Testing Many Parameters under Mixed Identification Strength

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Introduction

- Traditional Inference is distorted with
  - Large dimensional (k) parameters
  - Weakly identified parameters

- Previous work addresses
  - Parameter identification failure with small k
  - Large k without identification failure

- Some economic questions require considering both

- This test accommodates weak identification and large k

Introduction: Example Models

- Tests for No Omitted Non-linearity - Non-linear Mean Reversion in Exchange Rates
- Linear IV with many instruments estimated by LIML Linear IV LIML
- Nonlinear Binary Choice with many covariates

Introduction: Example Models

- Tests for Omitted Non-linearity

$$y_t = \delta' X_t + \sum_{d=1}^{k} \beta_d h(\pi_d, Z_{d,t}) + \varepsilon_t, \qquad E(\varepsilon_t | X_t, Z_t) = 0$$

- Linear IV with many instruments estimated by LIML

▸ Linear IV LIML

- Nonlinear Binary Choice with many covariates

Introduction: Example Models

- Tests for Omitted Non-linearity

$$y_t = \sum_{d=1}^{k} \left(\pi_d' X_t \cdot h(\beta_d, Z_{d,t}) \right) + \varepsilon_t, \qquad E(\varepsilon_t | X_t, Z_t) = 0$$

- Linear IV with many instruments estimated by LIML

▸ Linear IV LIML

- Nonlinear Binary Choice with many covariates

Introduction: Example Models

- Tests for Omitted Non-linearity
- Linear IV with many instruments estimated by LIML

Linear IV LIML

$$y_t = x_{1,t}\pi_1 + x_{2,t}\pi_2 + Z_t'\omega + u_t^*,$$
$$x_{1,t} = Z_{1,t}'\beta_1 + v_t, \qquad x_{2,t} = Z_{2,t}'\beta_2 + \eta_t$$
$$(u_t, v_t, \eta_t) \sim N(0, \Sigma)$$

- Nonlinear Binary Choice with many covariates

Introduction: Example Models

- Tests for Omitted Non-linearity
- Linear IV with many instruments estimated by LIML

Linear IV LIML

- Nonlinear Binary Choice with many covariates

$$y_i = 1(y_i^* > \varepsilon_i), \qquad \varepsilon_i \sim N(0,1)$$
$$y_i^* = \sum_{j=1}^{p} \beta_j h_j(X_{j,i}, \pi_j)$$

Initial Simulation Evidence

$$Y_t = \zeta + \beta_1 h(Z_t, \pi_b) + \lambda_1 h(X_{1,t}, \pi_1) + \cdots + \lambda_k h(X_{k,t}, \pi_k) + \varepsilon_t$$

$$H_0 : \lambda_i = 0 \quad \text{for every } i$$

Model	Linear	Non-Linear	Linear	Non-Linear
k	1	1	20	20
Traditional Wald Test	0.04	0.14	0.22	0.85
Traditional Max Test	0.05	0.11	0.05	0.12

: Rejection Frequencies, $J = 1000$, $\alpha = 0.05$, $n = 200$

idea:

- Use the max test to control distortion from large dimension
- Correct the distortion from identification failure by simulating the distribution

Literature

Max Tests

- Hansen (2005), Ghysels et al. (2017), Hill and Dennis (2018)
- Do not accommodate parameter identification failure

Weak Identification/Parameter Identification Failure

- Andrews and Cheng (2012a, 2013, 2014), Cheng (2015)
- Do not accommodate large dimensional parameters

Inference on Large Dimensional Parameters/Bootstrapping Maximum Statistics

- van de Geer et al. (2014), Caner and Kock (2018) Wooldridge and Zhu (Forthcoming)
- Chernozhukov et al. (2015, 2013, 2016), Zhang and Cheng (2017), Zhang and Wu (2017)
- Do not accommodate parameter identification failure

Outline

Construction

Consider a model

$$Y_t = g(X_t, \beta, \lambda) + \varepsilon_t, \qquad E[\varepsilon_t | X_t] = 0$$

- We want to test

$$H_0 : \lambda = 0_k$$

- Issue 1: $\dim(\lambda) = k$ is 'large'
- Results in an estimation problem

Construction

Issue 1:

- $dim(\lambda) = k$ is large $\Rightarrow$ estimation problem
- Solution: construct and estimate k *sub-models* by imposing $\lambda_j = 0$ for all but one j:

$$Y_t = g(X_t, \beta, (0, \ldots, \lambda_i, 0, \ldots, 0)) + \nu_{i,t}$$

- Then under H_0, $\nu_{i,t} = \varepsilon_t$ for every i
- Creates a trade-off: small dimensional *sub-models* can be estimated accurately, but we must combine k (many) of them:

$$\left.\begin{array}{c} Y_t = g(X_t, \beta, (\lambda_1, 0, \ldots, 0)) + \nu_{1,t} \\ \vdots \\ Y_t = g(X_t, \beta, (0, \ldots, 0, \lambda_k)) + \nu_{k,t} \end{array}\right\} \Rightarrow \left.\begin{array}{c} \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_k \end{array}\right\} \Rightarrow ?$$

Construction

- Issue 1: $\dim(\lambda) = k$ is 'large'
- Observe

$$\begin{aligned} \lambda = 0_k \quad & \textit{iff} \quad \lambda_i = 0 \quad \text{for every } i \\ & \textit{iff} \quad \max_{1 \leq i \leq k} |\lambda_i| = 0 \end{aligned}$$

- We use this relationship to construct our test

Construction

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$$\begin{aligned} \lambda = 0_k \qquad & \textit{iff} \qquad \lambda_i = 0 \qquad \text{for every } i \\ & \textit{iff} \qquad \max_{1 \le i \le k} |\lambda_i| = 0 \end{aligned}$$

- We use this relationship to construct our test:

$$\left. \begin{aligned} Y_t &= g(X_t, \beta, (\lambda_1, 0, \ldots, 0)) + \nu_{1,t} \\ &\vdots \\ Y_t &= g(X_t, \beta, (0, \ldots, 0, \lambda_k)) + \nu_{k,t} \end{aligned} \right\} \Rightarrow \left. \begin{matrix} \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_k \end{matrix} \right\} \Rightarrow \hat{\mathcal{T}} = \max_{1 \le i \le k} |\mathcal{N}_i \hat{\lambda}_i|$$

Construction

Issue 2: identification failure

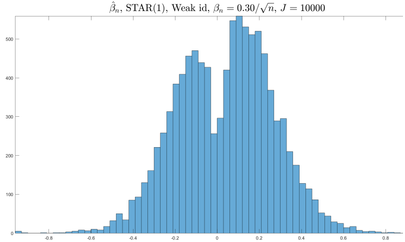
$$Y_t = g(X_t, \beta, \lambda, \pi_\beta, \pi_\lambda) + \varepsilon_t, \qquad E[\varepsilon_t | X_t] = 0$$

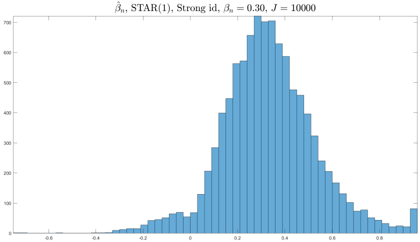
- $\pi_{\lambda,i}$ is unidentified when $\lambda_i = 0$.
- Similar for π_β
- We want to test

$$H_0 : \lambda_i = 0 \qquad \text{for every } i$$

- identification failure induces non-standard distributions: ▸ Histogram
- and from the previous slide, we have k of these! ▸ Histogram
- I provide a procedure for simulating these distributions and the resulting maximum statistic

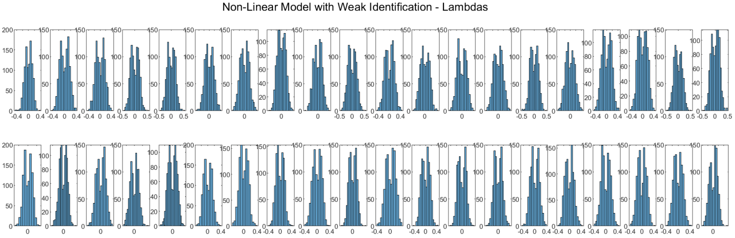
Identification Failure: Non-Standard Distribution



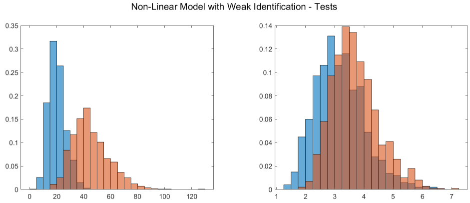


$$Y_t = \beta h(X_t, \pi) + \varepsilon_t$$

Identification Failure: Non-Standard Distribution



1) The non-standard distributions and 2) combining many of these distributions both affect our test statistics (blue: ‘standard’, orange: empirical):



Outline

Empirical Example - Exchange Rate Modeling

- Studies of PPP $\Rightarrow$ exchange rate adjustments resemble a unit root process within a band and a stationary process outside of that band. (Taylor et al., 2001; Obstfeld and Taylor, 1997)
- Large literature following Taylor et al. (2001) allow a smooth transition at the boundary of the band with a 'Smooth Transition Regression' Model
- Example: Kilic (2016) examines the first differenced model

$$\Delta q_t = \left[\delta_0 + \sum_{j=1}^{p} \delta_j \Delta q_{t-j}\right] h(\gamma_d, \Delta q_{t-d}) + u_t$$

where h is the exponential transition function

$$h(\gamma; x) = 1 - \exp(-\gamma x^2).$$

q_t is the demeaned log real exchange rate, and $u_t \sim iid(0, \sigma^2)$

Empirical Example - Exchange Rate Modeling

$$\Delta q_t = \left[\delta_0 + \sum_{j=1}^{p} \delta_j \Delta q_{t-j}\right] h(\gamma_d, \Delta q_{t-d}) + u_t$$
$$h(\gamma; x) = 1 - \exp(-\gamma x^2).$$

Two issues:

- The unknown value of d must be selected
  - (a large number of lags are available).
- Parameter identification failure occurs when $\gamma_d = 0$.

Empirical Example - Exchange Rate Modeling

In principle, the δ's can differ by d

$$\Delta q_t = \left[\delta_{d,0} + \sum_{j=1}^{p} \delta_{d,j} \Delta q_{t-j}\right] h(\gamma_d, \Delta q_{t-d}) + u_{d,t}$$

and there is an implied 'belief' that $\gamma_{\tilde{d}} = 0$ for any $\tilde{d} \neq d$.
This structure implies a full model:

$$\Delta q_t = \sum_{d=1}^{k} \left(\left[\delta_{d,0} + \sum_{j=1}^{p} \delta_{d,j} \Delta q_{t-j}\right] h(\gamma_d, \Delta q_{t-d}) \right) + \varepsilon_t$$

where we can formulate a null hypothesis that gives the appropriate restriction on this full model.

$$H_0 : \gamma_{\tilde{d}} = 0 \qquad \forall \tilde{d} \neq d$$

Empirical Example - Exchange Rate Modeling

This is the null for tests of no (omitted) nonlinearity. In general,

$$H_0 : \lambda = 0_k$$

where λ is a sub-vector of $\gamma = (\gamma_1, \dots, \gamma_d, \dots)$.

- How do we use the max test?
- Conveniently, the parsimonious models are already given. e.g. when $\lambda = \gamma$:

$$\Delta q_t = \left[\delta_{d,0} + \sum_{j=1}^{p} \delta_{d,j} \Delta q_{t-j}\right] h(\lambda_d, \Delta q_{t-d}) + u_{d,t}$$

  - for each $d = 1, \dots, k$

▸ When λ is a sub-vector

- Observe that $H_0 \Rightarrow u_{d,t} = \varepsilon_t$ for every d

Empirical Example - Exchange Rate Modeling

- let $\lambda = \gamma$,

$$F_t(\delta_{d}) = \delta_{d,0} + \sum_{j=1}^{p} \delta_{d,j} \Delta q_{t-j},$$

and consider $H_0 : \lambda = 0$:

Construct the max statistic via the parsimonious models:

$$\left.\begin{array}{c} \Delta q_t = F_t(\hat{\delta}_1) h(\hat{\lambda}_1, \Delta q_{t-1}) + u_{1,t} \\ \vdots \\ \Delta q_t = F_t(\hat{\delta}_k) h(\hat{\lambda}_k, \Delta q_{t-k}) + u_{k,t} \end{array}\right\} \Rightarrow \left.\begin{array}{c} \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_k \end{array}\right\} \Rightarrow \hat{\mathcal{T}} = \max_{1 \leq i \leq k} |\mathcal{N}_i \hat{\lambda}_i|$$

Outline

Max Test - A Little Theory

$$Y_t = \lambda_1 h(X_t, \pi_1) + \lambda_2 h(X_t, \pi_2) + \cdots + \lambda_k h(X_t, \pi_k) + \varepsilon_t$$

$$H_0 : \lambda = 0_k$$

We estimate k *sub-models*

$$Y_t = \lambda_i h(X_t, \pi_i) + \nu_{i,t}, \quad i = 1, \ldots, k$$

Then collect the estimators and form the max statistic

$$\hat{\mathcal{T}}_n = \max_{1 \leq i \leq k_n} |\mathcal{N}_i \hat{\lambda}_{i,n}|$$

where $\mathcal{N}_i = \sqrt{n}$ in this example.

Limiting Distribution

- If there are weakly identified parameters, then under some assumptions and H_0,

$$\left| \hat{\mathcal{T}}_n - \max_{1 \le i \le \mathring{k}_n} |S'_{(i),\lambda} \mathfrak{Z}_{(i)}(\pi^*_{(i),l_K}(b, \gamma_0); \gamma_0)| \right| \xrightarrow{p} 0$$

for some non-unique $\mathring{k}_n = o(n)$. Pointwise, we have

$$\mathfrak{Z}_{(i)}(\pi_{(i),l_K}; \gamma_0) = \begin{pmatrix} \tau_{(i)}(\pi^*_{(i),l_K}(b, \gamma_0)) - S_{l_K} b_{(i),l_K} \\ \pi^*_{(i),l_K}(b, \gamma_0) \end{pmatrix}$$

where S_{l_K} is the selection matrix that selects the columns corresponding to $\lambda_{(i),l_k}$.

Limiting Distribution

- if no parameters are weakly identified then under assumptions and H_0,

$$\left|\hat{\mathcal{T}}_n - \max_{1 \leq i \leq \mathring{k}_n} |S'_{(i),\lambda} \mathfrak{Z}_{(i)}(\gamma_0)|\right| \xrightarrow{p} 0$$

for some non-unique $\mathring{k}_n = o(n)$. Pointwise,

$$\mathfrak{Z}_{(i)}(\gamma_0) = H_{(i),K-1}(\gamma_0)^{-1} \mathcal{G}_{(i),\theta}(\gamma_0)$$

where $\mathcal{G}_{(i),\theta}(\gamma_0) \sim N(0, \Omega_{(i),\theta}(\gamma_0))$.

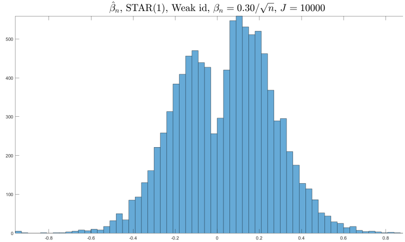
Limiting Distribution - Weak Identification

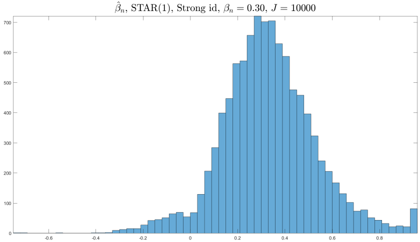
- Key take away - two different distributions based on whether π can be consistently estimated.
- Recall

$$Y_t = \lambda_i h(X_{i,t}, \pi_i) + \nu_{i,t}, \quad i = 1, \dots, k$$

- λ_i is "large" $\Rightarrow \pi_i$ is (strongly) identified
  - $\hat{\pi}_i$ consistently estimates π_i
- λ_i is "small"
  - $\Rightarrow \pi_i$ is weakly identified
  - $\hat{\pi}_i$ cannot consistent estimate π_i (noise dominates the signal)
  - $\Rightarrow \hat{\pi}_i$ has Nonstandard Distribution (π_i^*)
  - This nonstandardness propogates to the other estimators! $\Rightarrow \hat{\lambda}_i$ also has a Nonstandard Distribution!

Identification Failure: Non-Standard Distribution

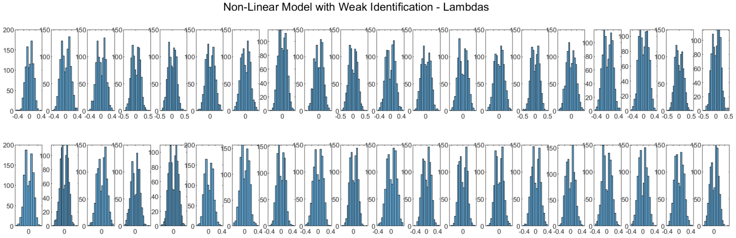




$$Y_t = \lambda h(X_t, \pi) + \varepsilon_t$$

Return to Intro

Identification Failure: Non-Standard Distribution



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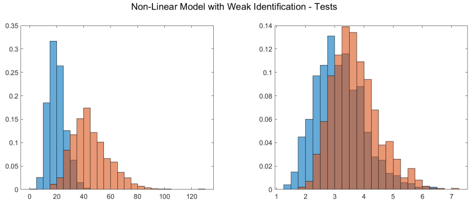

Limiting Distribution Note on Assumptions

- Needed restrictions on the functional form of the model
- There must be a parametric source of identification failure e.g. $y_t = \beta h(X_t, \pi) + \varepsilon_t$: $\beta = 0 \Rightarrow \pi$ is unidentified
- There can be more than one source of identification failure, but each can only affect its own parameters e.g. $y_t = \beta_1 h(X_{1,t}, \pi_1) + \beta_2 h(X_{2,t}, \pi_2) + \varepsilon_t$
- The derivatives of the criterion functions agree for the full and sub models

Outline

Inference

- Inference usually relies on calculation of asymptotic distribution. (de Haan, 1976)
- But EVT requires conditions that might be too restrictive (Hill and Dennis, 2018; Hill and Motegi, 2017)
  - (e.g. dependence properties, non-standard distributions)
- Instead, we provide a bootstrap procedure to simulate the distribution
- Bootstrapping this type of test is new (Chernozhukov et al., 2013, 2016; Zhang and Cheng, 2017; Zhang and Wu, 2017; Hill and Motegi, 2017; Hill and Dennis, 2018)
  We must account for
  - Dependence in the estimators (e.g. omitted variable bias)
  - Non-standard distributions (identification failure)

Inference

Recall 2 distributions

- We don't know which is correct - must bootstrap each individually
- 2 different critical values - How to combine them?
- ICS - Use data to determine if parameter is identified ICS Details
    - if so, then use identified cv
    - if not, take the larger of the 2 cvs

Inference Procedure

Gaussian Multiplier Bootstrap:

1. First, draw a Gaussian multiplier sequence Z_t
2. For each Parsimonious Model:
   a. Use ICS pretest to determine which parameters are strongly identified. Place the remainder in group I_K.
   b. Use Z_t to form key quantities $\mathfrak{Z}_i(\pi_{i,I_K};\gamma_0)$ and $\pi^*_{i,I_K}(b,\gamma_0)$. Inference Step 2b
   c. Collect the resulting values corresponding to λ, and use them to form the test statistic.
3. Repeat above steps M times
4. Form α-level critical values, $cv(b,\gamma_0)$.
5. Repeat on a grid of the nuisance parameters, and take the largest of the critical values.

Outline

Simulation Setup

Consider the additive non-linear model

$$Y_t = \zeta + \beta_1 h(\tilde{X}_{1,t}, \tilde{\pi}_1) + \lambda_1 h(X_{1,t}, \pi_1) + \cdots + \lambda_k h(X_{k,t}, \pi_k) + \varepsilon_t$$

with

$$h_j(X_{j,t}, \pi_j) = X_{j,t}\Big[1 - \exp(-c(Z_t - \pi_j)^2)\Big],$$

- Correlated Regressors: $X_t \sim N(0_{d_\beta}, \Sigma_x)$, $\varepsilon_t \sim$ iid $N(0,1)$.
- Set $c = 10$ for convenience,
- Consider $\beta_1 \in \{0, 1/\sqrt{n}, 1\}$
- Test $H_0 : \lambda_i = 0$ for every i

Simulation Results

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Null

$H_0 : \lambda_i = 0 \quad i = 1, \ldots, d_\beta - 1$

$n = 200,\ J = 10000,\ \alpha = 0.05$

	$k_{\lambda,n} = 1$			$k_{\lambda,n} = 20$		
β_1	0	$1/\sqrt{n}$	1	0	$1/\sqrt{n}$	1
Wald Test Standard	0.11	0.12	0.12	0.83	0.84	0.84
Max Test Standard	0.10	0.10	0.10	0.12	0.11	0.11
Max t-Test Standard	0.11	0.12	0.11	0.19	0.19	0.20
Wald Test BS1	0.06	0.06	0.06	0.68	0.68	0.69
Max Test BS1	0.04	0.04	0.04	0.03	0.03	0.03
Max t-Test BS1	0.06	0.06	0.06	0.13	0.12	0.13
Wald Test BS2	0.06	0.06	0.06	0.25	0.26	0.25
Max Test BS2	0.05	0.05	0.05	0.04	0.04	0.04
Max t-Test BS2	0.06	0.06	0.06	0.08	0.08	0.09

Histograms Full table

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Null

$H_0 : \lambda_i = 0 \quad i = 1, \ldots, d_\beta - 1$

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Wald Test BS2	0.06	0.06	0.06	0.25	0.26	0.25
Max Test BS2	0.05	0.05	0.05	0.04	0.04	0.04
Max t-Test BS2	0.06	0.06	0.06	0.08	0.08	0.09

Histograms Full table

Simulation Results

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Wald Test Standard	0.11	0.12	0.12	0.83	0.84	0.84
Max Test Standard	0.10	0.10	0.10	0.12	0.11	0.11
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Max t-Test BS1	0.06	0.06	0.06	0.13	0.12	0.13
Wald Test BS2	0.06	0.06	0.06	0.25	0.26	0.25
Max Test BS2	0.05	0.05	0.05	0.04	0.04	0.04
Max t-Test BS2	0.06	0.06	0.06	0.08	0.08	0.09

Histograms Full table

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Wald Test BS2	0.06	0.06	0.06	0.25	0.26	0.25
Max Test BS2	0.05	0.05	0.05	0.04	0.04	0.04
Max t-Test BS2	0.06	0.06	0.06	0.08	0.08	0.09

Histograms Full table

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Local Alternative
$H_{LA}: \lambda_1 = 1/\sqrt{n},\ \lambda_i = 0\ \forall i \geq 2$
$n = 200,\ J = 10000,\ \alpha = 0.05$

	$k_{\lambda,n} = 1$			$k_{\lambda,n} = 20$		
β_1	0	$1/\sqrt{n}$	1	0	$1/\sqrt{n}$	1
Wald Test Standard	0.21	0.22	0.21	0.91	0.91	0.91
Max Test Standard	0.19	0.20	0.19	0.25	0.25	0.25
Max t-Test Standard	0.20	0.20	0.21	0.50	0.50	0.50
Wald Test BS1	0.12	0.12	0.12	0.81	0.80	0.80
Max Test BS1	0.09	0.09	0.09	0.09	0.09	0.10
Max t-Test BS1	0.12	0.12	0.12	0.42	0.42	0.43
Wald Test BS2	0.13	0.13	0.13	0.39	0.39	0.39
Max Test BS2	0.11	0.11	0.11	0.11	0.10	0.11
Max t-Test BS2	0.13	0.13	0.13	0.34	0.33	0.33

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Local Alternative

$H_{LA}: \lambda_1 = 1/\sqrt{n}, \ \lambda_i = 0 \ \forall i \geq 2$

$n = 200, \ J = 10000, \ \alpha = 0.05$

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Max t-Test BS2	0.13	0.13	0.13	0.34	0.33	0.33

Simulation Results

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Local Alternative
$H_{LA} : \lambda_1 = 1/\sqrt{n}, \ \lambda_i = 0 \ \forall i \geq 2$
$n = 200, \ J = 10000, \ \alpha = 0.05$

	$k_{\lambda,n} = 1$			$k_{\lambda,n} = 20$		
β_1	0	$1/\sqrt{n}$	1	0	$1/\sqrt{n}$	1
Wald Test Standard	0.21	0.22	0.21	0.91	0.91	0.91
Max Test Standard	0.19	0.20	0.19	0.25	0.25	0.25
Max t-Test Standard	0.20	0.20	0.21	0.50	0.50	0.50
Wald Test BS1	0.12	0.12	0.12	0.81	0.80	0.80
Max Test BS1	0.09	0.09	0.09	0.09	0.09	0.10
Max t-Test BS1	0.12	0.12	0.12	0.42	0.42	0.43
Wald Test BS2	0.13	0.13	0.13	0.39	0.39	0.39
Max Test BS2	0.11	0.11	0.11	0.11	0.10	0.11
Max t-Test BS2	0.13	0.13	0.13	0.34	0.33	0.33

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Alternative
$H_A : \lambda_1 = 1, \lambda_i = 0\ \forall i \geq 2$
$n = 200,\ J = 10000,\ \alpha = 0.05$

	$k_{\lambda,n} = 1$			$k_{\lambda,n} = 20$		
β_1	0	$1/\sqrt{n}$	1	0	$1/\sqrt{n}$	1
Wald Test Standard	1.00	1.00	1.00	1.00	1.00	1.00
Max Test Standard	1.00	1.00	1.00	1.00	1.00	1.00
Max t-Test Standard	1.00	1.00	1.00	1.00	1.00	1.00
Wald Test BS1	1.00	1.00	1.00	1.00	1.00	1.00
Max Test BS1	1.00	1.00	1.00	1.00	1.00	1.00
Max t-Test BS1	1.00	1.00	1.00	1.00	1.00	1.00
Wald Test BS2	1.00	1.00	1.00	1.00	1.00	1.00
Max Test BS2	1.00	1.00	1.00	1.00	1.00	1.00
Max t-Test BS2	1.00	1.00	1.00	1.00	1.00	1.00

Conclusion

- Traditional Inference is distorted with
  - Large dimensional (k) parameters
  - Identification Failure
- Previous work addresses
  - identification failure
  - large k without identification failure
- Some economic questions require considering both
- Our max test accommodates weak identification and large k

Thank You!

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Appendix Slides follow

Appendix Slides follow

Max Test - General Setup

Appropriate for models estimated with M-estimators (under mixing and moment conditions):

$$Q_n = \frac{1}{n}\sum_{t=1}^{n} m_t(\theta)$$

The parsimonious models are defined by

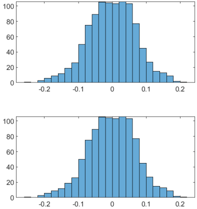
$$Q_{(i),n} = \frac{1}{n}\sum_{t=1}^{n} m_{(i),t}(\theta_{(i)})$$

where

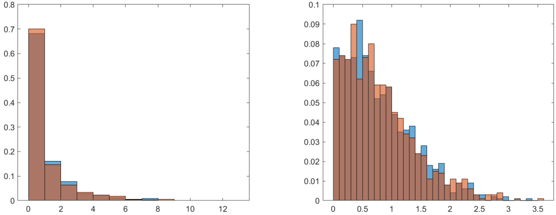
$$m_{(i),t}(\theta_{(i)}) = m_t\big(\underbrace{(\delta, 0, \ldots, \lambda_i, 0, \ldots, \tilde{\delta}_i)}_{\theta_{(i)}=[\theta]_{(i)}}\big)$$

Return to Example Setup

Linear Model Lambdas



Linear Model Tests



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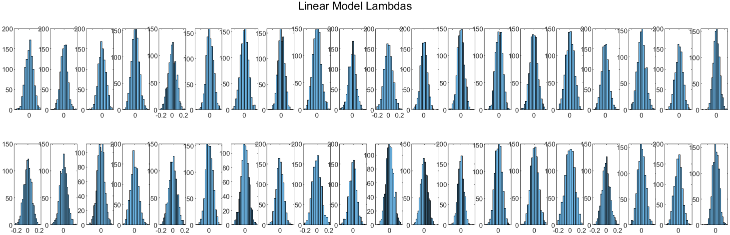

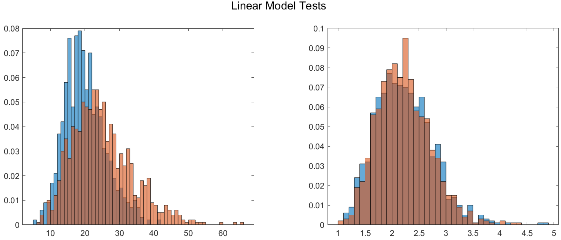

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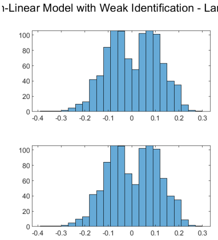
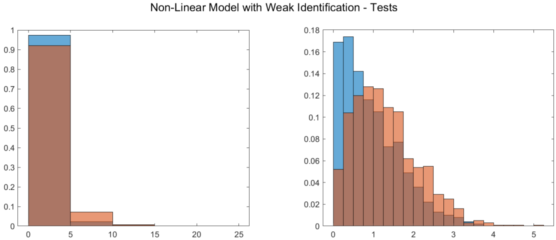
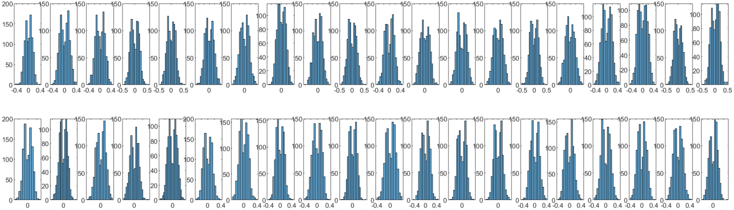


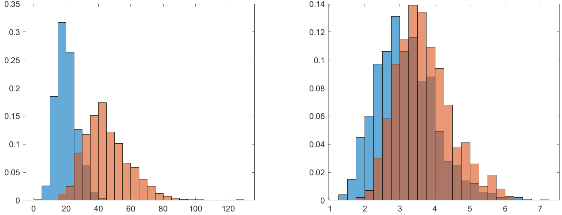


Return to Example Setup

Non-Linear Model with Weak Identification - Lambdas



Non-Linear Model with Weak Identification - Tests



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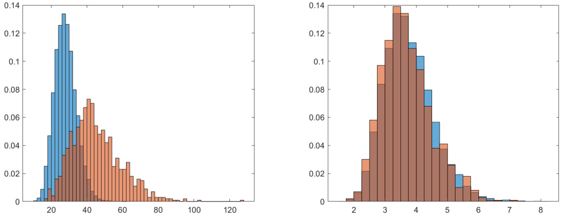
Non-Linear Model with Weak Identification - Tests



Return to Simulation Results

Empirical Example - Exchange Rate Modeling

- Parsimonious model when λ is a sub-vector of γ:

$$\Delta q_t = \left[\delta_{\tilde{d},0} + \sum_{j=1}^{p} \delta_{\tilde{d},j} \Delta q_{t-j}\right] h(\gamma_{\tilde{d}}, \Delta q_{t-\tilde{d}})$$
$$+ \left[\delta_{d,0} + \sum_{j=1}^{p} \delta_{d,j} \Delta q_{t-j}\right] h(\lambda_{d}, \Delta q_{t-d}) + u_{d,t}$$

  - for each $d = 1, \ldots, k$
- Observe that $H_0 \Rightarrow u_{d,t} = \varepsilon_t$ for every d

Return

Other Empirical Examples - LIML Linear IV

- Linear IV with many instruments typically estimated with LASSO (Belloni et al., 2010)
- Leeb and Pötscher (2008); Pötscher (2009); Belloni et al. (2016) note that this can be problematic with many weak instruments
- Some new inference procedures address this. For example, Belloni et al. (2014b,a) perform a follow-up study regarding the effect of legalized abortion on crime
- They alter Donohue and Levitt (2001) to include 284 variables with only 600 observations and illustrate 'LASSO-double-selection'
    - i) results based on a small set of intuitively selected controls differ from results obtained through formal variable selection
    - ii) accounting for nonlinear trends in the data affects the results
- We can use the Max Test to examine if the 'intuitively selected' or 'added fidelity' controls are relevant.

Other Empirical Examples - LIML Linear IV

- Linear IV estimated by LIML fits into the weak identification framework (Andrews and Cheng, 2012a,b).
- We extend this to allow many covariates and instruments and mixed identification strength.

- Related to LIML Linear IV weak instruments literature (Bound et al., 1996; Andrews et al., 2006; Chao and Swanson, 2007) etc...
- and the many weak instruments literature Andrews and Stock (2007) etc...
- and Post Selection Inference (Belloni et al., 2014b,a, 2016)

Other Empirical Examples - LIML Linear IV

Consider the structural model

$$y_t = x_{1,t}\pi_1 + x_{2,t}\pi_2 + Z_t'\omega + u_t^*,$$
$$x_{1,t} = Z_{1,t}'\beta_1 + v_t, \qquad x_{2,t} = Z_{2,t}'\beta_2 + \eta_t.$$

The reduced form equations are

$$y_t = Z_{1,t}'(\beta_1\pi_1 + \omega_1) + Z_{2,t}'(\beta_2\pi_2 + \omega_2) + u_t,$$
$$x_{1,t} = Z_{1,t}'\beta_1 + v_t, \quad x_{2,t} = Z_{2,t}'\beta_2 + \eta_t$$

- where $u_t = v_t^*\pi_1 + \eta_t^*\pi_2 + u_t^*$,
- and we assume $(u_t, v_t, \eta_t) \sim N(0, \Sigma)$.

Other Empirical Examples - LIML Linear IV

- We can use the Max Test to examine if the 'intuitively selected' or 'added fidelity' controls are relevant.
- e.g. $H_0 : (\beta_2, \omega_2) = 0_k$ for large k
- The Parsimonious models are the reduced form equations

$$\begin{aligned} y_t &= Z_{1,t}'(\beta_1\pi_1 + \omega_1) + Z_{2,i,t}(\beta_{2,i}\pi_2) + \tilde{u}_{(i),t}, \\ x_{1,t} &= Z_{1,t}'\beta_1 + v_t \\ x_{2,t} &= Z_{2,i,t}\beta_{2,i} + \tilde{\eta}_{(i),t} \end{aligned}$$

and

$$\begin{aligned} y_t &= Z_{1,t}'(\beta_1\pi_1 + \omega_1) + Z_{2,i,t}\omega_{2,i} + \tilde{w}_{(i),t}, \\ x_{1,t} &= Z_{1,t}'\beta_1 + v_t \end{aligned}$$

- let $\lambda = \beta_2$, and consider $H_0 : \lambda = 0$:

REWRITE THIS

$$\left.\begin{aligned} y_t &= Z'_{1,t}\beta_1\pi_1 + Z_{2,1,t}\lambda_1\pi_2 + u_t \\ x_{1,t} &= Z'_{1,t}\beta_1 + v_t \\ x_{2,t} &= Z_{2,1,t}\lambda_1 + \tilde{\eta}_{1,t} \\ &\vdots \\ y_t &= Z'_{1,t}\beta_1\pi_1 + Z_{2,k,t}\lambda_k\pi_2 + u_t \\ x_{1,t} &= Z'_{1,t}\beta_1 + v_t \\ x_{2,t} &= Z_{2,k,t}\lambda_k + \tilde{\eta}_{k,t} \end{aligned}\right\} \Rightarrow \left.\begin{matrix} \hat{\lambda}_1 \\ \vdots \\ \hat{\lambda}_k \end{matrix}\right\} \Rightarrow \hat{\mathcal{T}} = \max_{1\le i\le k} |\mathcal{N}_i\hat{\lambda}_i|$$

Back to Empirical Examples

Appendix - Grouping Notation

- Parameters are grouped based on identification strength.
- Weakly identified parameters are placed in group I_K

let $\mathcal{G}_{(i)}(\pi_{(i),l_K};\gamma_0)$ be a zero mean Gaussian process with covariance kernel $\Omega_{(i)}(\pi_{(i),l_K},\tilde{\pi}_{(i),l_K};\gamma_0)$, and define the processes

$$\tau_{(i)}(\pi_{(i),l_K};\gamma_0) = \Big[H_{(i),K}(\pi_{(i),l_K};\gamma_0)\Big]^{-1}\Big(\mathcal{K}_{(i),K}(\pi_{(i),l_K};\gamma_0)b_{(i),l_K} + \mathcal{G}_{(i)}(\pi_{(i),l_K}; \tag{1}$$

$$\chi_{(i)}(\pi_{(i),l_K};\gamma_0) = -\frac{1}{2}\tau_{(i)}(\pi_{(i),l_K};\gamma_0)'\Big[H_{(i),K}(\pi_{(i),l_K};\gamma_0)\Big]\tau_{(i)}(\pi_{(i),l_K};\gamma_0). \tag{2}$$

Theorem (5.4)

Let Assumptions 1-8 hold. Under $\gamma_n \to \gamma_0$,

a) *If* $I_K \neq \emptyset$, *where* I_K *indexes the weakly identified subvector of* $\pi_{(i)}$, *then*

$$n\left(Q^c_{(i),n}(\pi_{(i),I_k}) - Q_{(i),n}(\psi^0_{(i),K,n}, \pi_{(i),I_k})\right) \Rightarrow \chi_{(i)}(\pi_{(i),I_K};\gamma_0) \tag{3}$$

$$\begin{pmatrix} n^{1/2} B(\beta_{(i),K^-,n})(\hat{\psi}_{(i),K^-} - \psi_{(i),K^-,n}) \\ \hat{\pi}_{(i),I_K} \end{pmatrix} \xrightarrow{d} \begin{pmatrix} \tau_{(i)}(\pi^*_{(i),I_K}) - S_{I_K} b_{(i),I_K} \\ \pi^*_{(i),I_K} \end{pmatrix} \tag{4}$$

where S_{I_K} *is the selection matrix that selects the columns corresponding to* $\beta_{(i),I_k}$.

b) *if* $I_k = \emptyset$, *then no parameters are weakly identified, so* $\beta_{(i),K^-,n} = \beta_{(i),n}$ *and*

$$n\left(Q_{(i),n}(\hat{\theta}_{(i)}) - Q_{(i),n}(\theta_{(i),n})\right) \xrightarrow{d} \chi_{(i),\theta}(\gamma_0) \tag{5}$$

Max Tests

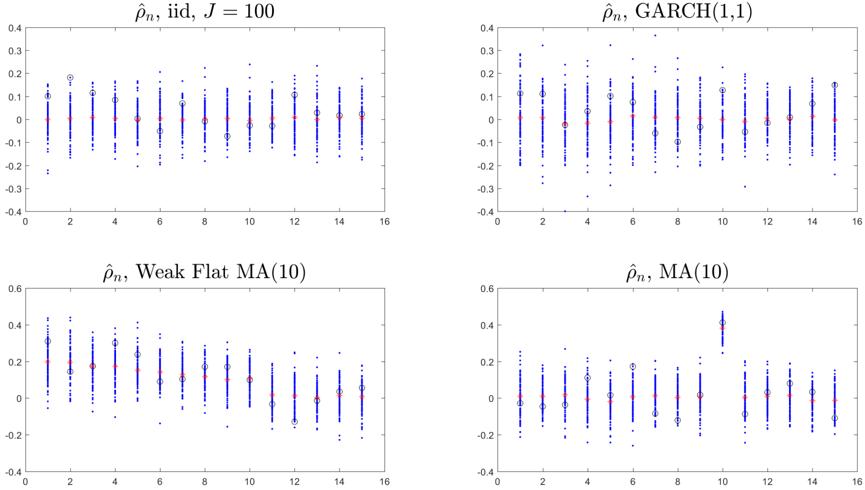
$$\hat{\mathcal{T}}_n = \max_{1 \leq i \leq k_n} |\sqrt{n}\hat{\lambda}_{i,n}|$$

- do not require inversion of a large covariance matrix
- Utilize the most informative of a sequence of estimators
- trade-off: ignore information from everything that is not the maximum (Hansen, 2005)

Example Example 2

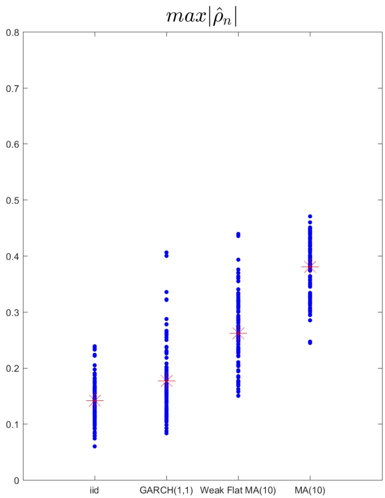
Return to Intro Return to Caveats

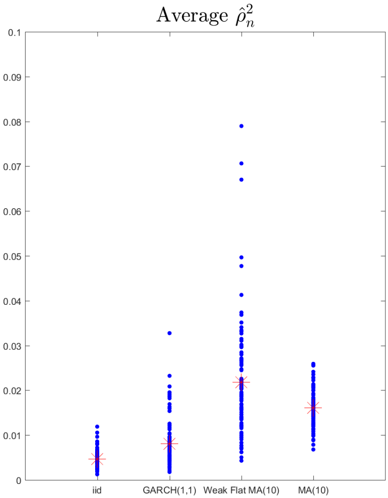
Max Tests - Correlation Examples



▸ Combining Correlations ▸ Go back

Max Tests - Max Correlation Examples





$$H_0 : \rho_n = 0 \quad \forall n$$

Go back

Appendix - Test Statistic

- Take away: Distributions are different $\Rightarrow$ possibility of distorted inference when ignoring possibility of identification failure
  - Under Strong Id, the limit is standard
  - Under Weak Id, $\hat{\pi}_n$ is not consistent, and the limiting distribution is complicated!
- We will bootstrap these distributions...

Bootstapping a Max Statistic?

Return to CV slide

Appendix - Inference Procedure Step 2b

1. The bootstrapped quantities with weakly identified parameters:

$$\hat{\mathcal{G}}^{bs}_{(i)}(\pi_{(i),l_K}) = \frac{1}{\sqrt{n}}\sum_{t=1}^{n} z_t\Big\{m_{(i),t}(\hat{\psi}^{0}_{(i),K^-,n}(\pi_{(i),l_K}), \pi_{(i),l_K}) - \frac{1}{n}\sum_{t=1}^{n} m_{(i),t}(\hat{\psi}$$

Use this to form the quantities

$$\hat{\tau}^{bs}_{(i)}(\pi_{(i),l_K};\gamma_0, b) = \left[\hat{H}_{(i),K}(\pi_{(i),l_K})\right]^{-1}\Big(\hat{\mathcal{K}}_{(i),K}(\pi_{(i),l_K};\gamma_0)b_{(i),l_K} + \hat{\mathcal{G}}^{bs}_{(i)}(\pi_{(i}$$

$$\hat{\chi}^{bs}_{(i)}(\pi_{(i),l_K};\gamma_0, b) = -\frac{1}{2}\hat{\tau}^{bs}_{(i)}(\pi_{(i),l_K};\gamma_0, b)'\left[\hat{H}_{(i),K}(\pi_{(i),l_K})\right]\hat{\tau}^{bs}_{(i)}(\pi_{(i),l_K};\gamma_0, b$$

Next, compute

$$\pi^{*,bs}_{(i),l_K}(\gamma_0, b) = \underset{\pi_{(i),l_K}\in\Pi_{(i),l_K}}{\operatorname{argmin}} \hat{\chi}^{bs}_{(i)}(\pi_{(i),l_K};\gamma_0, b).$$

Inference Procedure

Appendix - Bootstrap Overview

Weak Identification:

1. Simulate a random draw, $\pi^*_{(bs)}(b, \pi_0)$, from the distribution $\pi^*(b, \pi_0)$ using the z_t
2. Use $\pi^*_{(bs)}(b, \pi_0)$ to construct the components of our test statistic under weak identification, which are functions of π.
3. Use the draws z_t to construct the wild bootstrap version of the test statistic.
4. deal with nuisance parameters b and π_0

Go to the Bootstrap Algorithm for Weak Id

Strong Identification:

1. Construct the components of our test statistic using $\hat{\theta}_n$.
2. Use the draws z_t to construct the wild bootstrap version of the test statistic.

Go to the Bootstrap Algorithm for Strong Id

Appendix - Critical Value Computation Overview

1. Repeat the above procedures M times for each identification category.
2. Order the resulting test statistics within each category.
3. α-level critical values are the statistics in $[(1-\alpha)\cdot M]$th ordered positions
4. The critical value under weak identification depends on nuisance parameters. Sup over these nuisance parameters.

Go To Critical Value Computation Algorithm

Return to CV slide

Appendix - Putting the Critical Values Together

Robust Critical Values:

- Least Favorable (LF)
  - always take the larger of the critical values
  - $c_{1-\alpha}^{(LF)} = \max\{c_{1-\alpha}^{(w)}, c_{1-\alpha}^{(s)}\}$

- Identification-Category Selection (ICS)
  - data driven pre-test for the id category
  - Step 1: Use data to determine if $b = \lim_n \sqrt{n}\beta_n$ is finite
  - Step 2:
    - if we believe b is finite, use LF cv
    - otherwise, use (semi-) Strong identification cv

Go To ICS Details

Decision Rule: Reject the null hypothesis when $\hat{T}_n > c_{1-\alpha}^{(\cdot)}$.

Return to Inference

ICS Critical Values

The statistic used for category selection is

$$\mathcal{A}_n = (n\hat{\beta}_n'\hat{\Sigma}_{\beta\beta,n}^{-1}\hat{\beta}_n/d_\beta)^{1/2}$$

where $\hat{\Sigma}_{\beta\beta,n}$ is the upper left $d_\beta \times d_\beta$ block of $\hat{\Sigma}_n = \hat{J}_n^{-1}\hat{V}_n\hat{J}_n^{-1}$, the estimator of the strong identification covariance matrix $\Sigma(\gamma_0) = J^{-1}VJ^{-1}$. Let $\{\kappa_n : \ n \geq 1\}$ be a sequence of constants s.t.

$$\kappa_n \to \infty \qquad \text{and} \qquad \kappa_n/n^{1/2} \to 0$$

The ICS critical value is $c_{1-\alpha}^{(ICS)} = \begin{cases} c_{1-\alpha}^{(LF)} & \text{if } \mathcal{A}_n \leq \kappa_n \\ c_{1-\alpha}^{(s)} & \text{if } \mathcal{A}_n > \kappa_n \end{cases}$

Return to CV Overview

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Null
$n = 200$, $J = 10000$, $\alpha = 0.05$

	$k_{\lambda,n} = 1$						$k_{\lambda,n} = 20$					
b_1	0	1	2	5	10	14	0	1	2	5	10	14
Wald Test Standard	0.11	0.12	0.12	0.13	0.12	0.12	0.83	0.84	0.83	0.83	0.84	0.84
Max Test Standard	0.10	0.10	0.10	0.10	0.10	0.10	0.12	0.11	0.11	0.11	0.11	0.11
Max t-Test Standard	0.11	0.12	0.11	0.11	0.11	0.11	0.19	0.19	0.19	0.19	0.20	0.20
Wald Test BS1	0.06	0.06	0.06	0.06	0.06	0.06	0.68	0.68	0.67	0.68	0.69	0.69
Max Test BS1	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03
Max t-Test BS1	0.06	0.06	0.06	0.06	0.06	0.06	0.13	0.12	0.13	0.13	0.13	0.13
Wald Test BS2	0.06	0.06	0.06	0.06	0.06	0.06	0.25	0.26	0.26	0.26	0.25	0.25
Max Test BS2	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04	0.04
Max t-Test BS2	0.06	0.06	0.06	0.06	0.06	0.06	0.08	0.08	0.08	0.08	0.08	0.09
Wald Test Taylor	0.09	0.09	0.09	0.09	0.09	0.09	0.52	0.52	0.52	0.52	0.52	0.52
Max Test Taylor	0.60	0.60	0.60	0.60	0.60	0.60	0.99	0.99	0.99	0.99	0.99	0.99
Max t-Test Taylor	0.09	0.09	0.09	0.09	0.09	0.09	0.16	0.16	0.16	0.16	0.16	0.16

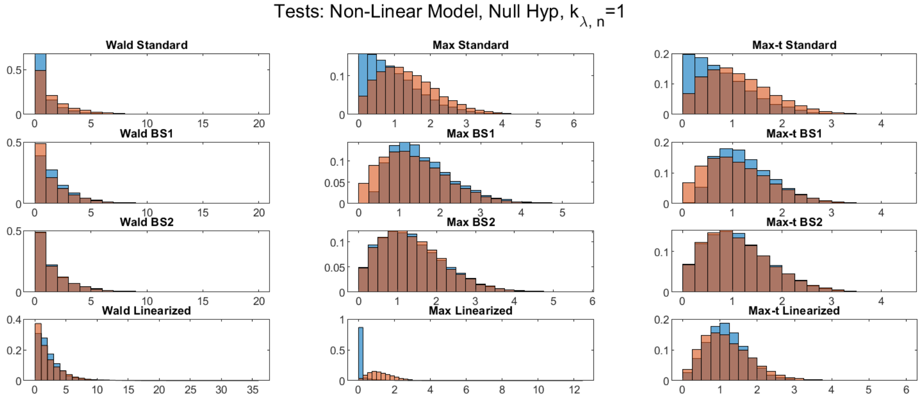
Back to table

Simulation Results

Rejection Frequencies, Experiment: 1, DGP: 2, Hyp: Local Alternative
$n = 200$, $J = 10000$, $\alpha = 0.05$

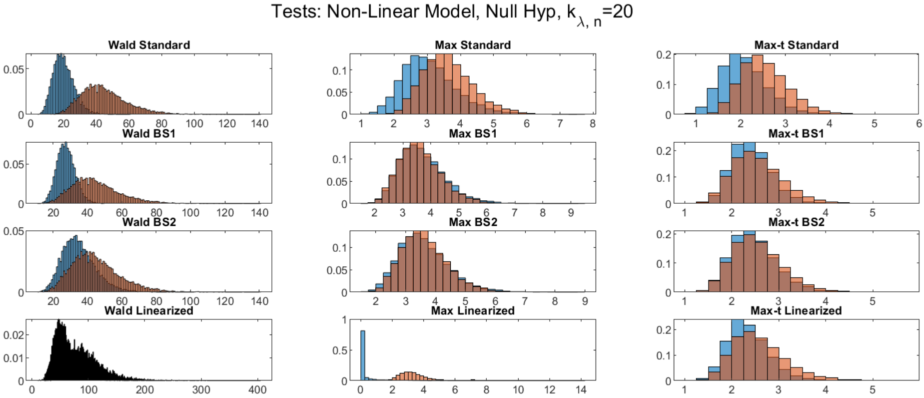
	$k_{\lambda,n} = 1$						$k_{\lambda,n} = 20$					
b_1	0	1	2	5	10	14	0	1	2	5	10	14
Wald Test Standard	0.21	0.22	0.20	0.22	0.21	0.21	0.91	0.91	0.90	0.91	0.91	0.91
Max Test Standard	0.19	0.20	0.20	0.20	0.19	0.19	0.25	0.25	0.25	0.26	0.25	0.25
Max t-Test Standard	0.20	0.20	0.21	0.21	0.20	0.21	0.50	0.50	0.51	0.50	0.50	0.50
Wald Test BS1	0.12	0.12	0.12	0.12	0.12	0.12	0.81	0.80	0.79	0.80	0.80	0.80
Max Test BS1	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10
Max t-Test BS1	0.12	0.12	0.12	0.12	0.12	0.12	0.42	0.42	0.42	0.43	0.43	0.43
Wald Test BS2	0.13	0.13	0.13	0.13	0.13	0.13	0.39	0.39	0.39	0.39	0.39	0.39
Max Test BS2	0.11	0.11	0.12	0.12	0.12	0.11	0.11	0.10	0.11	0.11	0.11	0.11
Max t-Test BS2	0.13	0.13	0.13	0.13	0.13	0.13	0.34	0.33	0.33	0.33	0.33	0.33
Wald Test Taylor	0.15	0.15	0.15	0.15	0.15	0.15	0.61	0.61	0.61	0.61	0.61	0.61
Max Test Taylor	0.71	0.71	0.71	0.71	0.71	0.71	1.00	1.00	1.00	1.00	1.00	1.00
Max t-Test Taylor	0.15	0.15	0.15	0.15	0.15	0.15	0.37	0.37	0.37	0.37	0.37	0.37

Simulation Results



Return to table

Simulation Results



Return to table