Demand risk management in clustered vehicle routing

06-815 Course Project - Proposal

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Vehicle routing problems (VRPs) and their extensions constitute some of the most studied problems in combinatorial optimization and operations research. In general terms, the VRP aims to define routes of vehicles to fulfill the service demands of customers placed at known locations. This group of problems has extensive scope of applications, such as transportation, network optimization, supply chain and distribution planning, etc. The capacitated vehicle routing problem (CVRP), first described by Dantzig and Ramser (1959), is concerned with the minimum cost delivery of a single product from a depot to customers through a specified number of capacity-constrained vehicles.

The Generalized Vehicle Routing Problem (GVRP) is a generalization of the CVRP in which nodes (customers) are partitioned into mutually exclusive and exhaustive node sets, called clusters (Pop et al., 2012). We are interested in finding optimal routed from the given depot to the number of predefined clusters which include exactly one node from each cluster.

Motivation

The classical GVRP is a deterministic optimization problem because it assumes that the customer demands are known with certainty ahead of planning the route. However, there are many real life scenarios where this is not the case. Consider the distribution of goods by sea to a number of customers situated in an archipelago as in Philippines, New Zealand, Indonesia, Italy, etc. A number of potential harbors is selected for each island and a fleet of ships is required to visit one harbor for every island. Goods from the ships are unloaded based on a demand which is realized when the ship visits reaches the island. As such, a decision has to be made when at the depot in regards to the amount of goods to be loaded at the beginning of the route. Since the cost of the route could be directly influenced by the amount of loaded goods, it is desirable to ensure that the minimum amount is loaded while satisfying all customer demands.

Another variant of the CVRP is the clustered capacitated vehicle routing problem (CCVRP). The customers are organized into clusters, similar to the GVRP. However, instead of visiting exactly one customer from each cluster, the vehicle must satisfy demands by visiting all the customers in a cluster consecutively. A potential application of this problem is the routing of fuel trucks to gas stations. Demand uncertainty in this case arises due to variability in

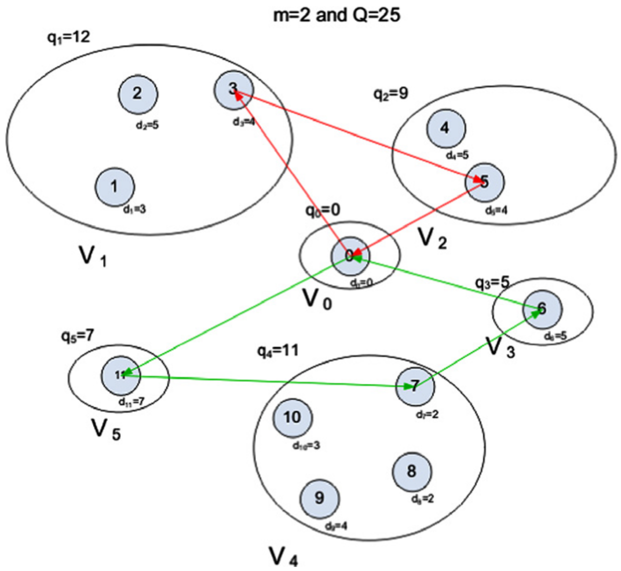

Figure 1: An example of a GVRP and a feasible solution (Pop et al., 2012)

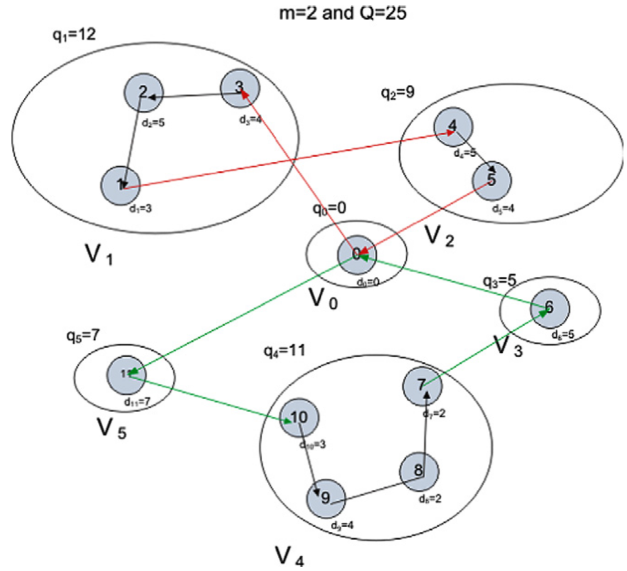

Figure 2: An example of a CCVRP and a feasible solution (Pop et al., 2012)

the amount of unsold fuel available at the gas station from the previous delivery. It is of interest to minimize the amount of fuel transported by the fuel truck for safety and cost reasons. Mail pickup and delivery in a multi-storey or multi-pavilion office complex is a similar application of the CCVRP (Laporte and Palekar, 2002). Examples of the GVRP and CCVRP are shown in Fig. 1 and 2 respectively.

Current state of the art

Mathematical programming approaches to these type of problems have so far focused on deterministic scenarios i.e. when customer demand and costs are known in advance. Both exact (Kara and Bektaş, 2003; Pop et al., 2012) and metaheuristic approaches (Noon and Bean, 1991; Ghiani and Improta, 2000; Pop et al., 2009) have not considered uncertainty scenarios which arise in real life situations. The objective of future work should therefore be the development of a comprehensive framework to handle uncertainty and risk in various parameters involved in capacitated vehicle routing with customers grouped into clusters.

Mitigating risk involves designing feasible routes that ensure that all possible realizations of uncertain parameters are accounted for, while minimizing the cost or other objective function. Robust optimization (RO) assumes that the uncertain data lies in a so-called “Uncertainty set”. Constraint violation is not allowed for any realization of data in the uncertainty set. RO results is a static, “here-and-now” solution that is often overly conservative. Adjustable RO (Gorissen et al., 2015) on the other hand results in a more flexible solution policy by adjusting future decisions based on the actual realizations of the uncertain parameters as they occur.

Hence the objective of the proposed work is the development of a comprehensive framework for both static and adjustable robust optimization for the mitigation of demand and cost risk in generalizations of CVRPs defined above. The definition of the uncertainty set is heavily dependent on past data. Past customer demand data can be used to describe either discrete or continuous uncertainty sets. MILP formulations previously described in literature (Pop et al., 2012; Kara and Bektaş, 2003) can be reformulated to ensure tractability with the RO frameworks.

Preliminary results

Preliminary studies conducted to compare exact formulations for the GVRP indicate that a modern flow-based formulation described by (Pop et al., 2012) exhibits sufficient LP relaxation tightness to be able to solve deterministic problems of up to 70 customers to optimality with sufficient efficiency. Other formulations such as the node-based one described by the same authors and one by (Kara and Bektaş, 2003) exhibit significantly higher difficulty to solve to optimality.

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