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Theory and Methodology

An efficient transformation of the generalized vehicle routing problem

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Abstract

The Generalized Vehicle Routing Problem (GVRP) is the problem of designing optimal delivery or collection routes, subject to capacity restrictions, from a given depot to a number of predefined, mutually exclusive and exhaustive clusters. In this paper we describe an efficient transformation of the GVRP into a Capacitated Arc Routing Problem (CARP) for which an exact algorithm and several approximate procedures are reported in literature. It constitutes the only known approach for solving the GVRP. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The well-known Vehicle Routing Problem (VRP) can be defined as the problem of determining optimal delivery or collection routes from a given depot to some geographically dispersed customers subject to operating restrictions.

The Generalized Vehicle Routing Problem (GVRP) is an extension of the VRP which can be formally defined as follows. Let $G(V,A)$ be a directed graph where V is the set of vertices and $A = \{(v_i, v_j): v_i, v_j \in V, v_i \neq v_j\}$ the set of arcs. A nonnegative cost c_{ij} is associated with each arc (v_i, v_j). If $c_{ij} = c_{ji}$ for all $v_i, v_j \in V$, the problem is described as symmetric and is usually defined on an undirected graph where each pair of opposite arcs (v_i, v_j), (v_j, v_i) is substituted by an edge (v_i, v_j). In what follows, arc costs are supposed to represent the least-cost paths between vertices (or otherwise to satisfy the triangle inequality). Vertex set V is partitioned into $m+1$ nonempty subsets (or *clusters*) $C_0, C_1, \ldots, C_m$. Cluster C_0 has only one vertex v_0, which represents the depot, while cluster C_h $(h = 1, \ldots, m)$ represents $r(h)$ possible locations of a customer having a nonnegative demand d_h. In addition, a fleet of homogeneous vehicles, having capacity Q ($\geqslant d_h$, $h = 1, \ldots, m$) is available at the depot. The GVRP is to find a set of shortest routes starting and ending at the depot, visiting each cluster C_h $(h = 1, \ldots, m)$ exactly once such that the

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sum of the demands of any route does not exceed Q.

The GVRP is a location-routing problem (Laporte, 1988) as it combines two kinds of simultaneous decisions: node selection and node sequencing.

Several real-world situations can be modeled as a GVRP. The post-box collection problem described in Laporte et al. (1989) becomes an asymmetric GVRP if more than one vehicle is required. Furthermore, the GVRP is able to model the distribution of goods by sea to a number of customers situated in an archipelago as in Venice (Italy), Greece and Croatia. In this application, a number of potential harbours is selected for every island and a fleet of ships is required to visit exactly one harbour for every island.

If $\sum_{h=1,\ldots,m} d_h \leqslant Q$, the GVRP coincides with the Generalized Traveling Salesman Problem (GTSP) whose aim is to determine a least-cost Hamiltonian circuit through the clusters (Noon and Bean, 1991, 1993; Fischetti et al., 1996, 1997; Laporte et al., 1996).

In this paper we describe an efficient transformation of the GVRP into a Capacitated Arc Routing Problem (CARP) for which an exact algorithm (Hirabayashi et al., 1992) and several approximate procedures (see Assad and Golden, 1995; Eiselt et al., 1995a, b for a review) are reported in literature. It constitutes the only known approach for solving the GVRP.

For the sake of simplicity, the transformation of the GVRP into a CARP will be described for the symmetric case.

2. The transformation

Let $G(V, E)$ be an undirected graph where V is a set of vertices and E is a set of edges. A depot, at which a given fleet of identical vehicles is based, is located at a vertex $v_0 \in V$. A subset R of edges are said to be *required*, i.e. they are to be serviced by a vehicle. A nonnegative cost c_e is associated with each edge $e \in E$. Furthermore, a positive demand d_e is attached to each edge $e \in R$. The CARP is to find a set of shortest postman tours such that:

1. each route starts and ends at the depot;
2. each required edge is traversed at least once and is serviced by exactly one vehicle;
3. the total demand serviced by any vehicle does not exceed vehicle capacity Q.

In a CARP feasible solution, every vertex is entered by an even number of *copies* of edges (Assad and Golden, 1995; Eiselt et al., 1995a). In what follows, this parity condition is formulated, for the sake of simplicity, using the word "edge" instead of "copy of an edge".

Let G_R be the subgraph induced by edges in R, V_h $(h = 1, \ldots, m)$ the set of nodes of the hth connected component of G_R and V_R the set of nodes v_i such that an edge (v_i, v_j) exists in R, i.e.

$$V_R = \bigcup_{h \in \{1,\ldots,m\}} V_h.$$

Transformation

Step 1. A loop of "very expensive" required edges is created with the vertices $v^{(h)_1}, \ldots, v^{(h)_{r(h)}}$ of cluster C_h $(h = 1, \ldots, m)$. To this end, an edge $(v_i^{(h)}, v_{i \bmod (r(h))+1}^{(h)})$, having cost equal to a large positive number M, is inserted into R $(i = 1, \ldots, r(h))$. In particular, if $r(h) = 1$, a required loop $x(v^{(h)_1}, v^{(h)_1})$ is introduced. These loops create m connected components of required edges. The remaining edges are non-required.

Step 2. Non-required intra-cluster edges are removed.

Step 3. The cost of each inter-cluster edge is increased by $M/2$, if an endpoint coincides with the depot, or by M otherwise.

Step 4. Strictly positive demands having sum equal to d_h are assigned to the edges of the loop corresponding to C_h $(h = 1, \ldots, m)$.

As most CARP algorithms require integer demands, vehicle capacity and cluster demands are to be scaled before step 5 is performed. To this purpose, if GVRP demands d_h are integer, both d_h $(h = 1, \ldots, m)$ and Q are to be multiplied by $\max_{h=1,\ldots,m} \lceil r(h)/d_h \rceil$.

The following proposition states that solving the CARP on the transformed graph provides the optimal GVRP solution (Fig. 1).

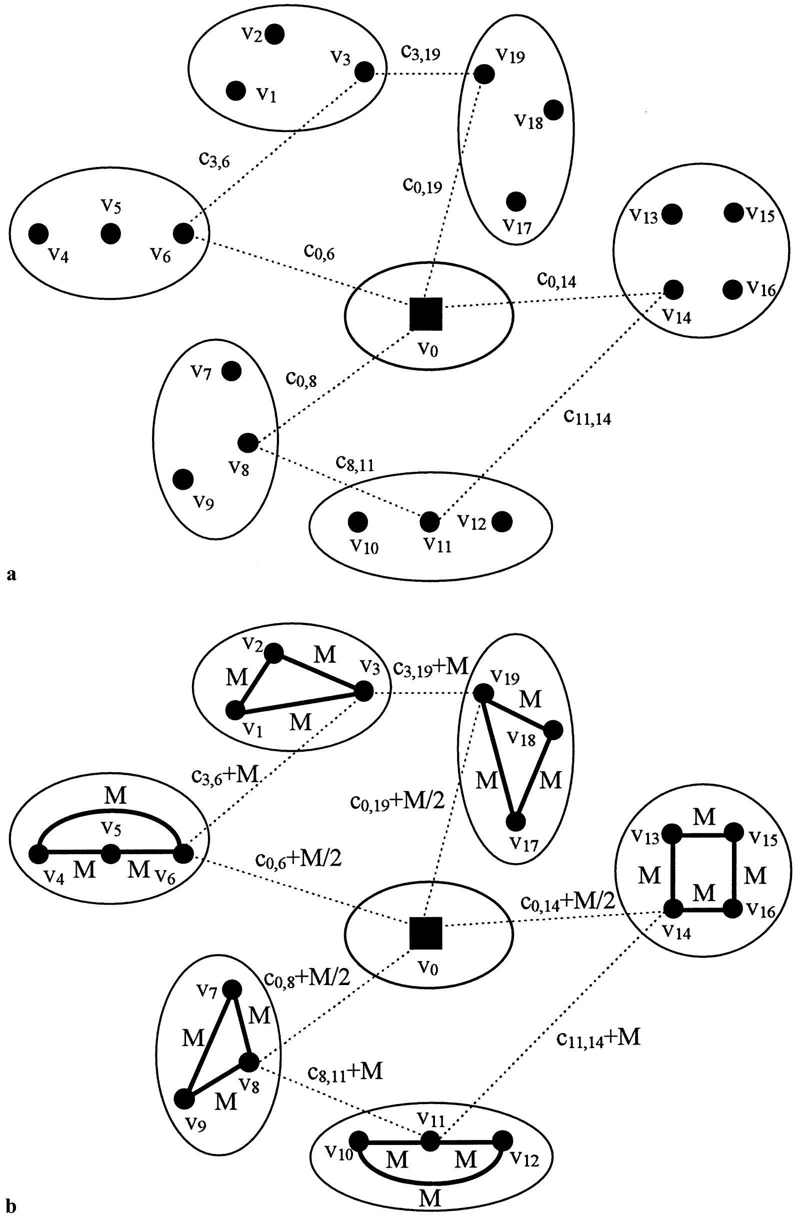

Fig. 1. (a) A sample GVRP solution. (b) The corresponding CARP solution on the transformed graph.

Proposition 1. *The CARP optimal solution on the transformed graph corresponds to the GVRP optimal solution on the original graph.*

Proof. In the transformed graph the connected components of G_R (which correspond to the clusters of the original graph) are Eulerian and $V = V_R \cup \{v_0\}$. As a consequence, in a CARP solution every vertex is entered by an even number of non-required edges. Because of large positive numbers M, a CARP *optimal* solution traverses each required edge exactly once and each connected

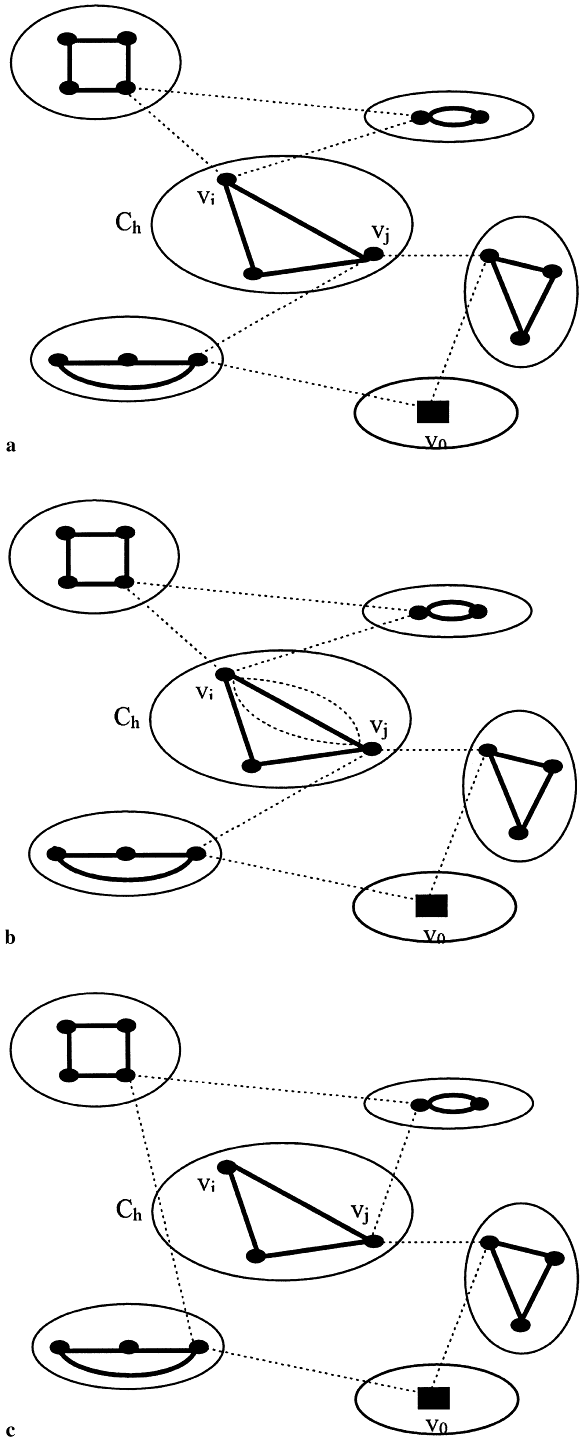

Fig. 2. The optimal CARP solution corresponds to the GVRP solution if $M = 2 \cdot c_{\mathrm{MAX}}$.

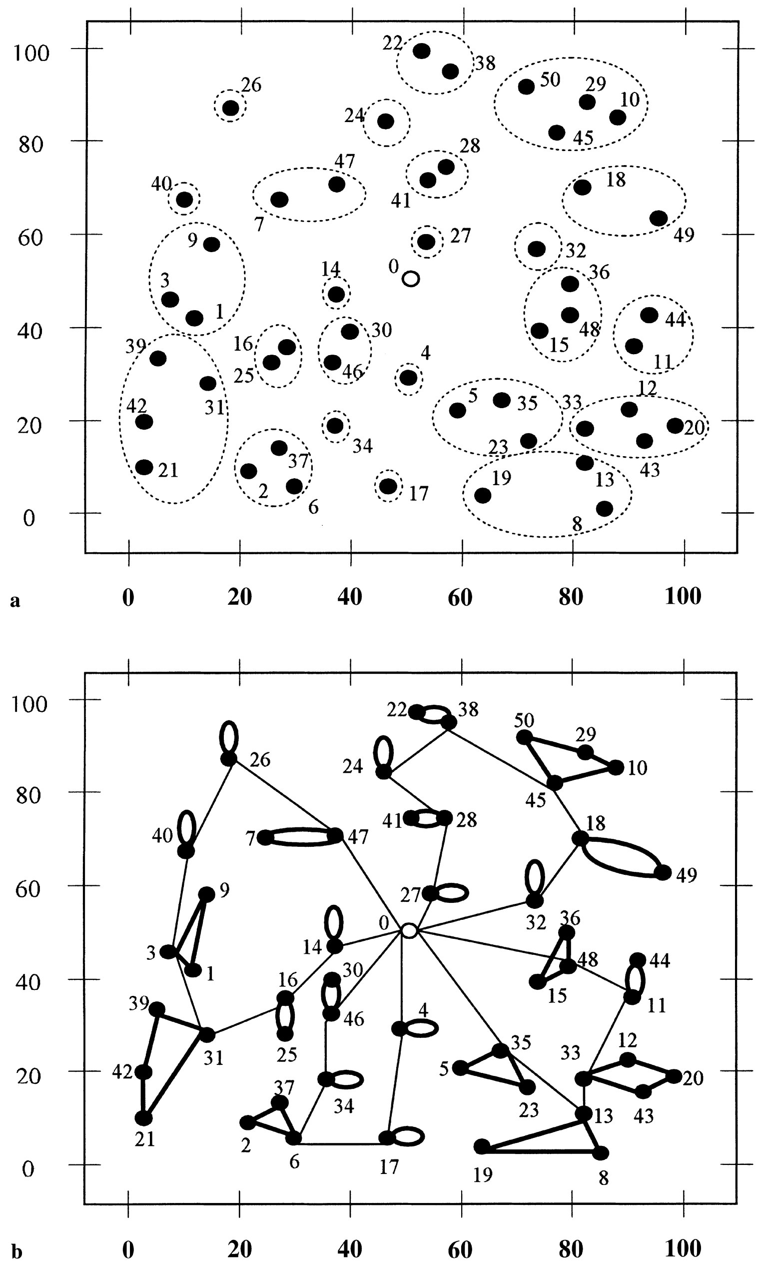

Fig. 3. (a) Sample GVRP instance. (b) The CARP solution on the transformed graph.

component of G_R is entered by exactly two non-required edges. Consequently such solution corresponds to a feasible GVRP solution and has cost equal to the cost of the corresponding GVRP solution plus the constant $M \cdot (|V| + m - 1)$. Vice versa, a CARP feasible solution with no feasible counterpart has more than $|V| + m - 1$ edges with large costs and, hence, cannot be optimal.

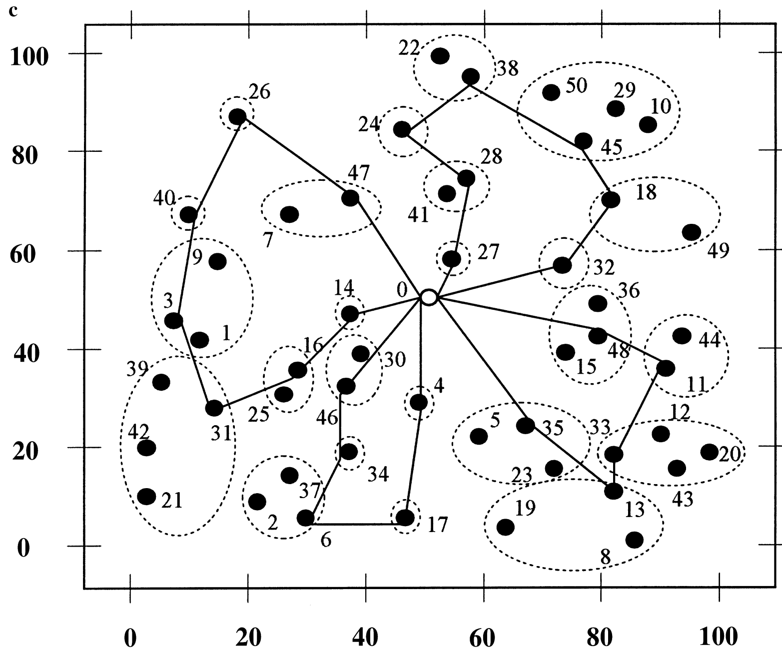

Fig. 3. (c) The corresponding GVRP solution on the original graph.

Constant M has to be chosen as large as exactly two edges enter each cluster in the CARP solution. The following proposition allows us to set M.

Proposition 2. *If M is set equal to $2 \cdot c_{\mathrm{MAX}}$, where c_{MAX} is the largest intra-cluster edge cost in the GVRP graph, the optimal CARP solution corresponds to the GVRP solution.*

Proof. In a CARP solution each component is entered by an even number $2k$ of edges, $k \in \{1, 2, \ldots\}$. Let C_h be a component such that $k \geqslant 2$. For the sake of simplicity, suppose $k = 2$ (Fig. 2(a)). Let v_i and v_j be the vertices of C_h entered by the inter-cluster edges. Modify this solution adding two copies of edge (v_i, v_j) (Fig. 2(b)). Then, use some shortcuts to get a solution having exactly two edges incident to C_h. Let z_s and z_f be the cost of the starting and the final solutions, respectively. The cost difference $z_f - z_s$ is equal to $2 \cdot c_{ij}$ minus the saving obtained using the shortcuts. As the saving is equal at least to $M/2 + M/2 = M$, the following inequalities hold:

$$z_f - z_s \leqslant 2 \cdot c_{ij} - M \leqslant 2 \cdot c_{\mathrm{MAX}} - M.$$

Hence, if M is set equal to $2 \cdot c_{\mathrm{MAX}}$, $z_f \leqslant z_s$ and the starting solution cannot be optimal.

3. A numerical example

We illustrate the above procedure on a numerical example (Fig. 3(a)). A GVRP instance is obtained by taking VRP test problem 7 from Araque et al. (1994). The node clustering is as follows:

$C_0 = \{0\}$, $C_1 = \{22, 38\}$,
$C_2 = \{26\}$, $C_3 = \{24\}$,
$C_4 = \{10, 29, 45, 50\}$, $C_5 = \{40\}$,
$C_6 = \{7, 47\}$, $C_7 = \{28, 41\}$,
$C_8 = \{18, 49\}$, $C_9 = \{1, 3, 9\}$,
$C_{10} = \{14\}$, $C_{11} = \{32\}$,
$C_{12} = \{21, 42, 39, 31\}$, $C_{13} = \{16, 25\}$,
$C_{14} = \{30, 46\}$, $C_{15} = \{4\}$,
$C_{16} = \{15, 36, 48\}$, $C_{17} = \{11, 44\}$,
$C_{18} = \{2, 37\}$, $C_{19} = \{34\}$,
$C_{20} = \{5, 23, 35\}$, $C_{21} = \{12, 20, 33, 43\}$,
$C_{22} = \{8, 13, 19\}$

Cluster demands d_h are obtained summing the demands of vertices $v_1^{(h)}, \dots, v_{r(h)}^{(h)}$ reported in Araque et al. (1994). To solve this GVRP, we apply CARPET heuristic (Hertz et al., 1996) to the auxiliary CARP problem (Fig. 3(b)). The solution found is shown in Fig. 3(c).

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