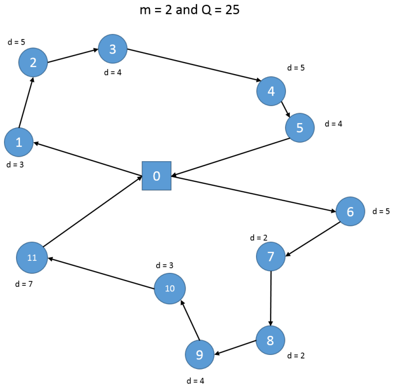
Node-based and flow-based formulations for the Generalized Vehicle Routing Problem

Jay Shah

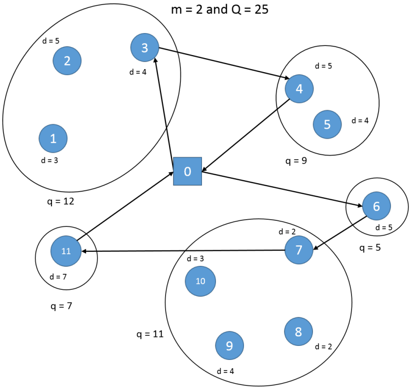
Carnegie Mellon University
06-815 Project

December 4, 2017

Problem definition



Capacitated VRP



Generalized VRP

General formulation

Variables

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is included in the tour of a vehicle,} \\ 0, & \text{otherwise} \end{cases}$$

$$w_{pr} = \begin{cases} 1 & \text{if there is a path from cluster } V_p \text{ to cluster } V_r, \\ 0, & \text{otherwise} \end{cases}$$

Node-based

u_p: load of a vehicle just after leaving the cluster V_p, $p \neq 0$, $p \in K$

Flow-based

y_{pr}: amount of goods delivered by vehicle just after leaving the cluster V_p if the vehicle goes from the cluster V_p to the cluster V_r

MILP formulation [1]

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

s.t. Cluster degree constraints

Cluster connectivity constraints

Capacity bounding constraints

Subtour elimination constraints

$$x_{i,j} \in \{0,1\}, \forall (i,j) \in A$$

[1] Pop, P. C., Kara, I., and Marc, A. H. New mathematical models of the generalized vehicle routing problem and extensions.
Applied Mathematical Modelling, 36(1):97 – 107 (2012).

General constraints

Constraints that are common between the two formulations:

- Cluster degree constraints:
  - exactly ONE outgoing arc to any other node belonging to other clusters
  - exactly ONE incoming arc to a cluster from any other node belonging to other clusters
  - m leaving arcs from and at most m entering arcs to the depot ($= m$ or $\leq m$)
- Cluster connectivity constraints:
  - entering and leaving nodes should be the same for each cluster
  - relation between w_{pr} and x_{ij}

Node-based formulation

u_p: load of a vehicle just after leaving the cluster V_p, $p \neq 0$, $p \in K$

- Capacity bounding constraints

$$u_p - \sum_{r \in K, r \neq p} q_r w_{rp} \geq q_p, \;\; p \neq 0, p \in K$$

$$u_p + (Q - q_p) w_{0p} \leq Q, \;\; p \neq 0, p \in K$$

$$u_p + \sum_{r \in K, r \neq p} q_r w_{pr} \leq Q, \;\; p \neq 0, p \in K$$

- Subtour elimination constraints

$$u_p - u_r + Q w_{pr} + (Q - q_p - q_r) w_{rp} \leq Q - q_r, \;\; p \neq r \neq 0; p, r \in K$$

Flow-based formulation

y_{pr}: amount of goods delivered by vehicle just after leaving the cluster V_p if the vehicle goes from the cluster V_p to the cluster V_r

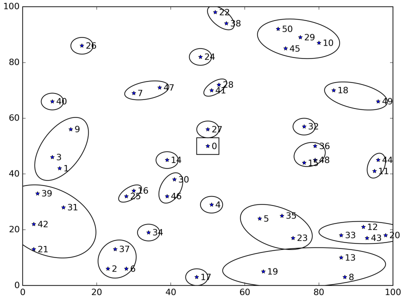
- Capacity bounding constraints

$$y_{rp} \leq (Q - q_p)w_{rp}, \; r \neq p; r, p \in K$$
$$y_{rp} \geq q_r w_{rp}, \; r \neq p; r, p \in K$$
$$\sum_{p=1}^{k} y_{p0} = \sum_{p=1}^{k} q_p$$

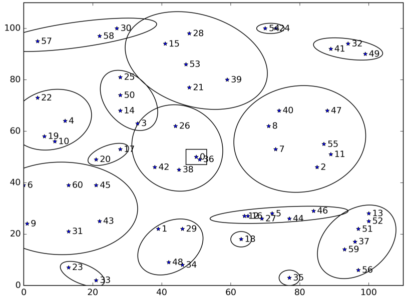
- Subtour elimination constraints

$$\sum_{p=0}^{k} y_{rp} - \sum_{p=0}^{k} y_{pr} = q_r, \; r \neq 0; r, p \in K$$

Test examples



GVRP1 [2]



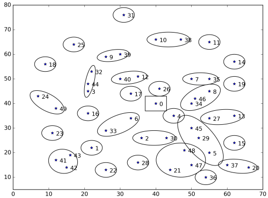
GVRP2 [3]

[2] Ghiani, G. and Improta, G. An effcient transformation of the generalized vehicle routing problem. *European Journal of Operational Research*, 122(1):11 – 17 (2000)

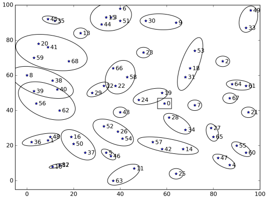
[3] Kara, I. and Bektaș, T. New mathematical models of the generalized vehicle routing problem and extensions. *Proc. of the 5th EURO/INFORMS Joint International Meeting* (2003).

New test examples

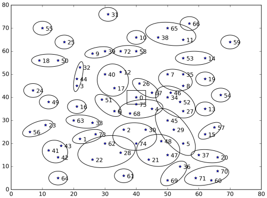
- New test examples generated by clustering CVRP datasets from CVRPLIB [4]
- Simple clustering algo: K-means
- Number of clusters such that max 4 customers in a cluster
- Number of vehicles and capacities not maintained



P-n50-k10



A-n69-k9

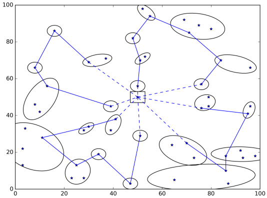


P-n76-k5

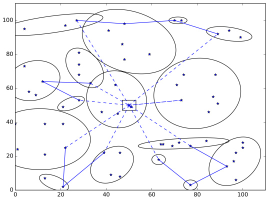
[4] http://vrp.atd-lab.inf.puc-rio.br/

Implementation

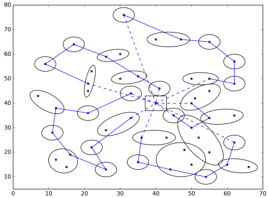
- Node-based and flow-based formulations implemented using CPLEX/C++ API
- Windows PC with an Intel i7 processor (2.90 GHz)
- Optimal solutions found for `GVRP1`, `GVRP2`, `P-n50-k10`, `A-n69-k9` using flow-based formulation
- `P-n76-k5` solved to 2.55% relative gap



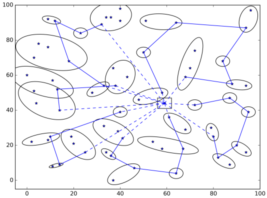
GVRP1



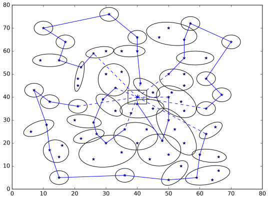
GVRP2



P-n50-k10



A-n69-k9



P-n76-k5

Root node relaxation

Table: Deviation between root node relaxation and optimal values

Instance	Best known objective	Formulation	Root node relaxation	% Deviation
GVRP1	527.813	flow	498.594	5.54
		node	449.358	14.86
GVRP2	557.564	flow	533.9	4.24
		node	545.365	2.19
P-n50-k10	417.742	flow	384.597	7.93
		node	338.098	19.07
A-n69-k9	756.131	flow	692.212	8.45
		node	560.44	25.88
P-n76-k5	500.955	flow	450.746	11.27
		node	399.788	21.30

Model comparision

Table: Model comparision for GVRP instances (time limit = 1 hour)

Instance	Formulation	Status	Objective value	Relative gap (%)	Number of constraints
GVRP1	flow	Optimal	527.813	0.01	2108
	node	Feasible	527.813	5.42	1482
GVRP2	flow	Optimal	557.564	0.00	1242
	node	Optimal	557.564	0.00	952
P-n50-k10	flow	Feasible	417.742	0.75	3094
	node	Feasible	417.742	14.69	2132
A-n69-k9	flow	Feasible	756.131	1.25	3385
	node	Feasible	763.045	18.59	2360
P-n76-k5	flow	Feasible	508.194	6.04	4896
	node	Feasible	595.149	31.20	3374

Outcomes

- New test cases represent more realistic benchmark instances for GVRP because of non-unit customer demands
- Zero-gap solutions found for two out of three new test cases
- Flow-based formulation outperforms node-based
- Formulations scale with number of customers rather than number of clusters

Thank you

Key features & assumptions

- The total demand of each cluster can be satisfied via any of its nodes
- The demand of each cluster q_k is the sum of demands of each customer in cluster k
- All vehicles are identical i.e. each vehicle has the same capacity Q
- NP-hard because it includes the generalized traveling salesman problem as a special case when $\mathrm{m} = 1$ and $\mathrm{Q} = \infty$

Table: Details of GVRPs

Instance	Customers	Clusters	Vehicles	Capacity	Total demand
GVRP1	50	24	4	15	50
GVRP2	60	16	6	15	60
P-n50-k10	49	30	5	200	951
A-n69-k9	68	31	6	150	845
P-n76-k5	75	38	5	280	1364

General formulation

Sets

- $V = \{0, 1, 2, ..., n\}$ set of customers
- $K = \{0, 1, 2, ..., k\}$ set of clusters
- $A = \{(i, j) | \ i, j \in V, i \neq j\}$ set of arcs (routes).

Parameters

- c_{ij} is the cost of traveling from node i to node j
- d_i is the demand of customer i
- $q_r = \sum_{i \in V_r} d_i, \ r \in K$ is the demand of the cluster V_r
- $M = \{1, \ldots, m\}$ set of identical vehicles
- Q is the capacity of each vehicle