.

5. Geometric Average Return

The stock price of a company rose by 13.75% in the first year, 35.70% in the second year, and 45.08% in the third year. However, due to micro-economic conditions, it declined by 8.80% in the fourth year and then further decreased by 25.13% in the fifth year. Determine the geometric average return and arithmetic average return for the five-year period.

Solution:

$t = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$13.75\% \uparrow \quad 35.70\% \uparrow \quad 45.08\% \uparrow \quad 8.80\% \downarrow \quad 25.13\% \downarrow$

$$\text{Geometric Average Return} = \left(1.1375 \times 1.3570 \times 1.4508 \times 0.9120 \times 0.7487\right)^{\frac{1}{5}} - 1$$

$$= (1.5291)^{\frac{1}{5}} - 1 = 0.0887 \quad \boxed{8.87\%}$$

$$\text{Arithmetic Average Return} = \frac{13.75 + 35.70 + 45.08 - 8.80 - 25.13}{5}$$

$$= \boxed{12.12\%}$$

5. Geometric Average Return

A stock has had returns of 27 percent, 12 percent, 32 percent, -12 percent, 19 percent, and -31 percent over the last six years. What are the arithmetic and geometric returns for the stock?

Solution:

27 12 32 -12 19 -31

ARITHEMATIC AVERAGE RETURN $= \frac{27+12+32-12+19-31}{6} = 7.83\%$

GEOMETRIC AVERAGE RETURN $= (1.27 \times 1.12 \times 1.32 \times 0.88 \times 1.19 \times 0.69)^{\frac{1}{6}} - 1$

$= 0.0522$ 5.22%

Remember, the geometric average return will always be less than the arithmetic average return if the returns have any variation.

6. Portfolio Risk and Return

Up to this point, our discussion has centered on assessing the risk and return of individual investments operating in isolation. However, it is rare for investors to allocate all their wealth into just one asset or investment. Instead, they typically build a portfolio, which involves assembling a collection of investments.

A **portfolio** refers to a combination of two or more securities or assets. For instance, a portfolio might consist of stocks, bonds, real estate, and commodities, all held together as part of an investor's overall investment strategy. Each investment within the portfolio contributes to its overall risk and return.

Return of Portfolio

The expected return of a portfolio is simply a weighted average of the expected returns of the securities constituting that portfolio. The weights are equal to the proportion of total funds invested in each security.

$$\text{EXPECTED RETURN OF PORTFOLIO} = \sum_{j=1}^{m} w_j \times \bar{r}_j$$

w_j = Weight of funds invested in Security j

$\bar{r}_j$ = Expected return for Security j

Risk of Portfolio

The total risk of a portfolio is measured by the standard deviation of the probability distribution of possible security returns. The portfolio standard deviation is given by:

$$\text{RISK OF PORTFOLIO}, \ \sigma_p = \sqrt{\sum_{j=1}^{m} \sum_{k=1}^{m} w_j \cdot w_k \cdot Cov(j,k)}$$

w_j, w_k = Proportion of funds invested in Security j and K.

$Cov(j,k)$ = Covariance between returns of j and K

m = Total number of Securities in Portfolio

$$Cov(j,k) = r \times \sigma_j \times \sigma_K$$

σ_j and σ_K = Standard Deviation of j and K. r = Correlation Coefficient

From the formula, we can see that the standard deviation (risk) for a portfolio depends not only on the variance (or standard deviation) of the individual securities but also on the covariances between various securities that have been paired.

Covariance measures the degree to which two variables change together. A positive covariance indicates that the variables tend to move in the same direction, while a negative covariance suggests they move in opposite directions.

$$Cov(1,2) = \sigma_1 \times \sigma_2 \times r$$

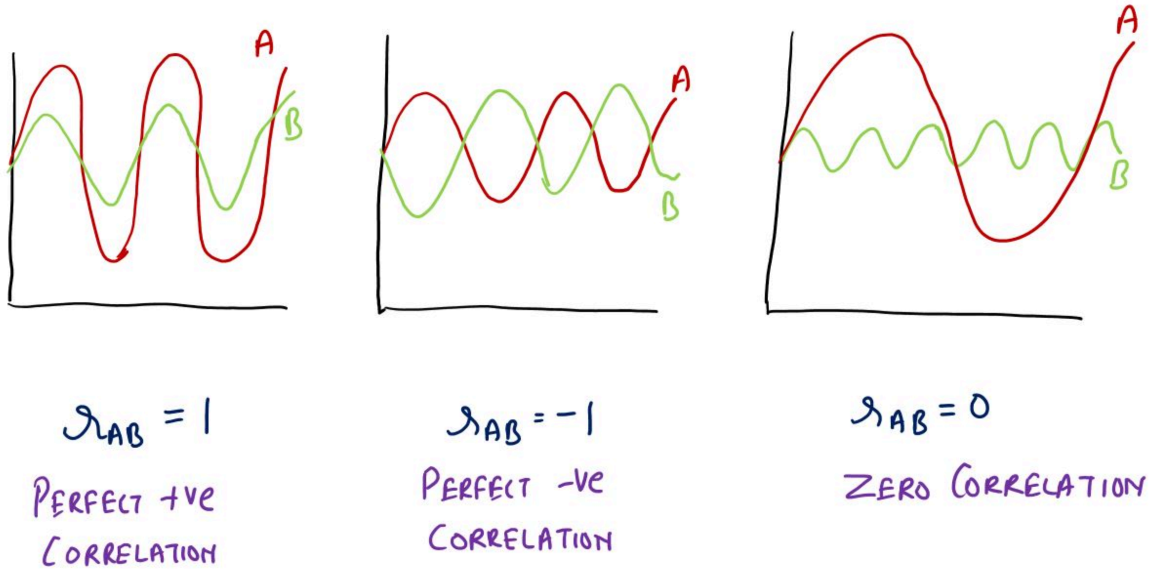
$Cov(1,2)$: Covariance between two securities

σ_1: Standard Deviation of Security 1

σ_2: Standard Deviation of Security 2

r: Coefficient of Correlation between 1 and 2

As the number of securities in a portfolio increases, the covariance terms become more important in comparison to the variance terms (or standard deviation terms). As a portfolio expands further to include all securities, covariance clearly becomes the more dominant factor.



The correlation coefficient (r) always lies in the range from −1.0 to +1.0. A positive correlation coefficient indicates that the returns from two securities generally move in the same direction, whereas a negative correlation coefficient implies that they generally move in opposite directions. The stronger the relationship, the closer the correlation coefficient is to one of the two extreme values. A zero correlation coefficient implies that the returns from two securities are uncorrelated; they show no tendency to vary together in either a positive or negative linear fashion.

If there are only 2 securities in the portfolio, then the standard deviation (risk) formula reduces to:

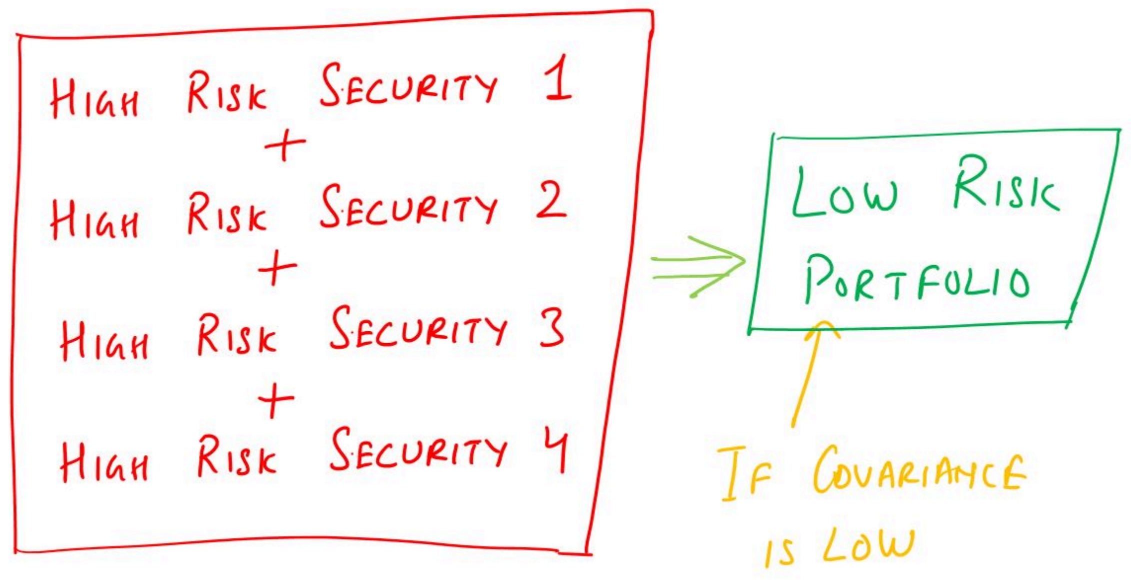
$$\sigma_P = \sqrt{w_1^2 \sigma_1^2 + 2 w_1 w_2 r \sigma_1 \sigma_2 + w_2^2 \sigma_2^2}$$

w_1, w_2 = Weights of securities 1 and 2

σ_1, σ_2 = Standard Deviation of 1 and 2

r = Correlation Coefficient between 1 and 2

We have seen that the riskiness of a portfolio depends much more on the paired security covariances than on the riskiness (standard deviations) of the separate security holdings. This means that a combination of individually high risky securities could still constitute a low-risk portfolio as long as securities do not move in lockstep with each other. In short, low covariances lead to low portfolio risk.



In simpler words, even if you have some risky securities, your overall portfolio could still be low risk if these securities don't all move in the same direction at the same time.

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6. Portfolio Risk and Return

Assume an investment manager has created a portfolio with the Stock A and Stock B. Stock A has an expected return of 20% and a weight of 30% in the portfolio. Stock B has an expected return of 15% and a weight of 70%. What is the expected return of the portfolio?

Solution:

$$\text{Expected Return on Portfolio} = \sum w_j \times \bar{r}_j$$

$$= w_1 \bar{r}_1 + w_2 \bar{r}_2$$

$$= 0.30 \times 0.20 + 0.70 \times 0.15$$

$$= 0.165 \quad 16.5\%$$

---

6. Portfolio Risk and Return

Two securities A and B generate the following sets of expected returns, standard deviations and correlation coefficient, with given weightage. Compute the expected return and the risk of a portfolio.

Securities	A	B
Return	20%	25%
Weightage, $\mathbf{w}$	40%	60%
Standard Deviation, σ	50	30
Correlation Coefficient, ρ_{AB}	-0.60	

Solution:

EXPECTED RETURN, $\bar{r}_p = w_1 \bar{r}_1 + w_2 \bar{r}_2$

$= 0.40 \times 20 + 0.60 \times 25 = 23\%$

RISK OF PORTFOLIO, σ_p

$$= \sqrt{w_1^2 \sigma_1^2 + 2 w_1 w_2 r \sigma_1 \sigma_2 + w_2 \sigma_2^2}$$

$$= \sqrt{0.40^2 \times 50^2 + 2 \times 0.40 \times 0.60 \times (-60) \times 50 \times 30 + 0.60^2 \times 30^2}$$

$$= \sqrt{400 - 432 + 324}$$

$$= \sqrt{292} = 17.09\%$$

6. Portfolio Risk and Return

Stock A has an expected value of annual return of 16 percent, with a standard deviation of 15 percent. Stock B has an expected value of annual return of 14 percent and a standard deviation of 12 percent. Further, the expected correlation coefficient between the two stocks is 0.40. If an investor decides to invest equal amounts in each of the two stocks, what would be the expected return and risk for the portfolio?

Solution:

Expected Return of Portfolio is:

$$\bar{r}_p = w_1 \bar{r}_1 + w_2 \bar{r}_2$$
$$= 0.50 \times 0.16 + 0.50 \times 0.14 = 0.15$$
$$15\%$$

Risk of Portfolio is:

$\bar{r}_1 = 0.16$ $\sigma_1 = 0.15$ $\bar{r}_2 = 0.14$ $\sigma_2 = 0.12$ $r = 0.40$

$w_1 = 0.50$ $w_2 = 0.50$

$$\sigma_p = \sqrt{w_1 w_1 \operatorname{Cov}(1,1) + w_1 w_2 \operatorname{Cov}(1,2) + w_2 w_1 \operatorname{Cov}(2,1) + w_2 w_2 \operatorname{Cov}(2,2)}$$
$$= \sqrt{w_1^2 \sigma_1^2 + w_1 w_2 r \sigma_1 \sigma_2 + w_2 w_1 r \sigma_1 \sigma_2 + w_2^2 \sigma_2^2}$$
$$= \sqrt{0.50^2 \times 0.15^2 + 0.50 \times 0.50 \times 0.40 \times 0.15 \times 0.12 + 0.50 \times 0.50 \times 0.40 \times 0.12 \times 0.15 + 0.50^2 \times 0.12^2}$$

$$= \sqrt{0.012825}$$
$$= 0.113 \quad 11.3\%$$

6. Portfolio Risk and Return

You have Rs 10,000 to invest in a stock portfolio. Your choices are Stock X with an expected return of 14 percent and Stock Y with an expected return of 9 percent. If your goal is to create a portfolio with an expected return of 12.9 percent, how much money will you invest in Stock X and in Stock Y ?

Solution:

EXPECTED RETURN $= w_1 \bar{r}_1 + w_2 \bar{r}_2$

$$0.129 = x \times 0.14 + (1-x) \times 0.09$$

$$\Longrightarrow x = 0.78$$

INVEST IN X = 78% of 10,000 = Rs 7800

INVEST IN Y = 10,000 − 7800 = Rs 2200

6. Portfolio Risk and Return

Based on the below information:

(i) What is the expected return on an equally weighted portfolio of these three stocks?

(ii) What is the variance of a portfolio invested 20 percent each in A and B, and 60 percent in C ?

State of Economy	Probability of Occurance	Rate of Return of Stock A	Rate of Return of Stock B	Rate of Return of Stock C
Boom	0.65	0.07	0.15	0.33
Bust	0.35	0.13	0.03	-0.06

Solution:

(i) What is the expected return on an equally weighted portfolio of these three stocks?

EXPECTED RETURN (BOOM) $= \frac{1}{3} \times 0.07 + \frac{1}{3} \times 0.15 + \frac{1}{3} \times 0.33$

$= 0.1833$

EXPECTED RETURN (BUST) $= \frac{1}{3} \times 0.13 + \frac{1}{3} \times 0.03 + \frac{1}{3} \times -0.06$

$= 0.0333$

EXPECTED RETURN OF PORTFOLIO $= 0.65 \times 0.1833 + 0.35 \times 0.0333$

$= 0.1308$ 13.08%

(ii) What is the variance of a portfolio invested 20 percent each in A and B, and 60 percent in C ?

EXPECTED RETURN (BOOM) $= 0.20 \times 0.07 + 0.20 \times 0.15 + 0.60 \times 0.33$

$= 0.2420$ 24.20%

EXPECTED RETURN (BUST) $= 0.20 \times 0.13 + 0.20 \times 0.03 + 0.60 \times (-0.06)$

$= -0.0040$ −0.40%

EXPECTED RETURN OF PORTFOLIO $= 0.65 \times 0.2420 + 0.35 \times (-0.0040)$

$= 0.1559$ 15.59%

VARIANCE $= 0.65 \times (0.2420 - 0.1559)^2 + 0.35(-0.0040 - 0.1559)^2$

$= 0.01377$

6. Portfolio Risk and Return

Based on the following information, calculate the expected return and standard deviation of each of the following stocks. Assume each state of the economy is equally likely to happen. What are the covariance and correlation between the returns of the two stocks?

State of Economy	Rate of Return of Stock A	Rate of Return of Stock B
Bear	0.102	-0.045
Normal	0.115	0.148
Bull	0.073	0.233

Solution:

EXPECTED RETURN

STOCK A $\Rightarrow \frac{1}{3} \times 0.102 + \frac{1}{3} \times 0.115 + \frac{1}{3} \times 0.073 = 0.0967$

STOCK B $\Rightarrow \frac{1}{3} \times (-0.045) + \frac{1}{3} \times 0.148 + \frac{1}{3} \times 0.233 = 0.1120$

VARIANCE AND STANDARD DEVIATION

STOCK A $(\sigma^2) = \frac{1}{3} \times (0.102 - 0.0967)^2 + \frac{1}{3} \times (0.115 - 0.0967)^2 + \frac{1}{3} \times (0.073 - 0.0967)^2 = 0.00031$

STOCK A $(\sigma) = \sqrt{0.00031} = 0.0176$ 1.76%

STOCK B $(\sigma^2) = \frac{1}{3} \times (-0.045 - 0.1120)^2 + \frac{1}{3} \times (0.148 - 0.1120)^2 + \frac{1}{3} \times (0.233 - 0.1120)^2$

$= 0.01353$

STOCK B $(\sigma) = \sqrt{0.01353} = 0.1163$ 11.63%

Covariance of A and B

$Cov(A, B) = \frac{1}{3}(0.102 - 0.0967)(-0.045 - 0.1120) + \frac{1}{3}(0.115 - 0.0967)(0.148 - 0.1120) + \frac{1}{3}(0.073 - 0.0967)(0.233 - 0.1120)$

$= -0.001014$

Correlation Coefficient between A and B

$$\rho = \frac{Cov(A,B)}{\sigma_A \cdot \sigma_B} = \frac{-0.001014}{0.0176 \times 0.1163} = -0.4964$$

6. Portfolio Risk and Return

Security F has an expected return of 10 percent and a standard deviation of 43 percent per year. Security G has an expected return of 15 percent and a standard deviation of 62 percent per year.

(i) What is the expected return on a portfolio composed of 30 percent of Security F and 70 percent of Security G ?

(ii) If the correlation between the returns of Security F and Security G is 0.25, what is the standard deviation of the portfolio described in part (i)?

Solution:

EXPECTED RETURN ON PORTFOLIO

$$= W_F \bar{r}_F + W_G \bar{r}_G$$

$$= 0.30 \times 0.10 + 0.70 \times 0.15 = 0.1350 \quad 13.50\%$$

VARIANCE OF PORTFOLIO

$$\sigma^2 = W_F^2 \sigma_F^2 + 2 W_F W_G r \sigma_F \sigma_G + W_G^2 \sigma_G^2$$

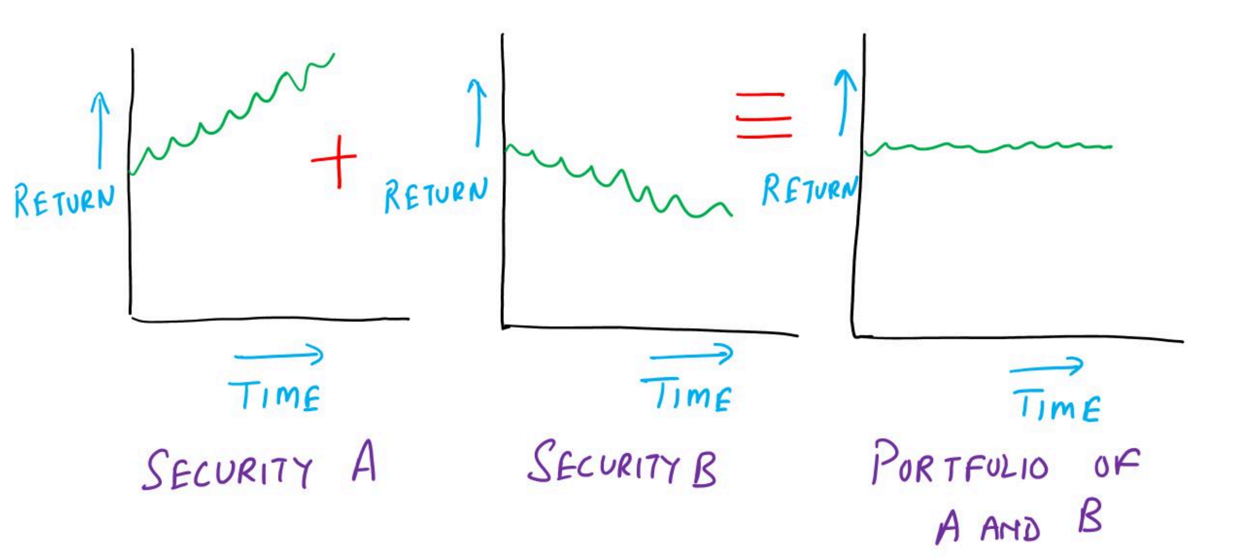
$$= 0.30^2 \times 0.43^2 + 2 \times 0.30 \times 0.70 \times 0.25 \times 0.43 \times 0.62 + 0.70^2 \times 0.62^2$$

$$= 0.23299$$

STANDARD DEVIATION σ $= \sqrt{0.23299} = 0.4827 = 48.27\%$

7. Diversification

In simple terms, diversification means spreading out your investments to lower the overall risk. It's like the saying "Don't put all your eggs in one basket." Instead of investing all your money in just one thing, you spread it across different investments.



But it is not just about splitting your money evenly among many investments. That's a basic idea, but it doesn't take into account how different investments might behave together. For example, if you put Rs 1,00,000 into 10 stocks all from the same industry, they might all go up and down at the same time. But if you put the same amount into 5 stocks from different industries, they might not all move in sync.

Effective diversification means putting together investments that don't move exactly the same way. For instance, if one investment tends to do well when the economy is strong but another does well when the economy is weak, they might balance each other out. So even if one goes down, the other might go up, reducing the overall ups and downs of your investments.

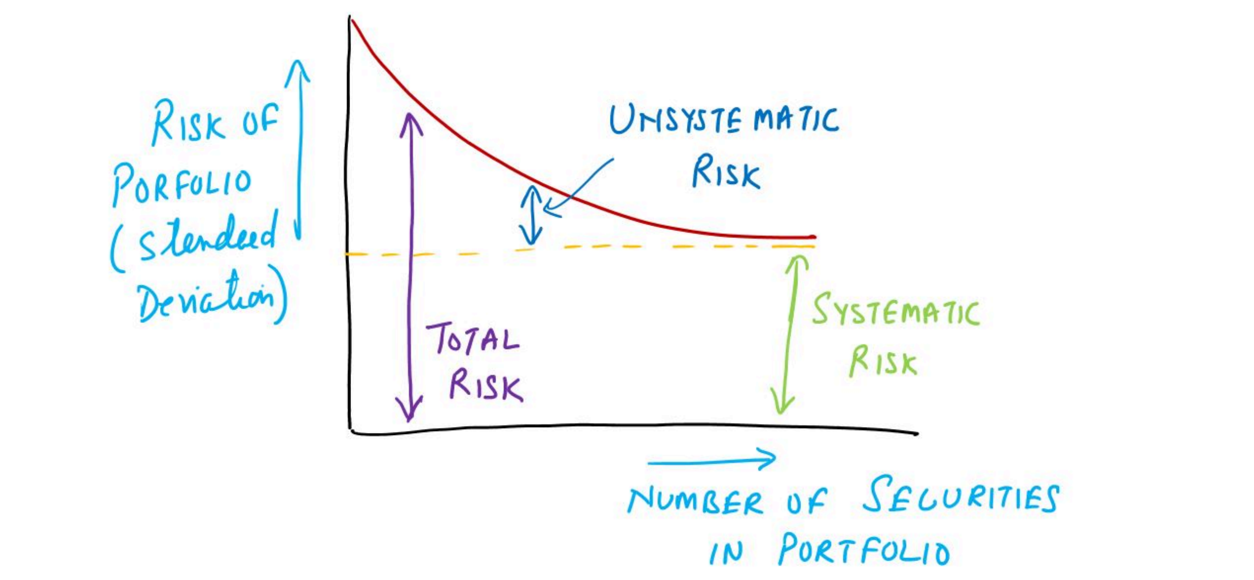
Benefits of diversification, in the form of risk reduction, occur ONLY WHEN the securities are not perfectly, positively correlated.

To conclude, the standard deviation of a portfolio of securities is less than the weighted average of the standard deviations of the individual securities as long as there is less than perfect positive correlation. The **diversification principle will work, only when the coefficient of correlation between two securities is less than 1.**

When you invest in different countries' financial markets instead of just one country, you also get more diversification. That's because different countries' economies don't always follow the same patterns. If one country's economy is struggling, another might be doing better. This helps spread out the risk even more, making your overall investment safer.

7. Diversification

We have learnt that as we add more securities to a portfolio, the overall risk of the portfolio tends to decrease, as long as these securities aren not perfectly positively correlated.



Starting with just one stock, the risk of the portfolio is basically the standard deviation of that single stock. But as we add more randomly chosen stocks to the portfolio, the total risk of the portfolio goes down. However, this reduction happens at a slower rate as we keep adding more stocks. We can see this is in the figure.

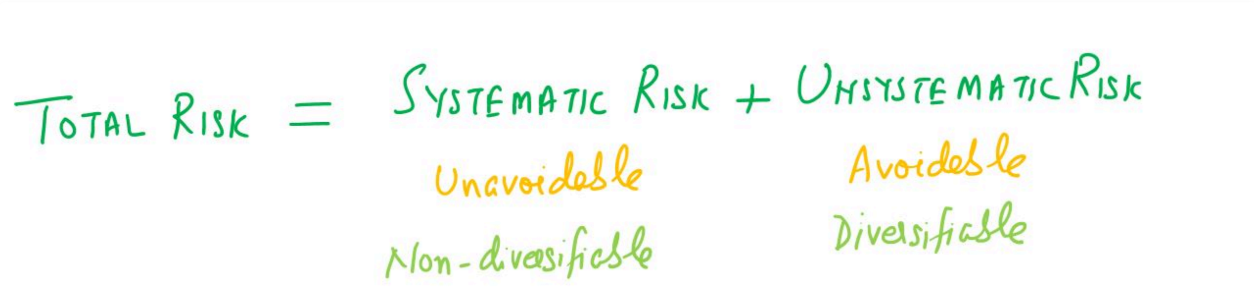
However, does this risk mitigation apply universally across all types of risk? **No**.

The total risk can be categorized into 2 types.

Systematic Risk

Firstly, there is **systematic risk**, which comes from factors affecting the entire market. These could be things like changes in the economy, new tax laws, or shifts in the global energy situation. These are risks that impact all securities and can't be avoided through diversification. It is also known as **market risk** or **un-diversifiable risk** or **volatility risk** or **relevant risk**.

Interest risk, *market risk*, and *inflationary risk* are all forms of systematic risk that affect the entire market or economy. These risks arise from factors like changes in interest rates, overall market trends, and fluctuations in inflation rates, impacting all securities in a similar manner.



Unsystematic Risk

Secondly, there is **unsystematic risk**, which is specific to individual companies or industries. It is not related to broader economic or political factors. For example, a company might face a strike from its workers, competition from a new rival, or become outdated due to a technological breakthrough. By diversifying your portfolio, you can reduce and sometimes even eliminate this type of risk if you diversify effectively. It is also known as **specific risk**, **diversifiable risk**, **idiosyncratic risk** or **residual risk**.

Business risk, *credit risk*, and *operational risk* are examples of unsystematic risk. These risks are specific to individual companies or industries and are not related to broader economic factors. Business risk relates to a company's operations and liquidity, credit risk pertains to the ability of a borrower to repay debt, and operational risk involves the day-to-day functioning of a business. These risks can be reduced through diversification.

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8. Capital Asset Pricing Model

We have learnt that the systematic risk, unlike unsystematic risk, can not be reduced through diversification. So, a security is expected to provide a return that matches its systematic risk. Basically, the higher the systematic risk of a security, the higher the return investors will demand from it.

This connection between expected return and systematic risk, and how securities are valued accordingly, lies at the core of **William Sharpe**'s Capital Asset Pricing Model (CAPM), which he developed in the 1960s. CAPM has been incredibly influential in the field of finance ever since its inception.

CAPM helps us make certain conclusions about risk and how big of a risk premium is needed to compensate for taking on risk.

Assumptions of CAPM
The assumptions underlying the CAPM are summarized below:
1. All investors focus on a single holding period, and they seek to maximize the expected utility of their terminal wealth by choosing among alternative portfolios on the basis of each portfolio’s expected return and standard deviation.
2. All investors can borrow or lend an unlimited amount at a given risk-free rate of interest.
3. All investors have identical estimates of the expected returns, variances, and covariances among all assets (that is, investors have homogeneous expectations).
4. All assets are perfectly divisible and perfectly liquid (that is, marketable at the going price).
5. There are no transactions costs.
6. There are no taxes.
7. All investors are price takers (that is, all investors assume that their own buying and selling activity will not affect stock prices).
8. The quantities of all assets are given and fixed.

There are two main types of investment returns we focus on with CAPM:

The first is a risk-free security return, where the return over the holding period is known without any doubt. Often, the rate on Treasury securities (Government T-Bills) with short to intermediate terms is used as a stand-in for the risk-free rate.

The second is the return of market portfolio of common stocks, which represents all available common stocks weighted by their total market values. Because the market portfolio can be complex to deal with, many people use proxies like the SENSEX or NIFTY index, which reflect the performance of major common stocks in a broad-based, market-value-weighted manner.

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8. Capital Asset Pricing Model

In CAPM, one of the terms used is Expected Return on Market Portfolio. What does it mean?

In a perfect world, all investors have access to information like data about past price movements and other publicly available information. Thus the estimates of the expected returns and variances for individual securities and the covariances between pairs of securities are likely to be same. This assumption is called **homogeneous expectations.**

If all investors have homogeneous expectations, then all investors would sketch out the same efficient frontier of risky assets because they would be working with the same inputs. Because the same risk-free rate would apply to everyone, all investors would be finally arriving at same portfolio of risky assets to be held. It is the **market portfolio**. The corresponding return is called Expected Return on Market Portfolio.

In practice, economists use a broad-based index such as the SENSEX as a proxy for the market portfolio.

8. Capital Asset Pricing Model

We have understood that, if we assume that financial markets are efficient and that investors as a whole are efficiently diversified, we need not worry about unsystematic risk.

The major risk associated with a stock becomes its systematic risk. The greater the risk of a stock, the greater the return expected. If we assume that unsystematic risk is totally diversified away, the required rate of return for stock s will be:

MARKET RISK PREMIUM
EXCESS MARKET RETURN

$$r_s = r_f + \beta \overbrace{(r_m - r_f)}^{\text{Market Risk Premium / Excess Market Return}}$$

Risk Free Return

Return on Stock

Risk Free Return

Beta, Risk of stock

Expected Return on Market Portfolio

Put another way, the required rate of return for a stock is equal to the return required by the market for a riskless investment plus a risk premium. This is famous **CAPM equation**.

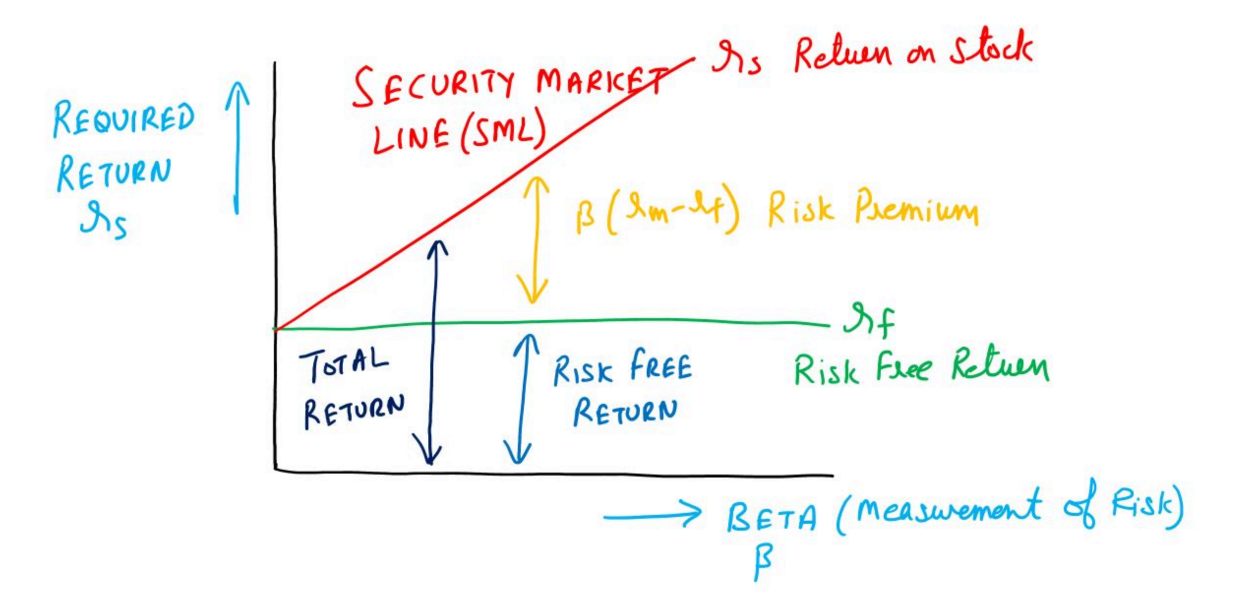
In turn, the risk premium is a function of:
(i) the expected market return less the risk-free rate, which represents the risk premium required for the typical stock in the market; and
(ii) the beta coefficient (which measures the riskiness of the stock).

Suppose that the expected return on Treasury securities is 8%, the expected return on the market portfolio is 13%, and the beta of a firm is 1.3. The required return on the stock would be:

$$
\begin{aligned}
r_s &= r_f + \beta(r_m - r_f) \\
&= 0.08 + 1.3(0.13 - 0.08) \\
&= 0.145 \quad 14.5\%
\end{aligned}
$$

What this tells us is that, based on CAPM computation, the market expects the firm to show a 14.5% annual return.

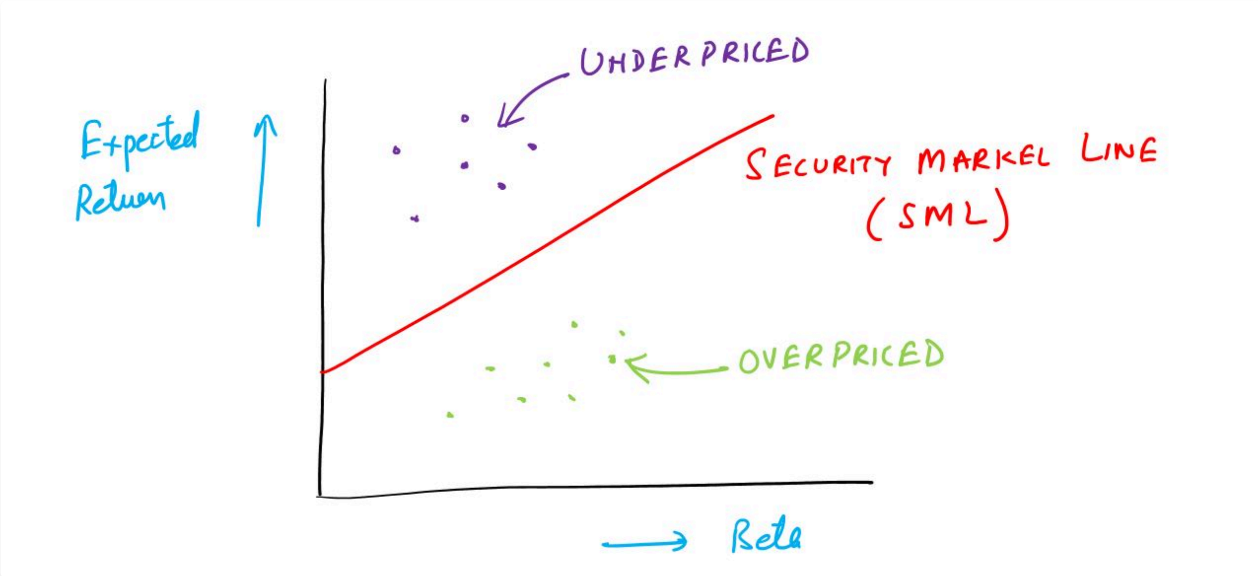
Security Market Line
The CAPM equation describes the relationship between an individual security's expected return and its systematic risk, as measured by beta. This linear relationship is known as the security market line (SML).



The expected return is shown on the vertical axis. Beta, our index of systematic risk, is on the horizontal axis. At zero risk, the security market line has an intercept on the vertical axis equal to the risk-free rate. Even when no risk is involved, investors still expect to be compensated for the time value of money. As risk increases, the required rate of return increases in the manner depicted in the figure.

8. Capital Asset Pricing Model

The concept of CAPM can be employed to assess whether a particular stock is accurately priced.



Underpriced Stock (Above the SML)

If a stock is plotted above the SML, it means the stock's actual expected return is higher than what CAPM suggests for its level of risk (beta). This indicates Higher return for the same risk compared to what the SML suggests. Investors will consider the stock undervalued or underpriced because it offers more return than expected for its risk level.

Let's say the SML suggests that for a stock with a beta of 1.5, the expected return should be 10%, but the actual return is 12%. The stock is above the SML, signaling that it's underpriced. This discrepancy makes it an attractive buy.

Overpriced Stock (Below the SML)

If a stock is plotted below the SML, it means the stock's actual expected return is lower than what CAPM suggests for its level of risk. This suggests Lower return for the same risk compared to what the SML suggests. Investors would see this as the stock being overvalued or overpriced because it offers less return than expected for its risk level.

If the SML suggests a stock with a beta of 1.5 should have an expected return of 10%, but the actual return is only 8%, it is below the SML. The stock appears overpriced, making it less attractive to investors.

Arbitrage

Securities that lie above the Security Market Line (SML) are considered **underpriced** because their expected returns exceed what would be expected given their level of risk, as indicated by their beta. According to the CAPM, which the SML represents, securities should offer a return that is proportionate to their level of systematic risk (beta). If a security's expected return exceeds what is indicated by its beta on the SML, it suggests that the market has undervalued the security.

Investors would seek to capitalize on this mispricing by purchasing these underpriced securities, which would drive their prices up until their expected returns align with their risk level as indicated by the SML. This process is referred to as arbitrage, where investors seek to exploit discrepancies in market pricing until equilibrium is reached.

8. Capital Asset Pricing Model

Answer following questions using CAPM approach:

(i) A stock has a beta of 1.15, the expected return on the market is 11 percent, and the risk-free rate is 5 percent. What must the expected return on this stock be?

(ii) A stock has an expected return of 10.2 percent, the risk-free rate is 4 percent, and the market risk premium is 7 percent. What must the beta of this stock be?

(iii) A stock has an expected return of 13.4 percent, its beta is 1.60, and the risk-free rate is 5.5 percent. What must the expected return on the market be?

Solution:

(i) A stock has a beta of 1.15, the expected return on the market is 11 percent, and the risk-free rate is 5 percent. What must the expected return on this stock be?

$$r_s = r_f + \beta\,(r_m - r_f)$$

$$?\quad 0.05\quad 1.15\quad 0.11\quad 0.05$$

$$r_s = 0.1190 \quad 11.90\%$$

(ii) A stock has an expected return of 10.2 percent, the risk-free rate is 4 percent, and the market risk premium is 7 percent. What must the beta of this stock be?

$$r_s = r_f + \beta\,\underbrace{(r_m - r_f)}_{0.07}$$

$$0.102\quad 0.04\quad ?$$

$$\beta = 0.89$$

(iii) A stock has an expected return of 13.4 percent, its beta is 1.60, and the risk-free rate is 5.5 percent. What must the expected return on the market be?

$$r_s = r_f + \beta\,(r_m - r_f)$$

$$0.134\quad 0.055\quad 1.60\quad ?\quad 0.055$$

$$r_m = 0.1044 \quad 10.44\%$$

8. Capital Asset Pricing Model

Stock Y has a beta of 1.35 and an expected return of 14 percent. Stock Z has a beta of 0.80 and an expected return of 11.5 percent. If the risk-free rate is 4.5 percent and the market risk premium is 7.3 percent, are these stocks correctly priced?

Solution:

FIRST METHOD (CAPM)

STOCK Y

$$r_s = r_f + \beta(r_m - r_f)$$
$$= 4.5 + 1.35 \times 7.3$$
$$= 14.36\%$$

Expected Return = 14%

$\Longrightarrow$ Overpriced

STOCK Z

$$r_s = r_f + \beta(r_m - r_f)$$
$$= 4.5 + 0.80 \times 7.3$$
$$= 10.34\%$$

Expected Return = 11.50%

$\Longrightarrow$ Underpriced

SECOND METHOD (TREYNOR INDEX)

STOCK Y

$$\text{Reward to Risk Ratio} = \frac{0.14 - 0.045}{1.35} = 0.0704$$

STOCK Z

$$\text{Reward to Risk Ratio} = \frac{0.115 - 0.045}{0.80} = 0.0875$$

$$\text{Reward to Risk Ratio for market} = \frac{0.073}{1} = 0.073$$

OVER PRICED

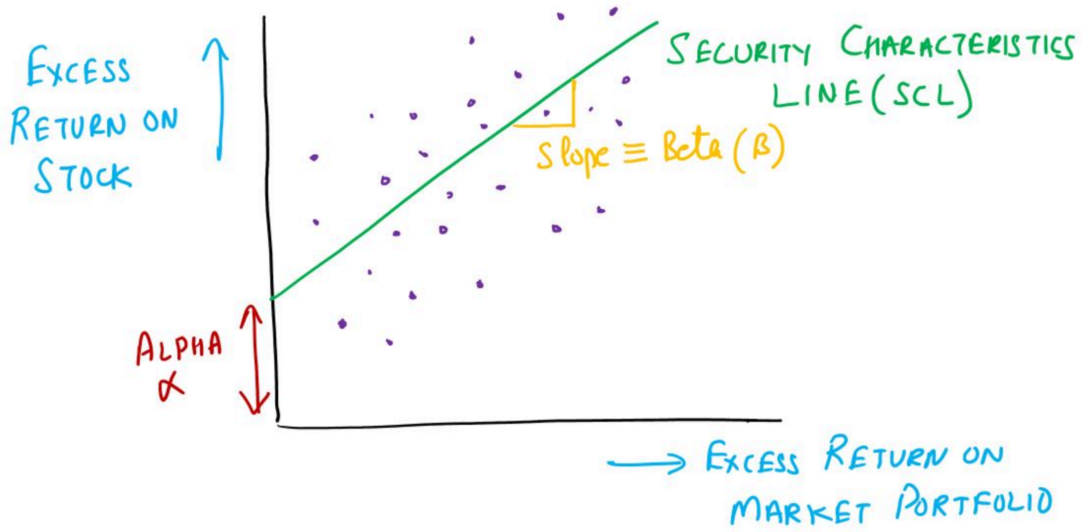
UNDER PRICED

---

9. Security Characteristic Line

The relationship between the returns of a security and the returns of the overall market portfolio, is measured by the Characteristic Line of that security. It is like drawing a line that shows how the performance of the security tends to move along with the performance of the market.

This line, known as the **Security's Characteristic Line** (SCL), shows the expected relationship between the excess returns of the stock (meaning its returns above the risk-free rate) and the excess returns of the market portfolio.



The dispersion of the data points about the characteristic line is a measure of the unsystematic risk of the stock. The wider the relative distance of the points from the line, the greater the unsystematic risk of the stocks: this is to say that the stock’s return has increasingly lower correlation with the return on the market portfolio. The narrower the dispersion, the higher the correlation and the lower the unsystematic risk. From before, we know that unsystematic risk can be reduced or even eliminated through efficient diversification.

Beta
The slope of SCL is what we call the beta. Beta gives us a sense of how much the security's returns tend to move in response to changes in the market returns. If the beta is higher than 1, it means the security's returns are more volatile compared to the market. If it's less than 1, the security's returns are less volatile than the market. And if it's exactly 1, it means the security's returns move in line with the market.

The greater the slope of the characteristic line for a stock, as depicted by its beta, the greater its systematic risk. This means that, for both upward and downward movements in market excess returns, movements in excess returns for the individual stock are greater or less depending on its beta. With the beta of the market portfolio equal to 1.0 by definition, beta is thus an index of a stock’s systematic or unavoidable risk relative to that of the market portfolio. This risk cannot be diversified away by investing in more stocks.

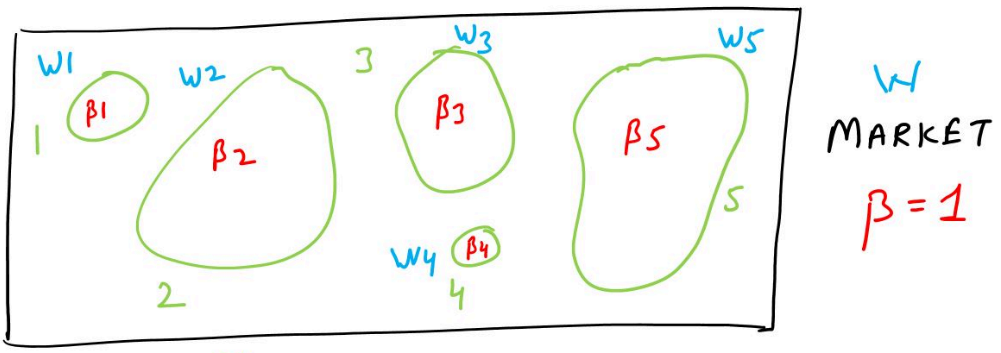
10. Beta

In the Capital Asset Pricing Model (CAPM), beta (β) measures the sensitivity of a stock's returns to changes in the overall market returns. Beta quantifies the systematic risk of an individual stock relative to the broader market.

$$\text{Beta of security } i = \frac{Cov(r_i, r_m)}{Var(r_m)} = \frac{r\,\sigma_i\,\sigma_m}{\sigma_m^2} = \frac{r\,\sigma_i}{\sigma_m}$$

r = Coefficient of correlation between security i and market

The beta is computed from the covariance of returns of a security with the market, divided by the variance of the market returns (standardized covariance of a security’s return with the return on the market portfolio).



$$\sum_{i=1}^{n} w_i \times \beta_i = 1$$

The average beta across all securities, weighted by their market values, equals 1.

This equation reflects that when securities are weighted by their market values, the resulting portfolio mirrors the complete market itself. As the market portfolio has a beta of 1 by definition, this equation intuitively follows. In essence, for every 1 percent movement in the market, the entire market must move 1 percent, indicating a balanced and representative reflection of market dynamics.

Beta measures the **systematic risk** of a security. Thus, diversified investors pay attention to the systematic risk of each security. However, they ignore the unsystematic risks of individual securities, since unsystematic risks are taken care of by diversification in a large portfolio.

10. Beta

Betas are estimated from the stock's Security Characteristic Line (SCL) by running a linear regression between past returns on the stock in question and past returns on some market index. We define betas developed in this manner as historical betas.

Let us understand the process of determining beta (\(\beta \)).

Initially, historical data on the returns of the firm's stock and the overall market (represented by a suitable index like the SENSEX) is gathered for a specific time period. Typically, daily, weekly, or monthly returns are considered.

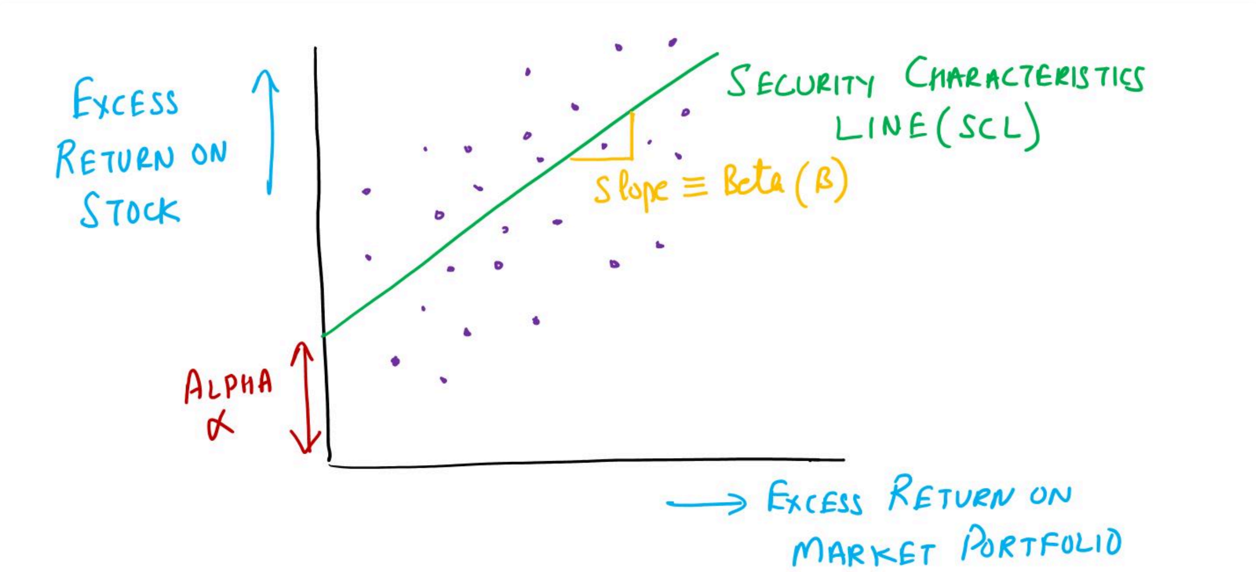
$$r_s = r_f + \beta(r_m - r_f)$$

$$\beta = \frac{r_s - r_f}{r_m - r_f}$$

$r_s - r_f$ ← Excess Return on Security

$r_m - r_f$ ← Excess Return on Market

Next, the excess returns of the firm's stock (or any security) are plotted on the Y-axis, while the corresponding excess returns on market portfolio are plotted on the X-axis.

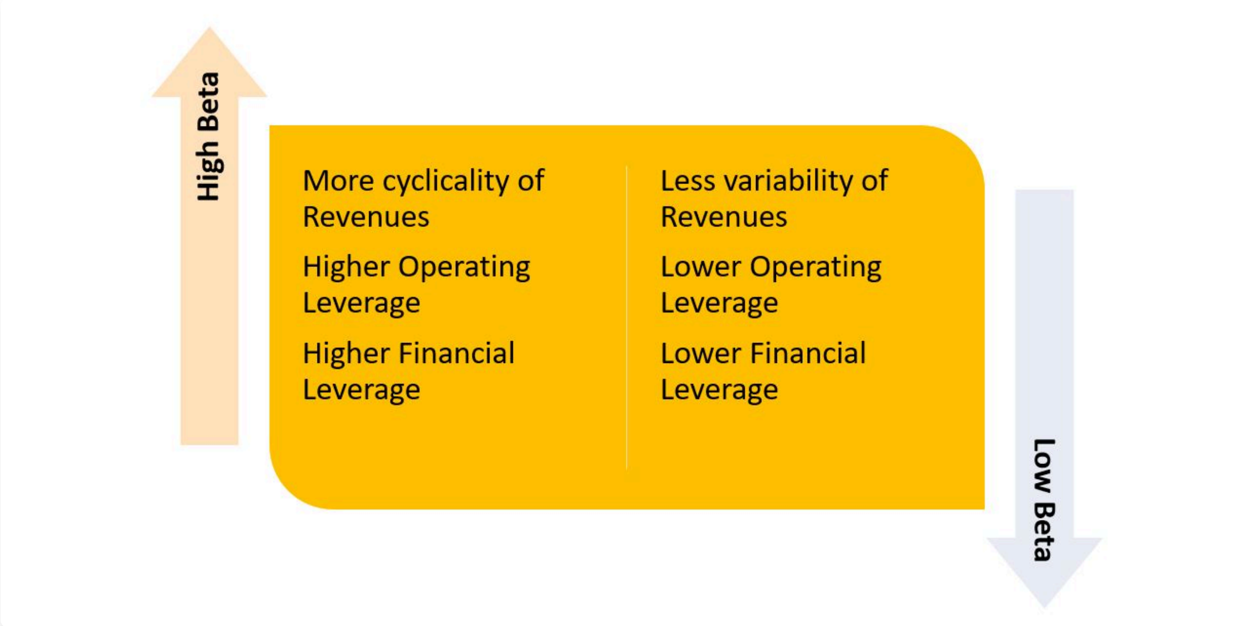


Each data point represents a pair of returns for a specific time period.

A linear regression analysis is then performed on the plotted data points. The goal is to fit a straight line through the scatter plot of returns, which best represents the relationship between the firm's returns and the market returns.

The slope of the regression equation represents the beta of the firm. It quantifies the systematic risk or volatility of the firm's stock relative to the market. A beta greater than 1 indicates higher volatility compared to the market, while a beta less than 1 suggests lower volatility.

10. Beta



Let us look at some of the determinants of Beta, in a firm.

1. Cyclicality of Revenues

Beta measures a stock's response to market movements, and thus, stocks in highly cyclical industries tend to have high betas. For instance, sectors like technology, retail, and automotive are prone to fluctuations with the business cycle, resulting in higher betas. Conversely, industries such as utilities, food, and airlines are less cyclical, leading to lower betas.

2. Operating Leverage

Firms with high fixed costs and low variable costs exhibit high operating leverage. This means that changes in sales volume have a significant impact on their profitability. Consequently, operating leverage magnifies the effect of revenue cyclicality on beta. If a firm shifts from variable to fixed costs in its production process, its beta will increase.

3. Financial Leverage

Increasing financial leverage, such as taking on more debt, also elevates beta. Higher financial leverage amplifies the firm's sensitivity to market movements, as it increases the financial risk borne by shareholders. Thus, firms with higher debt levels tend to have higher betas compared to those with lower debt levels.

10. Beta

When we look at the measured betas of individual securities, we often notice a trend where they tend to move back towards either the market portfolio beta of 1.0 or the beta of the industry they belong to. This tendency could be influenced by various economic factors affecting the company's operations and financing, as well as statistical factors.

To account for this tendency, financial institutions calculate what's known as an adjusted beta. This adjusted beta helps to provide a more accurate reflection of the security's risk.

For example, let's say the measured beta of a security is 1.4, and we believe it's reverting towards the market beta of 1.0. If we assign a weight of 0.67 to the measured beta and a weight of 0.33 to the market beta, the adjusted beta would be calculated as follows:

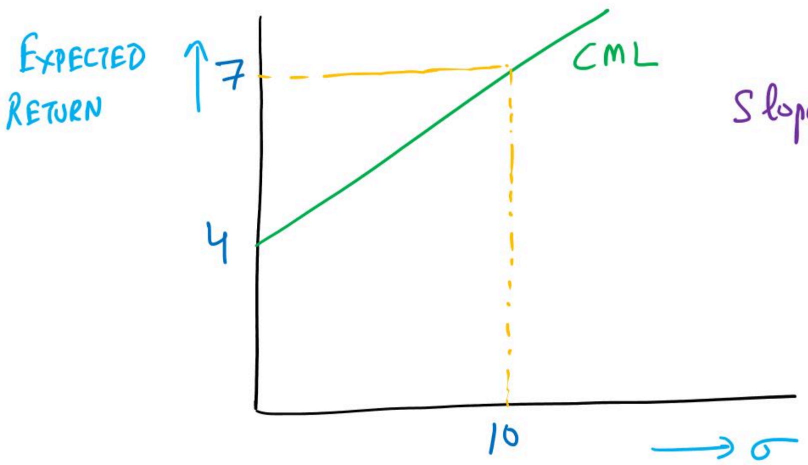
$$1.4 \times 0.67 + 1.0 \times 0.33 = 1.27$$

Adjusted Beta will be 1.27.

10. Beta

A portfolio that combines the risk-free asset and the market portfolio has an expected return of 7 percent and a standard deviation of 10 percent. The risk-free rate is 4 percent, and the expected return on the market portfolio is 12 percent. Assume the capital asset pricing model holds. What expected rate of return would a security earn if it had a 0.45 correlation with the market portfolio and a standard deviation of 55 percent?

Solution:



Slope of CML $= \dfrac{7-4}{10-0} = 0.30$

From CML line

Expected Return on Market portfolio

= Risk free return + Slope x Standard deviation of market portfolio

$$12 = 4 + 0.30 \times \sigma_M$$

$$\sigma_M = \frac{12-4}{0.30} = 26.67\%$$

$$\text{Beta of security } i = \frac{Cov(r_i, r_m)}{Var(r_m)}$$

$$= \frac{r \times \sigma_i \times \sigma_m}{\sigma_m^2} = \frac{r \times \sigma_i}{\sigma_m}$$

Coefficient of correlation between security & market returns

$$= \frac{0.45 \times 55}{26.67} = 0.93$$

Using CAPM

$$r_i = r_f + \beta(r_m - r_f) = 4 + 0.93(12 - 4)$$

$$= 11.43\%$$

11. Anomalies to the CAPM model

When researchers have tried to explain actual returns on securities, they've noticed some deviations from what the CAPM model predicts. These anomalies, or irregularities, shed light on areas where the model doesn't fully capture the real-world dynamics of the market.

One such anomaly is the **small-firm effect**, also known as the size effect. This anomaly suggests that stocks of smaller companies tend to provide higher returns compared to stocks of larger companies, even when other factors are taken into account.

Another irregularity is observed with stocks that have low price-to-earnings (P/E) ratios and low market-to-book-value ratios. These stocks often outperform those with higher ratios, which is contrary to what the CAPM model would predict.

Additionally, there are other anomalies that researchers have identified. For instance, there's something called the January effect, where holding a stock from December to January tends to result in higher returns compared to other similar-length periods. However, it's worth noting that while this effect has been observed over many years, it doesn't happen every single year.

These anomalies challenge the assumptions and predictions of the CAPM model, suggesting that there are additional factors at play in the stock market that influence returns beyond what the model accounts for.

12. Portfolio Management

A **Portfolio** refers to a collection of financial assets or investments (securities) owned by an individual, institution, or entity. These assets can include a wide range of investment types, such as:

- *Stocks:* Shares of ownership in a company.
- *Bonds:* Debt securities issued by companies or governments.
- *Real Estate:* Property investments.
- *Commodities:* Physical goods like gold, oil, or agricultural products.
- *Cash and Cash Equivalents:* Highly liquid investments like savings accounts or treasury bills.

A portfolio is often constructed to achieve specific financial goals, such as capital appreciation, income generation, or risk management. The combination of assets in a portfolio can be diversified to manage risk while maximizing returns.

Portfolio Management

Portfolio management refers to the process of making investment decisions to manage and grow an individual's or institution's portfolio in line with their financial goals, risk tolerance, and time horizon. Portfolio management involves selecting and overseeing investments that will meet these objectives, while also balancing risk and reward.

Components of Portfolio Management:

Asset Allocation: Determining the right mix of asset classes (e.g., stocks, bonds, real estate) in the portfolio. This decision is based on the investor's risk tolerance, time horizon, and financial goals. Asset allocation is crucial for balancing risk and ensuring diversification.

Diversification: Spreading investments across different assets or sectors to reduce exposure to any one particular risk. A diversified portfolio can mitigate losses from poorly performing investments while benefiting from well-performing ones.

Risk Management: Portfolio managers assess and manage risks based on the client's risk tolerance. This includes monitoring market conditions and adjusting the portfolio to avoid excessive risk or minimize losses during downturns.

Performance Monitoring and Rebalancing: Regularly reviewing the portfolio to ensure it aligns with the investor's goals and risk profile. Rebalancing may be needed to adjust the asset allocation back to its original state if market movements have caused a shift.

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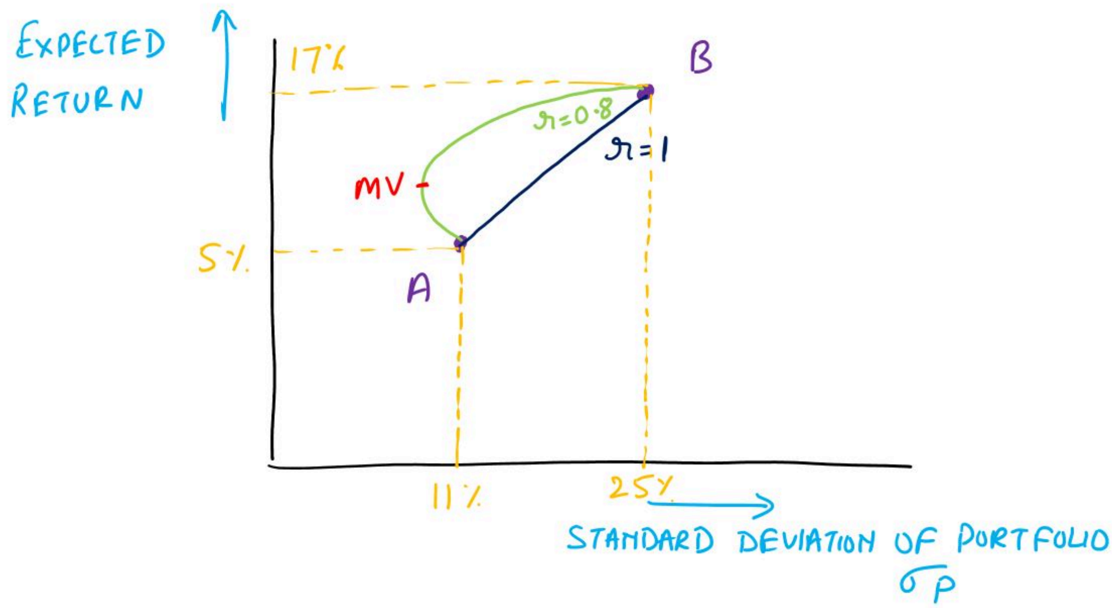
13. Efficient Frontier

Let us understand, how do we analyze portfolio of two risky securities.

Consider two securities, Alpha and Beta. Security Alpha has an expected return of 5% and a standard deviation (risk) of 11%, while security Beta has an expected return of 17% and a standard deviation of 25%. These data are plotted on a graph with standard deviation on the x-axis and expected return on the y-axis.



If the correlation coefficient between the two securities is 1, diversification does not offer any benefits. The set of portfolios, reflecting the weightage of Alpha and Beta, is represented by the straight line between points A and B.

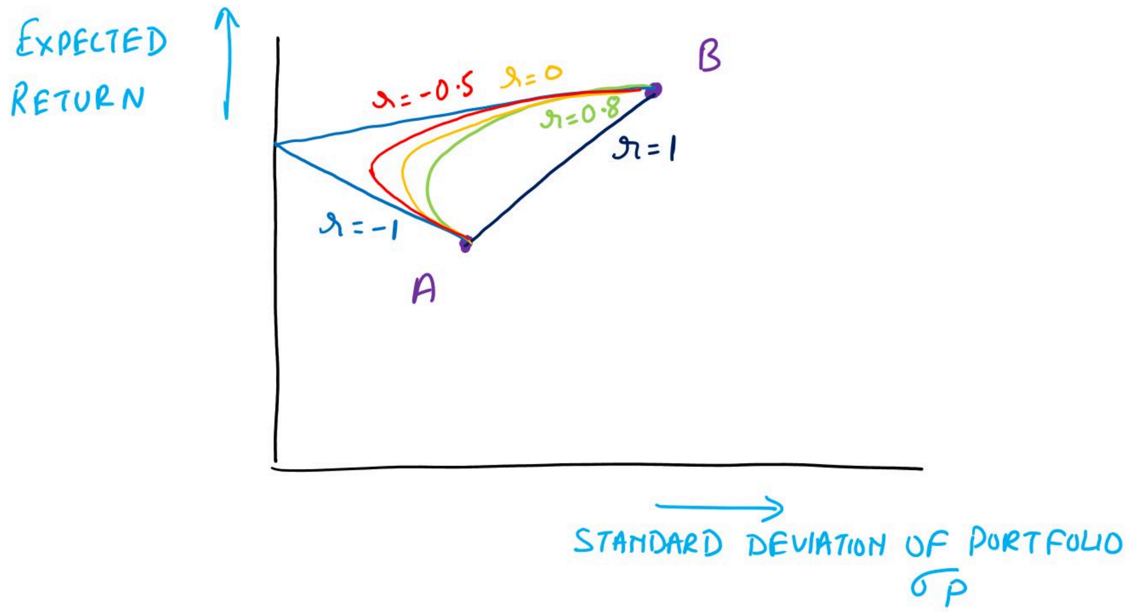
However, if the correlation coefficient is any value less than 1 (say 0.8), diversification will be beneficial. The set of portfolios with Alpha and Beta is represented by a curved line (backward-bending curve) from points A to B, located to the left of the straight line AB.

Point MV denotes the **minimum variance** portfolio, which represents the lowest possible risk. This point has minimum standard deviation, among all points on curved line between A and B.

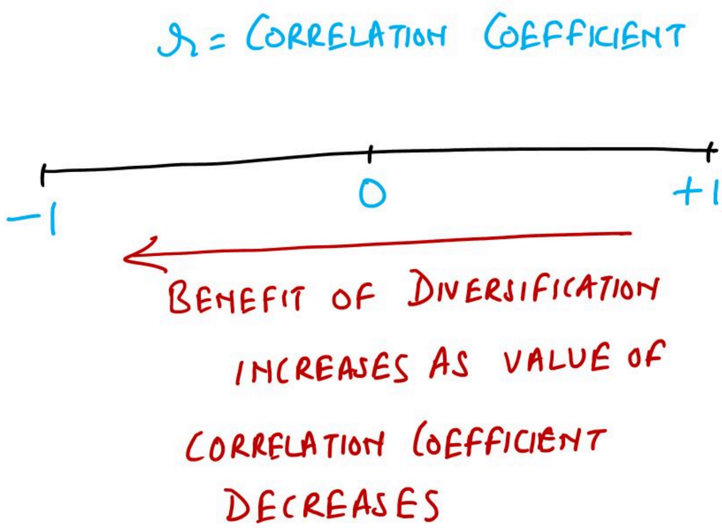
Investors considering a portfolio of Alpha and Beta face an opportunity set represented by the curved line (ligh green colour). They can create their portfolios by choosing any point on this curve. Those who are more tolerant of risk may choose points closer to B (favoring more investment in security Beta), while those less tolerant of risk may opt for points closer to MV (favoring investment in security Alpha).

It may be noted that the investor would never choose any point between A and MV. Investors would avoid portfolios with expected returns lower than that of the minimum variance portfolio (point MV). For instance, point 1, offering less expected return and higher standard deviation than point MV, would not be chosen.

Thus, while the entire curve from point A to point B is termed the **feasible set**, investors focus on the curve from point MV to point B. The portion of the curve between point MV to B is called the **efficient set or efficient frontier**.

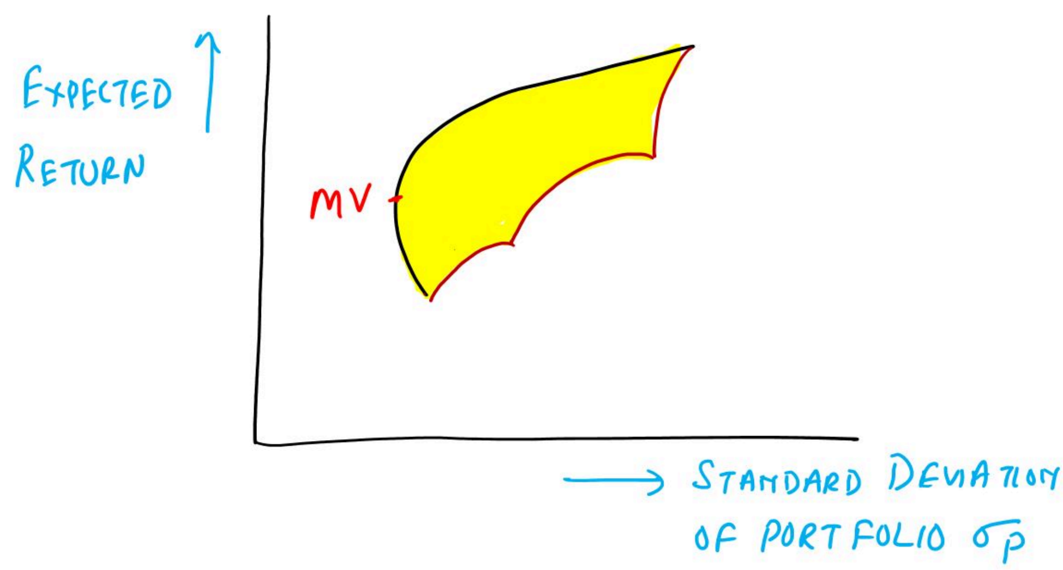


Let us understand, how the curve changes with change in the correlation coefficient between the two securities. As can be seen, the lower the correlation correlation coefficient, the more the curve bends. This indicates that the diversification effect rises as correlation coefficient declines. The greatest bend occurs in the limiting case where the correlation coefficient is equal to -1. This is perfect negative correlation.



13. Efficient Frontier

The preceding discussion focused on two securities, but as investors typically hold multiple securities, it's important to extend our analysis to encompass this scenario. The shaded region in the figure illustrates the opportunity set or feasible set when considering multiple securities, encapsulating all potential combinations of expected return and standard deviation for a portfolio.



While the combinations of portfolios are virtually limitless, they all fall within a confined area represented by the shaded region. No security or combination of securities can lie outside this area. In other words, investors cannot select a portfolio with an expected return higher than that delineated by the shaded region, nor can they choose a portfolio with a standard deviation lower than that indicated in the shaded area.

Any portfolio that falls outside the Efficient Frontier is considered sub-optimal for one of 2 reasons:
(i) it carries too much risk relative to its return, or
(ii) too little return relative to its risk.

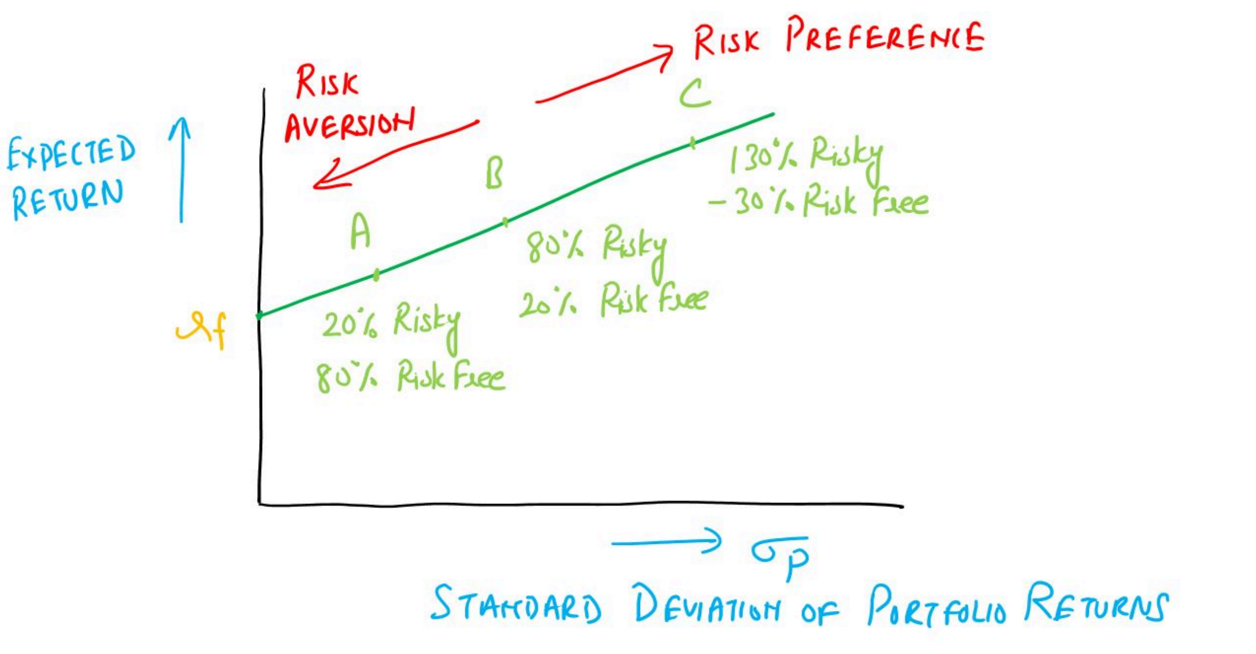
A portfolio that lies below the Efficient Frontier doesn’t provide enough return when compared to the level of risk. Portfolios found to the right of the Efficient Frontier have a higher level of risk for the defined rate of return.

At every point on the Efficient Frontier, investors can construct at least one portfolio from all available investments that features the expected risk and return corresponding to that point.

14. Combining risk-free and risky securities

What happens, when we create a portfolio composed of both risk-free and risky securities?

When plotting the relationship between the expected return of a portfolio and its risk (measured by standard deviation of portfolio returns), we observe a linear relationship represented by a line on a graph. This line shows the trade-off between risk and return for portfolios composed of both risk-free and risky securities.

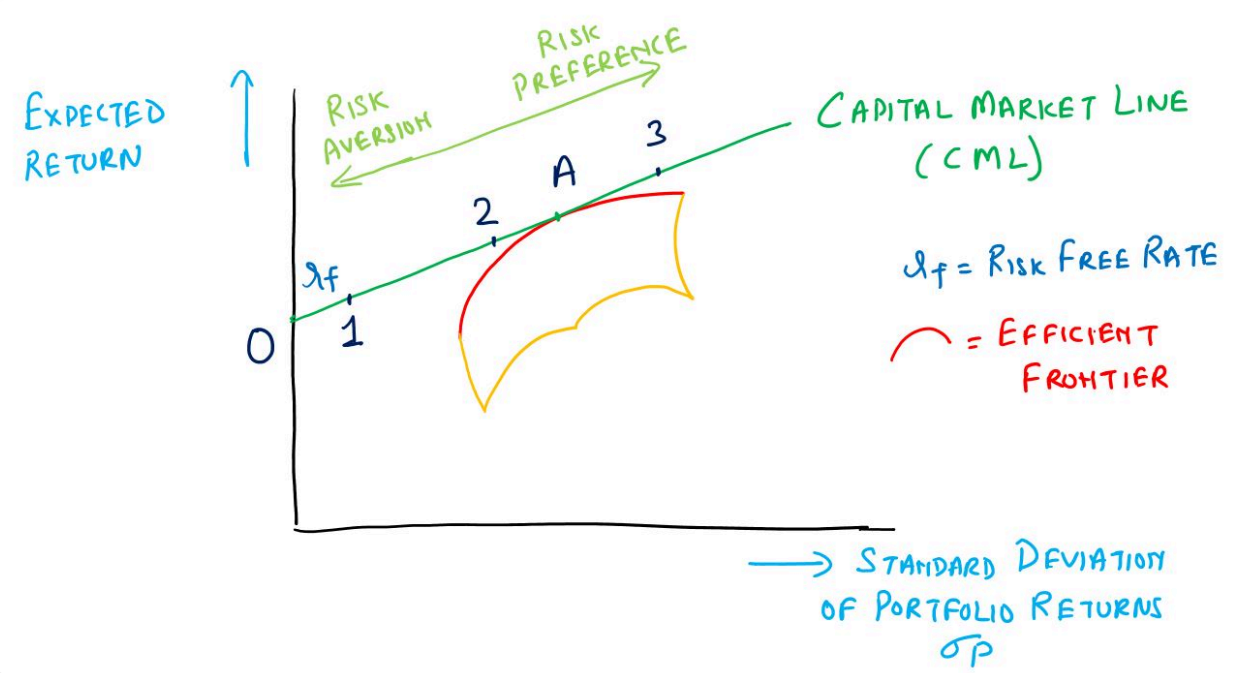


As the weightage of risky securities in the portfolio increases, both the standard deviation of portfolio returns and the expected portfolio return increase. This is because riskier securities typically offer higher expected returns but also introduce more variability in returns, thus increasing the overall risk of the portfolio. Look at point A and B in the figure.

Moreover, if an investor has the ability to borrow at the risk-free rate, they can leverage their investment by borrowing funds to invest in risky securities. This means that the weightage of risky securities in the portfolio can exceed 100%. This is shown by point C. Leveraging allows investors to potentially amplify their returns, but it also magnifies the risks associated with the portfolio.

15. Optimal Portfolio

The process of creating an optimal portfolio consists of two steps, also known *Separation Principle*.



First Step
The investor calculates the efficient frontier of risky securities, represented by the red curve in the Figure, with the help of the following data points:
(i) Expected returns and variances of individual securities.
(ii) Covariances between pairs of securities.

Then, he determines Point A, which is the point of tangency between the risk-free rate (point O) and the efficient frontier of risky securities, forming line OA. Point A represents the portfolio of risky assets that the investor will hold.

The green line OA is called the **Capital Market Line (CML)**. This line can be viewed as the efficient rontier of all securities, both risky and riskless. A well diversified portfolio (with zero unsystematic risk), should lie on this line.

It may be noted that Point A is determined solely from his estimates of returns, variances, and covariances, without considering personal characteristics of the investor such as the degree of risk aversion. The risk of investor is considered in second step.

Second Step
The investor now decides how to combine Point A, his portfolio of risky securities, with the riskless securities. He may invest some funds in the riskless asset and some in Portfolio A. This results in a point anywhere on the line between O and A, depending on the allocation of funds between the risk-free and risky securities.

If point O is chosen, investors have 100% of their funds invested in the risk-free asset, representing zero risk.

Investors at Point A have 100% of their funds invested in the portfolio of risky securities.

If the investor is risk-averse, he may choose a point near O, indicating a higher weightage of the risk-free security in the portfolio. For example, Point 1 in the figure has a 90% weightage of the risk-free security and 10% for the risky security.

If the investor prefers risk, he may opt for a point near A, indicating a higher weightage of the risky security in the portfolio. For instance, Point 2 in the figure has a 10% weightage of the risk-free security and 90% for the risky security.

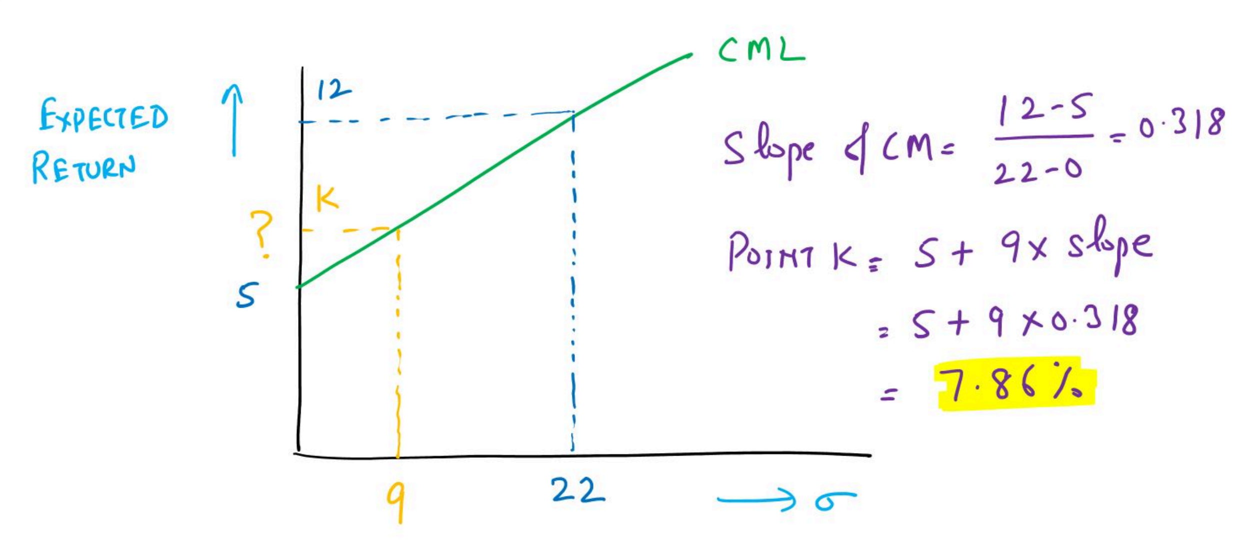
Alternatively, the investor may end up at a point on Line OA beyond A, such as Point 3, indicating a negative weightage of the risk-free security (-30%) and a higher weightage (130%) for the risky security. Here, the investor can borrow funds at the risk-free rate to invest in risky assets.

This is how investors construct an optimal portfolio by balancing risk preferences to maximize returns while minimizing overall portfolio risk through diversification.

15. Optimal Portfolio

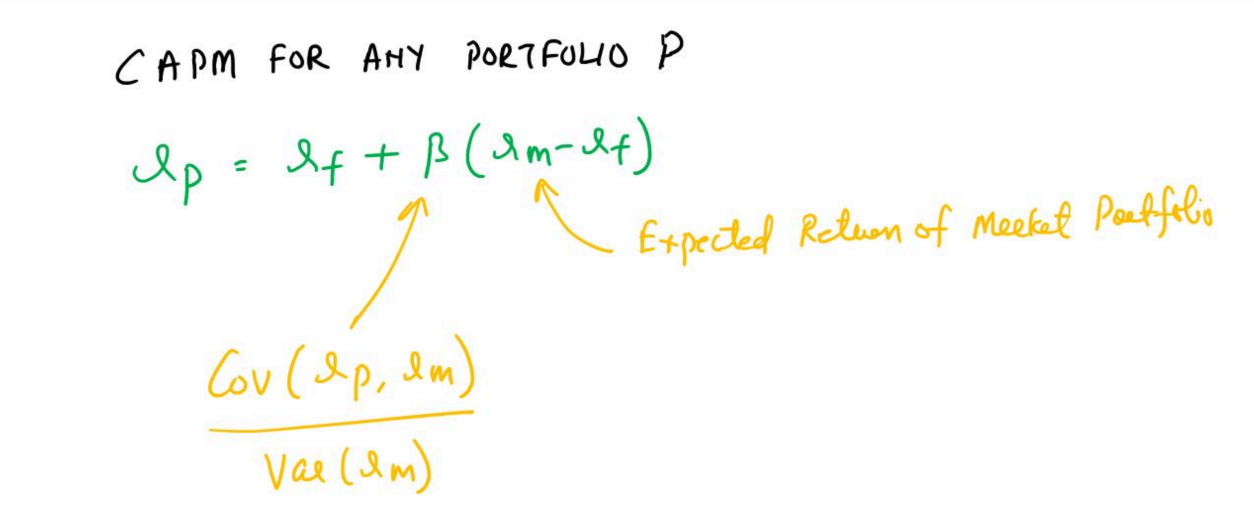
The market portfolio has an expected return of 12 percent and a standard deviation of 22 percent. The risk-free rate is 5 percent. What is the expected return on a well-diversified portfolio with a standard deviation of 9 percent?

Solution:



16. CAPM for Portfolios

In our discussion of the Capital Asset Pricing Model (CAPM), we examined its applicability to individual securities.



But does this relationship extend to portfolios, which consist of a group of securities?

Yes, it does.

The beta of a portfolio is merely a weighted average of the betas of the individual securities comprising the portfolio.

Therefore, the CAPM remains valid for portfolios, just as it does for individual securities.

16. CAPM for Portfolios

You own a stock portfolio invested 10 percent in Stock Q, 35 percent in Stock R, 20 percent in Stock S, and 35 percent in Stock T. The betas for these four stocks are 0.75, 1.90, 1.38, and 1.16, respectively. What is the portfolio beta?

Solution:

Beta of Portfolio $= w_1\beta_1 + w_2\beta_2 + w_3\beta_3 \ldots$

$= 0.10 \times 0.75 + 0.35 \times 1.90 + 0.20 \times 1.38 + 0.35 \times 1.16$

$= \boxed{1.42}$

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16. CAPM for Portfolios

You own a portfolio equally invested in a risk-free asset and two stocks. If one of the stocks has a beta of 1.65 and the total portfolio is equally as risky as the market, what must the beta be for the other stock in your portfolio?

Solution:

Beta for market = 1.0

$\Longrightarrow$ Beta for our portfolio = 1

Beta for Risk Free Asset = 0   Let Beta of Risky Asset = x

$1.0 = \frac{1}{3} \times 0 + \frac{1}{3} \times 1.65 + \frac{1}{3} \times x$

$\Longrightarrow x = 1.35$

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16. CAPM for Portfolios

A stock has a beta of 1.13 and an expected return of 12.1 percent. A risk-free asset currently earns 5 percent.
(i) If a portfolio of the two assets has an expected return of 10 percent, what is its beta?
(ii) If a portfolio of the two assets has a beta of 2.26, what are the portfolio weights? How do you interpret the weights for the two assets in this case?

Solution:

$\beta = 1.13 \qquad r_s = 12.1\% \qquad r_f = 5\%$

(i) If a portfolio of the two assets has an expected return of 10 percent, what is its beta?

Expected Return on Portfolio = 0.10

Assume, weight of Risky Asset = w_1

" " Risk Free Asset = $1 - w_1$

$0.10 = 0.121 \times (1 - w_1) + 0.05 \times w_1$

$\Rightarrow w_1 = 0.2958$

$1 - w_1 = 0.7042$

Beta of Portfolio = $0.2958 \times 0 + 0.7042 \times 1.13$

$= 0.796$

(ii) If a portfolio of the two assets has a beta of 2.26, what are the portfolio weights? How do you interpret the weights for the two assets in this case?

Portfolio Beta = 2.26

$2.26 = x \times 1.13 + (1 - x) \times 0$

Weight of Risk Free Asset

Weight of Risky Asset

Beta of Risk Free Asset

$x = 2$

PORTFOLIO
- 200% in Risky stock
- −100% in Risk Free Asset

16. CAPM for Portfolios

Suppose the risk-free rate is 4.2 percent and the market portfolio has an expected return of 10.9 percent. The market portfolio has a variance of 0.0382. Portfolio Z has a correlation coefficient with the market of 0.28 and a variance of 0.3285. According to the capital asset pricing model, what is the expected return on Portfolio Z?

Solution:

Standard Deviation of Portfolio Z $(\sigma_Z) = \sqrt{0.3285} = 0.5731$

Standard Deviation of market $(\sigma_m) = \sqrt{0.0382} = 0.1954$

Beta of Portfolio $= \dfrac{Cov(r_m, r_Z)}{Var(r_m)} = \dfrac{\sigma_m \times \sigma_Z \times r}{\sigma_m^2} = \dfrac{r \times \sigma_Z}{\sigma_m}$

$$= \frac{0.28 \times 0.5731}{0.1954} = 0.82$$

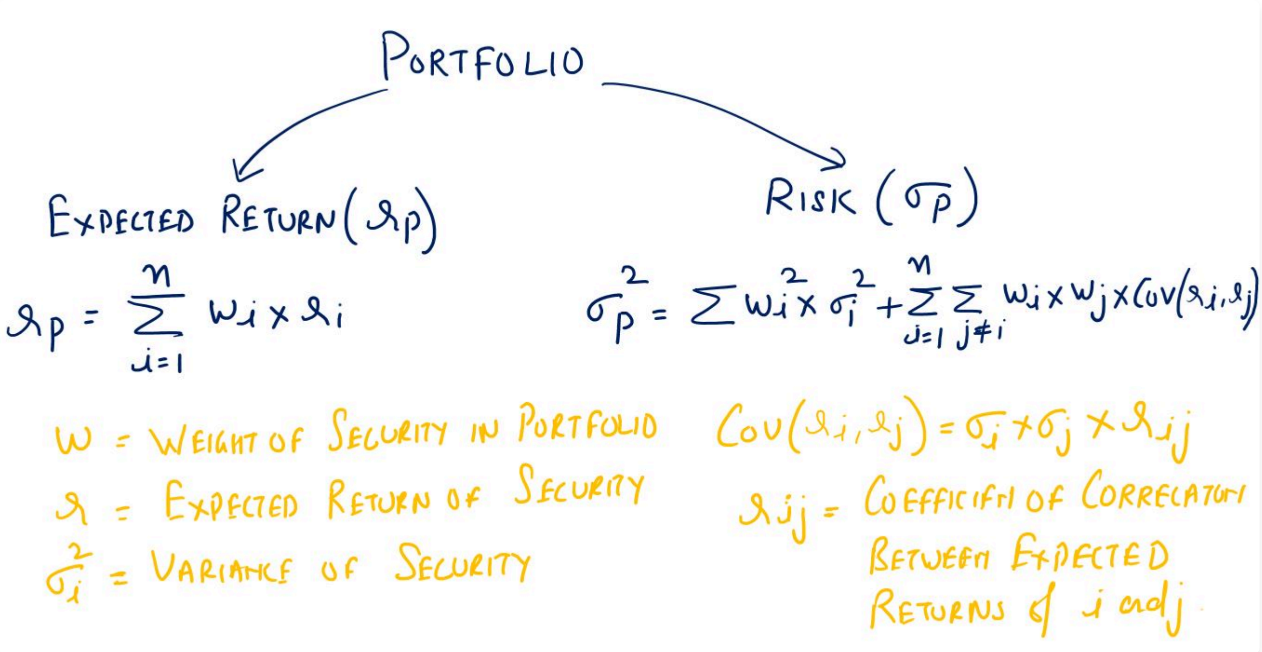
USING CAPM

$r_Z = r_f + \beta(r_m - r_f) = 0.042 + 0.82(0.109 - 0.042)$

$= 0.0970$ 9.70%

17. Modern Portfolio Theory

The techniques of mean-variance portfolio optimization, which allow an investor to find the portfolio with the highest expected return for any level of variance (or risk), were developed in an article, "Portfolio Selection," published in the Journal of Finance in 1952 by Harry Markowitz. Markowitz's approach has evolved into one of the main methods of portfolio optimization used on Wall Street. In recognition for his contribution to the field, Markowitz was awarded the Nobel Prize for economics in 1990.



Instead of focusing on the risk of each individual asset, Markowitz demonstrated that a diversified portfolio is less volatile than the total sum of its individual parts. While each asset itself might be quite volatile, the volatility of the entire portfolio can actually be quite low. Markowitz created a theory that allows an investor to mathematically trade off risk tolerance and reward expectations, resulting in the ideal portfolio. This theory is named Modern Portfolio Theory (MPT).

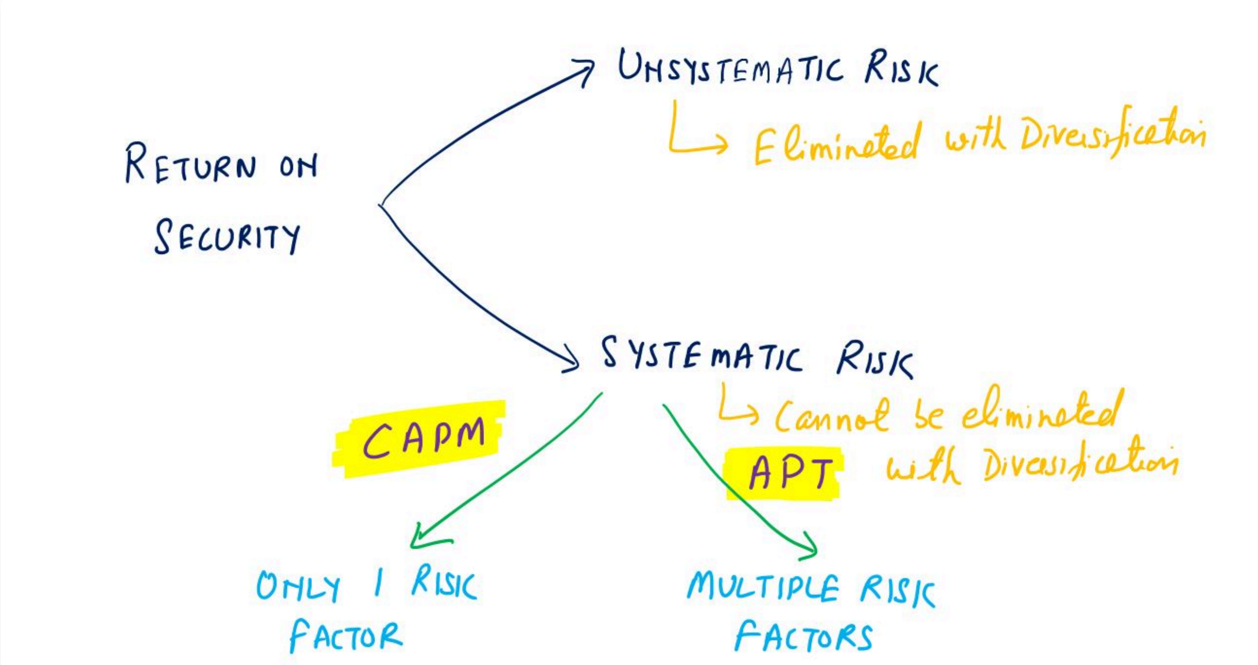
Markowitz's work made clear that it is a security's covariance with an investor's portfolio that determines its incremental risk, and thus an investment's risk cannot be evaluated in isolation. He also demonstrated that diversification provided a "free lunch"— the opportunity to reduce risk without sacrificing expected return. In later work Markowitz went on to develop numerical algorithms to compute the efficient frontier for a set of securities.

Thus, the Modern Portfolio Theory (MPT) refers to an investment theory that allows investors to assemble an asset portfolio that maximizes expected return for a given level of risk. The theory assumes that investors are risk-averse; for a given level of expected return, investors will always prefer the less risky portfolio.

The MPT is based on following assumptions:

- Investors are rational and desire to maximise their returns with the money available for investment.
- The investors have free access to fair information of returns and risk.
- The markets are efficient and absorb the information quickly and perfectly.
- Investors are risk averse and are in search of maximising returns and minimising risk.
- Standard deviation or variance and expected returns are the basis for investors to take the decision.
- Investor prefers higher returns for a given level of risk.

18. Arbitrage Pricing Theory



So far, we have understood that total risk of a stock (or any security) risk consists of two parts, systematic risk and unsystematic components. While diversification can mitigate unsystematic risk, systematic risk remains unaffected.

Hence, in portfolios, a stock's systematic risk becomes important to consider. The systematic risk is mesaured by Beta. Further, the capital asset pricing model (CAPM) implies the expected return on a security is linearly related to its beta.

Arbitrage Pricing Theory (APT) also attempts to determine the expected return of an asset. But unlike the Capital Asset Pricing Model (CAPM), which considers only one systematic risk factor (beta mesauring 'return on market portfolio'), the APT takes into account several factors that influence systematic risk, such as interest rates, inflation rates, GNP and economic indicators.

The arbitrage pricing theory (APT) was developed by the economist Stephen Ross in 1976, as an alternative to the capital asset pricing model (CAPM).

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18. Arbitrage Pricing Theory

The risk associated with any given stock can be dissected into two primary components: systematic risk and unsystematic risk.

$$r = r_f + \beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3 + \ldots\ldots$$

r_f = RISK FREE RETURN

$\beta_1, \beta_2, \beta_3 \ldots$ = SENSITIVITY OF RETURN TO F_1, F_2, F_3 FACTORS [BETA COEFFICIENTS]

$F_1, F_2, F_3 \ldots$ = FACTORS OF SYSTEMATIC RISK (RISK PREMIUM WITH EACH FACTOR)

r = EXPECTED RETURN

Unsystematic aspects of the returns are specific to a company. As more securities are added to a portfolio, the unsystematic risks of the individual securities offset each other. A fully diversified portfolio has no unsystematic risk.

However, systematic risks factors can not be eliminated with diversification. This is so because all companies are susceptible to the same systematic risk factors. For examples changes in factors like inflation, interest rate, GNP etc will impact all firms. This are factors of systematic risk.

For instance, unexpected changes in inflation impact most companies to some degree.

But what is the degree to which a particular stock's return reacts to unforeseen shifts in inflation?

We gauge the impact of systematic risks such as inflation on a stock by utilizing the **beta coefficient**. This coefficient, measures impact of systematic risk factor on return of a stock.

If a company’s stock is positively related to the risk of a factor (say inflation), that stock has a positive beta coefficient. If it is negatively related to the factor, its beta coefficient is negative; and if it is uncorrelated with the factor, its beta coefficient is zero.

Remember that, in Capital Asset Pricing Model (CAPM) also, we had a Beta. But that beta mesaured the impact of only one risk factor (return on market portfolio). But the Arbitrage Pricing Theory (APT) incorporates various types of systematic risks (not just one), leading to multiple beta coefficients in its equation.

Examples of systematic risks influencing stock returns encompass inflation, Gross National Product (GNP), and interest rates. These are called Factors. Therefore this model is also called **factor model**.

Ross suggests the following 5 economic factors that may account for the risk under APT:

1. Changes in expected inflation
2. Unanticipated changes in inflation
3. Unanticipated changes in industrial production
4. Unanticipated changes in the yield differential between low- and high-grade bonds (the default risk premium)
5. Unanticipated changes in the yield differential between long-term and short-term bonds (the term structure of interest rates)

The first three factors primarily affect the cash flow of the company, and hence its dividends and growth in dividends. The last two factors affect the market capitalization, or discount, rate.

NOTE

Consider an investor, who is evaluating a Bread making company using Arbitrage Pricing Theory (APT) model for estimating the required return on the company's stock. He plans to incorporate three risk factors: the risk premium on the stock market, the inflation rate, and the price of wheat.

Assessing his choice of risk factors, it appears that the market risk premium and inflation rates are suitable selections. These factors are commonly acknowledged as fundamental drivers of market behavior and are likely to influence stock returns. However, the inclusion of the price of wheat may not be optimal. While wheat price is undoubtedly a risk factor for company specifically, it may not qualify as a broad market risk factor applicable to all stocks. Instead, it represents a firm-specific risk factor rather than a market-wide one.

For a more comprehensive model, it would be advisable to incorporate macroeconomic risk factors such as interest rates, Gross Domestic Product (GDP), energy prices, and industrial production, among others. These factors have broader implications across various industries and are more likely to capture systematic risks that affect the entire market.

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18. Arbitrage Pricing Theory

In our Arbitrage Pricing theory (APT) model, we created a model by analyzing impact of multiple factors of systematic risk on the return of a stcok. This model is also called Factor Model. We can use inflation, GNP, and the change in interest rates as examples of systematic sources of risk, or factors.

ONE FACTOR MODEL ← also called Market Model

$$r = \bar{r} + \beta(r_m - \bar{r}_m) + \varepsilon$$

r_m = Return on Market Portfolio

$$r = \bar{r} + \beta F + \varepsilon$$

SINGLE FACTOR

In practice, researchers frequently use a **one-factor model** for stock returns. They do not use all of the sorts of economic factors; instead they use an index of stock market returns—like the SENSEX (NIFTY), or even a more broadly based index with more stocks in it—as the single factor. In this case, the model is called Market Model.

If we are creating a portfolio of stocks when each of the stocks follows a one-factor model, the equation of portfolio return is given by:

Stock 1

$$r_1 = \bar{r} + \beta_1 F + \varepsilon_1$$

Stock 2

$$r_2 = \bar{r} + \beta_2 F + \varepsilon_2$$

Stock 3

$$r_3 = \bar{r} + \beta_3 F + \varepsilon_3$$

PORFOLIO OF 1, 2 and 3

$$r_p = w_1 r_1 + w_2 r_2 + w_3 r_3$$

Weights of 1, 2 and 3 are w_1, w_2, w_3

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18. Arbitrage Pricing Theory

Arbitrage Pricing Theory (APT) states that if two securities have the same beta coefficients (coefficients of systematic risk factors in APT equation), they should provide the same expected return.

However, if this is not the case and one security offers a higher expected return than another, investors will rush to buy the security with the higher return and sell the one with the lower return. As a result, prices will adjust as arbitragers recognize the mispricing and engage in transactions to exploit it. The price of the security with the higher expected return will rise, causing its return to decrease. Conversely, the price of the security with the lower expected return will fall, leading to an increase in its expected return. This process continues until both securities have the same expected return.

The APT suggests that rational market participants will exploit any opportunities for arbitrage profits, leading to market equilibrium where expected returns for all securities are linearly related to the various beta coefficients. Therefore, arbitrage forms the foundation for equilibrium pricing according to APT.

By using APT, arbitrageurs aim to capitalize on deviations from fair market value, identifying discrepancies between expected and actual asset returns and leveraging them for potential profit.

18. Arbitrage Pricing Theory

In an article, Eugene Fama and Kenneth French looked empirically at the relationship among common stock returns and a firm's market capitalization (size), market-to-book-value ratio and beta.

Testing stock returns over the 1963–1990 period, they found that the size and market-to-book-value variables are powerful predictors of average stock returns. When these variables were used first in a regression analysis, the added beta variable was found to have little additional explanatory power. This led Professor Fama, to claim that beta – as sole variable explaining returns – is "dead." Thus Fama and French launched a powerful attack on the ability of the CAPM to explain common stock returns, suggesting that a firm's market value (size) and market-to-book-value ratio are the appropriate proxies for risk.

The Fama–French three-factor model is a practical application of Ross's arbitrage pricing theory, rooted in empirical evidence.

When constructing this model, researchers observed two prominent trends in the cross-section of average stock returns:
(i) Small companies, characterized by lower market equity, typically exhibit higher average stock returns compared to larger companies.
(ii) Value companies, typically defined by a higher ratio of book equity to market equity, tend to have superior average returns compared to growth companies.

According to the Fama–French three-factor model, expected returns are determined by:

$$r_s = r_f + \overbrace{\beta\,(r_m - r_f)}^{\text{FACTOR 1}} + \overbrace{s_i \cdot r_{SMB}}^{\text{FACTOR 2}} + \overbrace{h_i \cdot r_{HML}}^{\text{FACTOR 3}}$$

Similar to CAPM

Loading on SMB

Loading on HML

The **size factor, SMB** (small minus big), is the difference between the returns on a portfo lio of small stocks and a portfolio of big stocks.

The **value factor, HML** (high minus low), is the difference between the returns on a portfolio of high book-to market stocks and a portfolio of low book-to-market stocks.

18. Arbitrage Pricing Theory

A researcher has determined that a two-factor model is appropriate to determine the return on a stock. The factors are the percentage change in GNP and an interest rate. GNP is expected to grow by 3.6 percent, and the interest rate is expected to be 3.1 percent. A stock has a beta of 1.3 on the percentage change in GNP and a beta of -0.75 on the interest rate. If the expected rate of return on the stock is 12 percent, what is the revised expected return on the stock if GNP actually grows by 3.2 percent and the interest rate is 3.4 percent?

Solution:

$$r = \bar{r} + \beta_1 F_1 + \beta_2 F_2$$

12   1.3   3.2-3.6   0.75   3.4-3.1

EXPECTED RETURN   GNP   INTEREST RATE

$$= 12 + 1.3(3.2-3.6) - 0.75(3.4-3.1)$$

$$= 11.26\%$$

18. Arbitrage Pricing Theory

Suppose a three-factor model is appropriate to describe the returns of a stock. Information about those three factors is presented in the following chart:

Factor	Beta	Expected Value	Actual Value
GDP	0.006821	14,011	13,982
Inflation	-0.90	2.80%	2.6%
Interest Rate	-0.32	4.80%	4.6%

(a) What is the systematic risk of the stock return?
(b) Suppose unexpected bad news about the firm was announced that causes the stock price to drop by 1.1 percent. If the expected return on the stock is 12.8 percent, what is the total return on this stock?

Solution:

(a) What is the systematic risk of the stock return?

SYSTEMATIC RISK

$$= \beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3$$

$$= 0.006\,(13982 - 14011) - 0.90\,(2.60 - 2.80) - 0.32\,(4.60 - 4.80)$$

$$= -19.54\%$$

(b) Suppose unexpected bad news about the firm was announced that causes the stock price to drop by 1.1 percent. If the expected return on the stock is 12.8 percent, what is the total return on this stock?

$$r = \underbrace{\bar{r}}_{12.8} + \underbrace{\text{SYSTEMATIC RISK}}_{-19.54} + \underbrace{\text{UNSYSTEMATIC RISK}}_{-1.1\%\ \text{(PRICE DROP)}}$$

$$= -7.84\%$$

18. Arbitrage Pricing Theory

Suppose a factor model is appropriate to describe the returns on a stock. The current expected return on the stock is 10.5 percent. Information about those factors is presented in the following chart:

Factor	Beta	Expected Value	Actual Value
Growth in GNP	1.87	2.1%	2.6%
Inflation	-1.32	4.3	4.8

(a) What is the systematic risk of the stock return?
(b) The firm announced that its market share had unexpectedly increased from 23 percent to 27 percent. Investors know from past experience that the stock return will increase by 0.45 percent for every 1 percent increase in its market share. What is the unsystematic risk of the stock?
(c) What is the total return on this stock?

Solution:

(a) What is the systematic risk of the stock return?

$$\text{SYSTEMATIC RISK} = \beta_1 . F_1 + \beta_2 . F_2$$
$$= 1.87(2.6-2.1) - 1.32(4.8-4.3)$$
$$= 0.27\%$$

(b) The firm announced that its market share had unexpectedly increased from 23 percent to 27 percent. Investors know from past experience that the stock return will increase by 0.45 percent for every 1 percent increase in its market share. What is the unsystematic risk of the stock?

$$\text{UNSYSTEMATIC RISK} = 0.45(27-23)$$
$$= 1.80\%$$

(c) What is the total return on this stock?

$$r = \bar{r} + \text{Systematic Risk} + \text{Unsystematic Risk}$$
$$= 10.50 + 0.27 + 1.80 = 12.58\%$$

18. Arbitrage Pricing Theory

Suppose stock returns can be explained by a two-factor model. The firm-specific risks for all stocks are independent. The following table shows the information for two diversified portfolios:

	β_1	β_2	E(R)
Portfolio A	0.85	1.15	16%
Portfolio B	1.45	-0.25	12%

If the risk-free rate is 4 percent, what are the risk premiums for each factor in this model?

Solution:

$$r = \bar{r} + \beta_1 F_1 + \beta_2 F_2$$

$$16 = 4 + 0.85\,F_1 + 1.15\,F_2$$

$$12 = 4\% + 1.45\,F_1 - 0.25\,F_1$$

Solving for F_1 and F_2

$F_1 = 6.49\%$ $\quad F_2 = 5.64\%$

18. Arbitrage Pricing Theory

The following three stocks are available in the market:

	β	E(R)
Stock A	1.20	10.5%
Stock B	0.98	13%
Stock C	1.37	15.7%
Market	1.00	14.2%

Assume the market model is valid. A portfolio has weights of 30 percent Stock A, 45 percent Stock B, and 25 percent Stock C. Suppose the return on the market is 15 percent and there are no unsystematic surprises in the returns. What is the return on each stock? What is the return on the portfolio?

Solution:

RETURN ON A

$= 10.5 + 1.20(15 - 14.2) = 11.46\%$

RETURN ON B

$= 13 + 0.98(15 - 14.2) = 13.78\%$

RETURN ON C

$= 15.70 + 1.37(15 - 14.2) = 16.80\%$

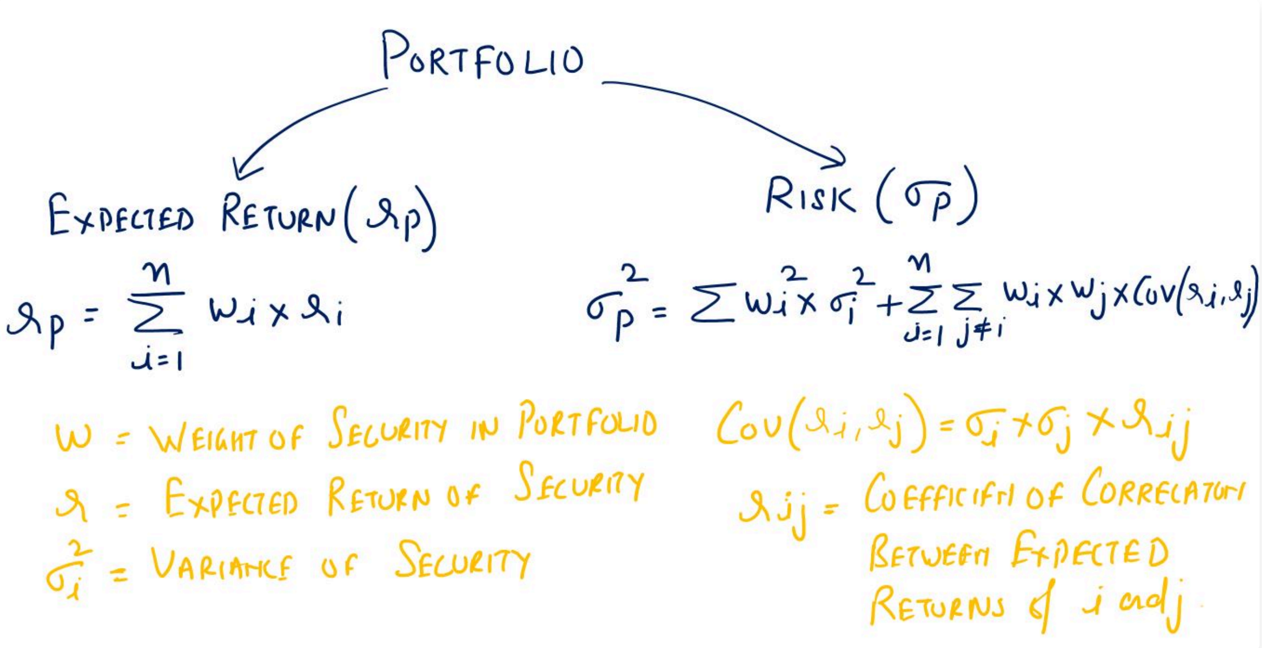
RETURN ON PORTFOLIO

$= 0.30 \times 11.46 + 0.45 \times 13.78$

$+ 0.25 \times 16.80 = 13.84\%$

19. Sharpe Single Index Model

Modern Portfolio Theory (MPT) (developed by Harry Markowitz) provides a systematic framework for constructing an optimal portfolio of investments that maximizes returns for a given level of risk, or minimizes risk for a given expected return. This is known as the mean-variance criterion.



In MPT, the expected return of a portfolio is the weighted average of the expected returns of the individual securities in the portfolio. The risk (measured as variance or standard deviation) of the portfolio depends not only on the individual risks of each security but also on the correlations (covariance) between the returns of the securities.

Further we understand that, through diversification, investors can reduce portfolio risk without necessarily sacrificing return, as long as the securities are not perfectly correlated.

The Markowitz Portfolio Selection Model requires a large amount of input data because it factors in both the expected returns and the covariances between all the securities in the portfolio.

Specifically, if a portfolio has n securities, the MPT model requires:

Thus, the Markowitz Model becomes computationally complex as the number of securities increases, as the amount of required data grows rapidly. This complexity is one of the reasons simpler models, like the Sharpe Single Index Model, were later developed to make the process more manageable.

Sharpe Single Index Model

The Sharpe Single Index Model, introduced by William Sharpe, is a simplified version of the more complex Markowitz Modern Portfolio Theory. It provides a way to estimate the expected return and risk of a portfolio by using a single factor, typically the return of a market index (such as the Nifty 50 or S&P 500). Instead of calculating the covariance between every pair of stocks in the portfolio (as in Markowitz's model), the Single Index Model assumes that the correlation between stocks is due to their common relationship with the broader market.

The Sharpe Single Index Model assumes that the returns of individual stocks are influenced by the overall movement of the market, represented by a market index. For example, the stock returns of companies like Tata Motors are assumed to be driven primarily by the performance of the market as a whole (Nifty 50).

The model posits a linear relationship between the return of a stock and the return of the market index. The stock return can be decomposed into two components:
(i) The part of the return that is attributable to the market.
(ii) The stock-specific return, which is independent of the market.

The return of a stock is given below:

$$r_i = \alpha_i + \underbrace{\beta_i . r_m}_{\text{SYSTEMATIC RISK}} + \underbrace{\varepsilon_i}_{\text{UNSYSTEMATIC RISK}}$$

r_i = RETURN OF SECURITY
α_i = PART OF RETURN WHICH IS INDEPENDENT OF MARKET
β_i = SENSITIVITY OF STOCK RETURN TO MARKET RETURN
r_m = RETURN OF MARKET INDEX
ε_i = RANDOM ERROR TERM

Sharpe identified this relationship by regressing the return on security (dependent variable) on the corresponding return on the market portfolio (independent variable).

The Single Index Model (SIM) states that the returns on a stock are a linear combination of its alpha plus the beta multiplied by the returns on the market index, plus a residual on any given day that represents the random movements on the firm on that day.

Portfolio Return and Risk

The return of a portfolio in the Single Index Model is calculated by aggregating the weighted returns of all stocks, where each stock's return is affected by its sensitivity (beta) to the market.

Expected Return of Portfolio

$$r_p = \alpha_p + \beta_p \times r_m$$

$$\alpha_p = \sum_{i=1}^{n} w_i \times \alpha_i$$

$$\beta_p = \sum_{i=1}^{n} w_i \times \beta_i$$

α_i = Stock Specific Return of Security i

β_i = Beta of Security i

r_m = Expected Return on Market

The risk of portfolio is split into two parts:

- Systematic Risk: The portion of risk attributed to market movements (cannot be diversified away).
- Unsystematic Risk: The stock-specific risk, which can be reduced through diversification.

Risk of Portfolio

$$\sigma_p^2 = \beta_p^2 \cdot \sigma_m^2 + \sum w_i^2 \cdot \sigma_{ei}^2$$

σ_m^2 = Variance of Market Return

β_p^2 = Weighted Beta of Portfolio $\left(\sum w_i \cdot \beta_i\right)$

σ_{ei}^2 = Unsystematic Variance of Security i

In the Sharpe Single Index Model, *stock-specific risk or unsystematic risk of security* is the portion of the risk that is unique to the individual stock and is unrelated to the overall market movements. It arises from factors that are specific to the company, such as management decisions, product performance, or industry-specific issues.

Unsystematic risk can be reduced or eliminated through diversification. By holding a portfolio of multiple stocks, the impact of stock-specific events (positive or negative) on the overall portfolio becomes smaller. In a well-diversified portfolio, the stock-specific risks of individual securities tend to cancel each other out, leaving only systematic risk (market-related risk).

In a fully diversified portfolio (with many securities), the aggregate unsystematic risk approaches zero. This is because the random fluctuations in individual stock prices become less impactful, and the portfolio's overall risk will primarily reflect systematic risk, represented by the market's volatility.

In a portfolio with n securities, the Single Index Model requires (3n + 2) estimates, which is a significant reduction compared to the n(n + 3)/2 estimates required by the Markowitz Model.

n SECURITIES	n SECURITIES
n EXPECTED RETURNS	n α_i
n VARIANCE TERMS	n β_i
$^nC_2$ COVARIANCE TERMS	n σ_i
	1 r_m
	1 σ_m^2
$2n + \frac{n(n-1)}{2}$ TERMS	$3n+2$ TERMS
MPT	SINGLE INDEX MODEL

The main difference between Sharpe Single Index model and APT is that the Sharpe Single Index model (market model) assumes that only one factor, usually a stock market aggregate, is enough to explain stock returns, while APT relies on multiple factors to explain returns.

20. Sharpe ratio

The Sharpe Ratio is used to measure the performance of an investment (such as a stock, mutual fund, or portfolio) compared to a risk-free asset, after adjusting for risk. It is one of the most widely used tools to assess the risk-adjusted return of an investment. The ratio helps investors understand how much excess return they are receiving for the extra risk they are taking on compared to a risk-free investment (such as government bonds).

$$\text{SHARPE RATIO} = \frac{\text{EXCESS RETURN OVER RISK FREE RETURN}}{\text{TOTAL RISK OF PORTFOLIO}} = \frac{r_p - r_f}{\sigma_p}$$

r_p = EXPECTED RETURN OF PORTFOLIO

r_f = RISK FREE RETURN

σ_p = RISK OF PORTFOLIO (TOTAL RISK)

A higher Sharpe Ratio indicates that an investment has a better risk-adjusted return. In other words, it is generating more return for each unit of risk taken.
A lower Sharpe Ratio suggests that the risk is not being adequately compensated by the returns, meaning the investor is taking on more risk for relatively less return.
A negative Sharpe Ratio means that the risk-free rate is higher than the portfolio’s return, indicating that the investor would have been better off investing in a risk-free asset.

Investors can use the Sharpe Ratio to compare multiple investments with varying risk levels. For instance, if two portfolios have similar returns but one has a higher Sharpe Ratio, it means the higher-ratio portfolio achieved the same return with less risk.

The Sharpe Ratio accounts for total risk, which includes both systematic and unsystematic risk. However, in a well-diversified portfolio, unsystematic risk (the risk specific to individual assets) can be minimized or eliminated. This is a limitation of the Sharpe Ratio because it doesn't differentiate between risks that can be diversified away and those that cannot.

Additionally, the Sharpe Ratio provides an absolute value, which is useful for comparing different investments or portfolios. However, it doesn't offer much insight when viewed in isolation, as it lacks context about whether the return is good or bad relative to the risk.

It’s also important to note that the Sharpe Ratio represents the slope of the Capital Allocation Line (CAL), which indicates the trade-off between risk and return.

21. Treynor Ratio

The Treynor Ratio is used to evaluate the risk-adjusted return of an investment portfolio, similar to the Sharpe Ratio.

$$\text{TREYNOR RATIO} = \frac{\text{EXCESS RETURN OVER RISK FREE RETURN}}{\text{SYSTEMATIC RISK OF PORTFOLIO}} = \frac{r_p - r_f}{\beta_p}$$

r_p = EXPECTED RETURN OF PORTFOLIO

r_f = RISK FREE RETURN

β_p = SYSTEMATIC RISK OF PORTFOLIO

However, the key difference is that while the Sharpe Ratio considers total risk (systematic risk + unsystematic risk, measured by standard deviation), the Treynor Ratio only accounts for only systematic risk (market risk), which is measured by beta. Beta represents how much an asset or portfolio moves in relation to the overall market.

A higher Treynor Ratio indicates that the portfolio is generating a higher return per unit of systematic risk. This suggests better performance relative to the market.
A lower Treynor Ratio indicates that the portfolio's return is not adequately compensating for the level of market risk taken.

Since the Treynor Ratio only looks at market risk, it is most useful when comparing well-diversified portfolios, where unsystematic risk has already been minimized.

All assets are expected to have the same reward-to-risk ratio, meaning that their risk-adjusted returns should be proportional to their systematic risk (beta). This concept is derived from the Capital Asset Pricing Model (CAPM) and the linearity of the Security Market Line (SML).

If the Treynor Ratio of an asset is greater than the Treynor Ratio for the market portfolio, it indicates that the asset is underpriced relative to its risk level. In other words, its expected return is higher than what is justified by its systematic risk. This suggests that the asset plots above the SML and is considered undervalued.

Conversely, if the Treynor Ratio of an asset is less than the Treynor Ratio for the market portfolio, it suggests that the asset is overpriced relative to its risk level. Its expected return is lower than what is justified by its systematic risk, indicating that the asset plots below the SML and is considered overvalued.

22. M Squared

M-Squared (M^2), or Modigliani-Modigliani Measure, is used to compare the performance of a portfolio to a benchmark, adjusting for risk. It was developed by Nobel laureates Franco Modigliani and his grand-daughter Leah Modigliani.

$$\text{M-SQUARED} = \left(\frac{r_p - r_f}{\sigma_p} \right) \times \sigma_m - (r_m - r_f)$$

r_p = EXPECTED PORTFOLIO RETURN

r_f = RISK FREE RETURN

σ_p = RISK OF PORTFOLIO (TOTAL RISK)

r_m = EXPECTED RETURN ON MARKET

σ_m = RISK OF MARKET

M^2 builds upon the Sharpe Ratio but expresses the results in percentage terms, making it easier to understand and compare across different portfolios and benchmarks.

It accounts for total risk, which includes both systematic and unsystematic risk.

Positive M^2: If the M^2 of the portfolio is higher than that of the benchmark, the portfolio has outperformed the benchmark on a risk-adjusted basis.
Negative M^2: If the M^2 is lower than the benchmark, the portfolio has underperformed on a risk-adjusted basis.

Unlike the Sharpe Ratio, which provides a ratio that is harder to interpret, M^2 expresses performance in percentage terms, making it easier to compare with benchmark returns.
