Econometrics II
Tutorial No. 1

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Outline

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Summary

Key terms

- **Binary Response Model:** A model for a binary (or dummy, i.e. with two possible outcomes 0 and 1) dependent variable.
- **Response Probability:** In a binary response model, the probability that the dependent variable takes on the value one, conditional on explanatory variables.
- **Linear Probability Model:** The multiple linear regression model with a binary dependent variable, where the response probability is linear in the parameters.

  [bad idea! the probability can be estimated outside the $[0, 1]$ interval]

Key terms - cont'd

- **Logit Model:** A model for binary response where the response probability is the logit function evaluated at a linear function of the explanatory variables.

$$G(z) = \frac{1}{1 - \exp(-z)} = \frac{\exp(z)}{1 + \exp(z)}.$$

- **Probit Model:** A model for binary responses where the response probability is the standard normal cumulative distribution function (CDF) evaluated at a linear function of the explanatory variables.

$$G(z) = \Phi(z) = \int_{-\infty}^{z} \phi(v)dv = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right) dv.$$

- **Latent Variable Model:** A model where the observed dependent variable is assumed to be a function of an underlying latent, or unobserved, variable. [interpretation of binary logit/probit model]

Key terms - cont'd

- **Partial Effect at the Average (PEA):** In models with nonconstant partial effects, the partial effect evaluated at the average values of the explanatory variables.

  [Substitute averages $\bar{x}_1, \ldots, \bar{x}_k$, where k is the number of regressors.]

- **Average Partial Effect (APE):** For nonconstant partial effects, the partial effect averaged across the specified population.

  $[\frac{1}{n}\sum_{i=1}^{n}\frac{\partial \mathbb{P}(y_i=1|x_i)}{\partial x_j} = \frac{1}{n}\sum_{i=1}^{n} g(\hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \cdots + \hat{\beta}_k x_{ik}) \cdot \beta_j]$

Key terms - cont'd

- **Akaike Information Criterion (AIC):** A general measure for relative quality of models estimated with maximum likelihood, computed as

  $$AIC = -2\frac{\ln L}{n} + 2\frac{k}{n},$$

  where $\ln L$ is the maximum value of likelihood, n is the number of observations and k is the number of parameters.
- **Schwarz Criterion (SC):** A general measure for relative quality of models estimated with maximum likelihood, computed as

  $$SC = -2\frac{\ln L}{n} + \ln(n)\frac{k}{n},$$

  where $\ln L$ is the maximum value of likelihood, n is the number of observations and k is the number of parameters.

Key terms – cont'd

- **Percent Correctly Predicted (Hit Rate):** In a binary response model, the percentage of times the prediction of zero or one coincides with the actual outcome. Percentage of observations with $\tilde{y}_i = y_i$, where

$$\tilde{y}_i = \begin{cases} 1, & \text{if } \hat{\mathbb{P}}(y_i = 1|x_i) = G(x_i'\hat{\beta}) > c, \\ 0, & \text{if } \hat{\mathbb{P}}(y_i = 1|x_i) = G(x_i'\hat{\beta}) \leq c, \end{cases}$$

where c is typically chosen as 0.5.

Extra Topics

The Perfect Classifier Problem

Recall – the loglikelihood:

$$\begin{aligned}\ln L(\beta) &= \ln p(y_1,\ldots,y_n|x_1,\ldots,x_n)\\ &= \sum_{i=1}^{n}\Big\{\underbrace{y_i \ln[G(x_i'\beta)]}_{(*)} + \underbrace{(1-y_i)\ln[1-G(x_i'\beta)]}_{(**)}\Big\}. \qquad (1)\end{aligned}$$

We have

$$0 < G(x_i'\beta) < 1,$$

hence

$$-\infty < \ln[G(x_i'\beta)] < 0.$$

Notice that

$$y_i = 1 \Rightarrow (*) < 0 \;\&\; (**) = 0,$$
$$y_i = 0 \Rightarrow (*) = 0 \;\&\; (**) < 0.$$

Perfect fit:

$$y_i = 1 \iff G(x_i'\beta) = 1,$$
$$y_i = 0 \iff G(x_i'\beta) = 0.$$

This could happen only when

$$y_i = 1 \iff x_i'\beta = \infty, \tag{2}$$
$$y_i = 0 \iff x_i'\beta = -\infty. \tag{3}$$

We say that the loglikelihood (1) is *bounded above by* 0, and it achieves this bound if (2) and (3) hold.

Now, suppose that there is some linear combination of the independent variables, say $x_i'\beta^\bullet$, such that

$$y_i = 1 \iff x_i'\beta^\bullet > 0, \tag{4}$$

$$y_i = 0 \iff x_i'\beta^\bullet < 0. \tag{5}$$

In other words, there is some range of the regressor(s) for which y_i is always 1 or 0.

Then, we say that $x_i'\beta^\bullet$ describes a **separating hyperplane** (see Figure 2.1) and there is **complete separation** of the data.

$x_i'\beta^\bullet$ is said to be a **perfect classifier**, since it allows us to predict y_i with perfect accuracy for every observation.

Problem?

Yes, for ML estimation!

Then, it is possible to make the value of $\ln L$ arbitrarily close to 0 (the upper bound) by choosing β arbitrarily large (in an absolute sense)[1].

Hence, no finite ML estimator exists.

[1] Formally: by setting $\beta = \gamma\beta^{\bullet}$ and letting $\gamma \to \infty$.

Computer arithmetic

This is exactly what any nonlinear maximization algorithm will attempt to do if there exists a vector $\beta^\bullet$ for which conditions (4) and (5) are satisfied.

Because of the numerical limitations, the algorithm will eventually terminate (with some numerical error) at a value of $\ln L$ slightly less than 0.

This is likely to occur in practice when the sample is very small, when almost all of the y_i are equal to 0 or almost all of them are equal to 1, or when the model fits extremely well.

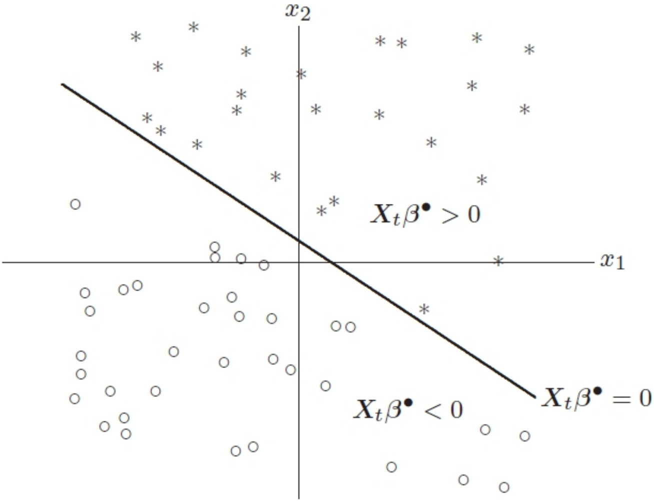


Figure 2.1: Figure 11.2 from Davidson and MacKinnon (1999), "Econometric Theory and Methods": A perfect classifier yields a separating hyperplane.

Simulation from the latent variable model

Consider the latent variable model

$$\begin{aligned} y_i^* &= \beta_0 + \beta_1 x_i + e_i, \\ e_i &\sim \mathcal{N}(0,1), \\ y_i &= \begin{cases} 1, & \text{if } y_i^* > 0, \\ 0, & \text{if } y_i^* \leq 0 \end{cases} \end{aligned}$$

Suppose that $x_i \sim \mathcal{N}(0,1)$*. We will generate* $5,000$ *samples of* 20 *observations on* (x_i, y_i) *pairs in the following way:*

- $1,000$ *assuming that* $\beta_0 = 0$ *and* $\beta_1 = 1$*;*
- $1,000$ *assuming that* $\beta_0 = 1$ *and* $\beta_1 = 1$*;*
- $1,000$ *assuming that* $\beta_0 = -1$ *and* $\beta_1 = 1$*;*
- $1,000$ *assuming that* $\beta_0 = 0$ *and* $\beta_1 = 2$*;*
- $1,000$ *assuming that* $\beta_0 = 0$ *and* $\beta_1 = 3$*.*

For each of the $5,000$ *samples, we will attempt to estimate a probit model.*

We are interested in the following question:

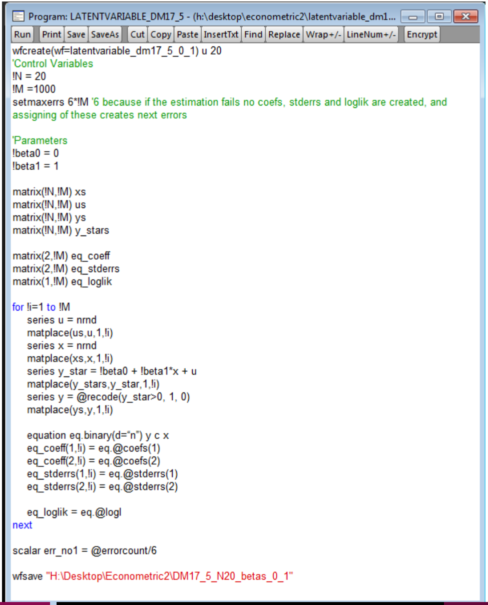
In each of the five cases, what proportion of the time does the estimation fail because of perfect classifiers?

We also want to explain why there will be more failures in some cases than in others.

Next, we will repeat this exercise for five sets of $1,000$ *samples of size* 40*, with the same parameter values.*

This will allow us to draw a conclusion about the effect of sample size on the perfect classifier problem.

EViews code for the first case ($N = 20$ with β_0 and β_1).


Program: LATENTVARIABLE_DM17_5 - (h:\desktop\econometric2\latentvariable_dm1...
Run
Print
Save
SaveAs
Cut
Copy
Paste
InsertTxt
Find
Replace
Wrap+/-
LineNum+/-
Encrypt

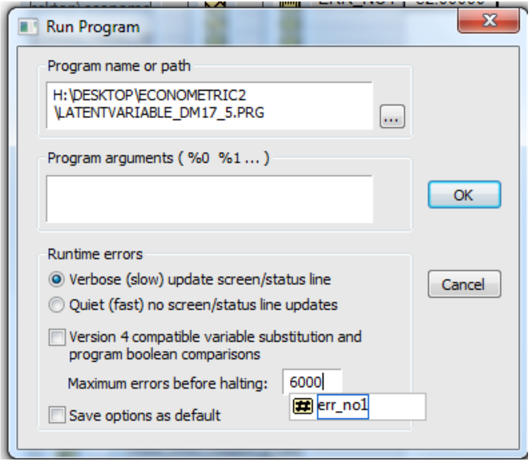
```
wfcreate(wf=latentvariable_dm17_5_0_1) u 20
'Control Variables
!N = 20
!M =1000
setmaxerrs 6*!M '6 because if the estimation fails no coefs, stderrs and loglik are created, and assigning of these creates next errors

'Parameters
!beta0 = 0
!beta1 = 1

matrix(!N,!M) xs
matrix(!N,!M) us
matrix(!N,!M) ys
matrix(!N,!M) y_stars

matrix(2,!M) eq_coeff
matrix(2,!M) eq_stderrs
matrix(1,!M) eq_loglik

for !i=1 to !M
    series u = nrnd
    matplace(us,u,1,!i)
    series x = nrnd
    matplace(xs,x,1,!i)
    series y_star = !beta0 + !beta1*x + u
    matplace(y_stars,y_star,1,!i)
    series y = @recode(y_star>0, 1, 0)
    matplace(ys,y,1,!i)

    equation eq.binary(d=“n”) y c x
    eq_coeff(1,!i) = eq.@coefs(1)
    eq_coeff(2,!i) = eq.@coefs(2)
    eq_stderrs(1,!i) = eq.@stderrs(1)
    eq_stderrs(2,!i) = eq.@stderrs(2)

    eq_loglik = eq.@logl
next

scalar err_no1 = @errorcount/6

wfsave "H:\Desktop\Econometric2\DM17_5_N20_betas_0_1"
```

If you are interested, you can check the results of each probit estimation: the coefficients estimates, their standard errors and the loglikelihood values are stored in matrices `eq_coeff`, `eq_stderrs` and `eq_logl`, respectively.

But what we are truly after, is the error count variable, `err_no1`, which reports how many times an estimation error occurred.

Notice, that we used the command `setmaxerr` to set the maximum number of error that the program may encounter before execution is halted.

Alternatively, you can specify it in the box showing up after clicking on the `run` button.



Run Program

Program name or path: H:\DESKTOP\ECONOMETRIC2\LATENTVARIABLE_DM17_5.PRG

Program arguments (%0 %1 ...):

Runtime errors

● Verbose (slow) update screen/status line

○ Quiet (fast) no screen/status line updates

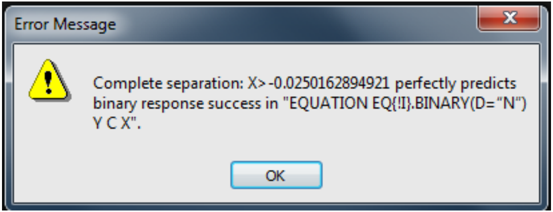
□ Version 4 compatible variable substitution and program boolean comparisons

Maximum errors before halting: 6000

err_no1

□ Save options as default

Without changing the value of maximum error allowed, the program would shortly break with the error message reporting the perfect separation problem.



Complete separation: X>-0.0250162894921 perfectly predicts binary response success in "EQUATION EQ{!I}.BINARY(D="N") Y C X".

The proportion of the time that perfect classifiers were encountered for each of the five cases and each of the two sample sizes:

Parameters	$n = 20$	$n = 40$
$\beta_0 = 0,\ \beta_1 = 1$	0.012	0.000
$\beta_0 = 1,\ \beta_1 = 1$	0.074	0.001
$\beta_0 = -1,\ \beta_1 = 1$	0.056	0.002
$\beta_0 = 0,\ \beta_1 = 2$	0.143	0.008
$\beta_0 = 0,\ \beta_1 = 3$	0.286	0.052

The proportion of samples with perfect classifiers increases as both β_0 and β_1 increase in absolute value. When $\beta_0 = 0$, the unconditional expectation of y_i is 0.5.

As β_0 increases in absolute value, this expectation becomes larger, and the proportion of 1s in the sample increases.

As β_1 becomes larger in absolute value, the model fits better on average, which obviously increases the chance that it fits perfectly.

The results for parameters $(1, 1)$ are almost identical to those for parameters $(-1, 1)$ because, with x_i having mean 0, the fraction of 1s in the samples with parameters $(1, 1)$ is the same, on average, as the fraction of 0s in the samples with parameters $(-1, 1)$.

Comparing the results for $n = 20$ and $n = 40$, it is clear that the probability of encountering a perfect classifier falls very rapidly as the sample size increases.

Lecture Problems

Lecture Problems: Exercise 1.

This exercise is about the reason why we can use the standard normal (or standard logistic) distribution. Consider the binary probit model

$$\begin{aligned}\mathbb{P}(y_i = 1|x_i) &= \Phi(\beta_0 + \beta_1 z_i),\\ \mathbb{P}(y_i = 0|x_i) &= 1 - \Phi(\beta_0 + \beta_1 z_i),\end{aligned}$$

where $\Phi(\cdot)$ *is the cumulative distribution function (CDF) of the standard normal distribution. This stems from the assumption that*

$$y_i^* = \beta_0 + \beta_1 z_i + e_i,$$

where e_i *is an error term with standard normal distribution (independent of* x_i*), where*

$$y_i = \mathbb{I}\{y_i^* > 0\} = \begin{cases} 1 & \textit{if } y_i^* > 0,\\ 0 & \textit{if } y_i^* \leq 0.\end{cases}$$

Lecture Problems: Exercise 1(1)

Suppose that we would assume that $e_i \sim \mathcal{N}(\mu, \sigma^2)$*, where* μ *and* σ^2 *are parameters to be estimated (instead of setting* $\mu = 0$ *and* $\sigma = 1$*).*

(1) Show that

$$\mathbb{P}(y_i = 1 | x_i) = \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right),$$

$$\mathbb{P}(y_i = 0 | x_i) = 1 - \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right).$$

Hint: use the 'standard' trick that $\frac{e_i - \mu}{\sigma} \sim \mathcal{N}(0,1)$ *if* $e_i \sim \mathcal{N}(\mu, \sigma^2)$*.*

$$
\begin{aligned}
\mathbb{P}(y_i = 1|x_i) &= \mathbb{P}(y_i^* > 0|x_i) \\
&= \mathbb{P}(x_i'\beta + e_i > 0|x_i) \\
&= \mathbb{P}(e_i > -x_i'\beta|x_i) \\
&= \mathbb{P}\left(\frac{e_i-\mu}{\sigma} > \frac{-x_i'\beta-\mu}{\sigma}\middle| x_i\right) \\
&\overset{\text{(symm)}}{=} \mathbb{P}\left(\frac{e_i-\mu}{\sigma} < \frac{x_i'\beta+\mu}{\sigma}\middle| x_i\right) \\
&\overset{\text{(cont)}}{=} \mathbb{P}\left(\frac{e_i-\mu}{\sigma} \leq \frac{x_i'\beta+\mu}{\sigma}\middle| x_i\right) \\
&\overset{\text{(ind)}}{=} \mathbb{P}\left(\frac{e_i-\mu}{\sigma} \leq \frac{x_i'\beta+\mu}{\sigma}\right) \\
&= \Phi\left(\frac{x_i'\beta+\mu}{\sigma}\right).
\end{aligned}
$$

Here: $x_i'\beta = \beta_0 + \beta_1 z_i$, so that

$$\mathbb{P}(y_i = 1|x_i) = \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right).$$

Further, we have either $y_i = 0$ or $y_i = 1$, so that

$$\mathbb{P}(y_i = 0|x_i) + \mathbb{P}(y_i = 1|x_i) = 1,$$

so that

$$\mathbb{P}(y_i = 0|x_i) = 1 - \mathbb{P}(y_i = 1|x_i) = 1 - \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right).$$

Lecture Problems: Exercise 1(2)

(2) What happens to $\mathbb{P}(y_i = 1|x_i)$ *if we change* β_0 *and* μ *into* $\beta_0 + 1$ *and* $\mu - 1$*?*

We have

$$\mathbb{P}(y_i = 1|x_i) = \Phi\left(\frac{(\beta_0 + 1) + \beta_1 z_i + (\mu - 1)}{\sigma}\right) = \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right),$$

so nothing happens to $\mathbb{P}(y_i = 1|x_i)$ in this case.

Therefore β_0 and μ are **not identified**!

The model with parameters $\beta_0 + a$ and $\mu - a$ $(-\infty < a < \infty)$ is the same Data Generating Process (DGP) as the model with parameters β_0 and μ.

It yields the same probabilities $\mathbb{P}(y_i = 0|x_i)$ and $\mathbb{P}(y_i = 1|x_i)$ for each observation $\Rightarrow$ the same Bernoulli distributions and the same properties of the y_i (conditionally upon x_i).

Even if we would have infinitely many observations, we could not distinguish between the model with parameters β_0 and μ and the model with parameters $\beta_0 + a$ and $\mu - a$.

Therefore we can set $\mu = 0$ *without loss of generality*.

Lecture Problems: Exercise 1(3)

(3) What happens to $\mathbb{P}(y_i = 1|x_i)$ *if we change* β_0, β_1, μ *and* σ^2 *into* $2\beta_0$, $2\beta_1$, 2μ *and* 2σ*?*

We have

$$\mathbb{P}(y_i = 1|x_i) = \Phi\left(\frac{2\beta_0 + 2\beta_1 z_i + 2\mu}{2\sigma}\right) = \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right),$$

so nothing happens to $\mathbb{P}(y_i = 1|x_i)$ in this case.

Therefore β_0, β_1, μ and σ are **not identified**!

The model with parameters $b \cdot \beta_0$, $b \cdot \beta_1$, $b \cdot \mu$ and $b \cdot \sigma$ is $(b > 0)$ is the same DGP as the model with parameters β_0, β_1, μ and σ.

It yields the same probabilities $\mathbb{P}(y_i = 0|x_i)$ and $\mathbb{P}(y_i = 1|x_i)$ for each observation ⇒ the same Bernoulli distributions and the same properties of the y_i (conditional upon x_i).

Even if we would have infinitely many observations, we could not distinguish between the model with parameters β_0, β_1, μ and σ and the model with parameters $b \cdot \beta_0$, $b \cdot \beta_1$, $b \cdot \mu$ and $b \cdot \sigma$.

Therefore we can set $\sigma = 1$ *without loss of generality.*

Lecture Problems: Exercise 1(4)

(4) What is the difference in $\mathbb{P}(y_i = 1|x_i)$ *between the model with parameters* β_0, β_1, μ *and* σ^2 *and the model with parameters* $b \cdot (\beta_0 + a)$, $b \cdot \beta_1$, $b \cdot (\mu - a)$ *and* $b \cdot \sigma$ *(with* $-\infty < a < \infty$ *and* $b > 0$*)?*

$$\mathbb{P}(y_i = 1|x_i) = \Phi\left(\frac{b \cdot (\beta_0 + a) + b \cdot \beta_1 z_i + b \cdot (\mu - a)}{b \cdot \sigma}\right)$$
$$= \Phi\left(\frac{\beta_0 + \beta_1 z_i + \mu}{\sigma}\right).$$

So there is no difference in $\mathbb{P}(y_i = 1|x_i)$ between the model with parameters β_0, β_1, μ and σ and the model with parameters $b \cdot (\beta_0 + a)$, $b \cdot \beta_1$, $b \cdot (\mu - a)$ and $b \cdot \sigma$ (with $-\infty < a < \infty$ and $b > 0$).

Therefore β_0, β_1, μ and σ are **not identified**!

We we can set $\mu = 0$ and $\sigma = 1$ *without loss of generality.*

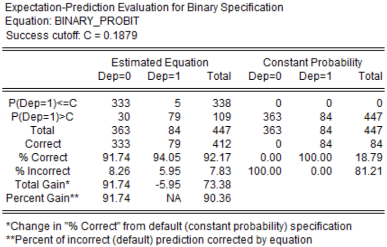
Only after imposing these restrictions $\mu = 0$ and $\sigma = 1$, the parameters β_0 and β_1 are identified: a different value of (β_0, β_1) implies a different distribution of y_i (conditional upon x_i).

Lecture Problems: Exercise 2(1)

The data are in the EViews file ***bank_employees.wf1****.*

(1) Change the threshold from 0.5 *to* $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$*. Compare the percentage correctly predicted between the binary probit and binary logit model.*

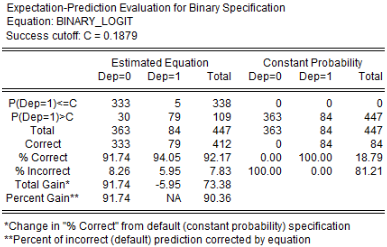
We have $n = 447$ observations, where $y_i = 0$ for 363 observations and $y_i = 1$ for 84 observations.
So $\overline{y} = 84/447 = 0.1879$.



Expectation-Prediction Evaluation for Binary Specification
Equation: BINARY_PROBIT
Success cutoff: C = 0.1879

	Estimated Equation			Constant Probability		
	Dep=0	Dep=1	Total	Dep=0	Dep=1	Total
P(Dep=1)<=C	333	5	338	0	0	0
P(Dep=1)>C	30	79	109	363	84	447
Total	363	84	447	363	84	447
Correct	333	79	412	0	84	84
% Correct	91.74	94.05	92.17	0.00	100.00	18.79
% Incorrect	8.26	5.95	7.83	100.00	0.00	81.21
Total Gain*	91.74	-5.95	73.38			
Percent Gain**	91.74	NA	90.36			

*Change in "% Correct" from default (constant probability) specification
**Percent of incorrect (default) prediction corrected by equation



Expectation-Prediction Evaluation for Binary Specification
Equation: BINARY_LOGIT
Success cutoff: C = 0.1879

	Estimated Equation			Constant Probability		
	Dep=0	Dep=1	Total	Dep=0	Dep=1	Total
P(Dep=1)<=C	333	5	338	0	0	0
P(Dep=1)>C	30	79	109	363	84	447
Total	363	84	447	363	84	447
Correct	333	79	412	0	84	84
% Correct	91.74	94.05	92.17	0.00	100.00	18.79
% Incorrect	8.26	5.95	7.83	100.00	0.00	81.21
Total Gain*	91.74	-5.95	73.38			
Percent Gain**	91.74	NA	90.36			

*Change in "% Correct" from default (constant probability) specification
**Percent of incorrect (default) prediction corrected by equation

Percentage correctly predicted $= 92.17\%$ in both models.

Note: this threshold 0.1878 (instead of 0.5) implies that we predict $\tilde{y}_i = 1$ more often (and $\tilde{y}_i = 0$ less often).

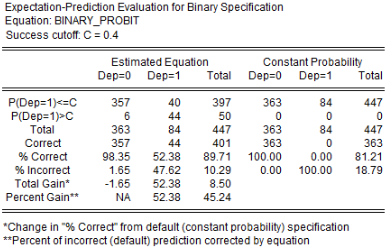
Now we have 109 predictions $\tilde{y}_i = 1$ instead of 50.

In this case this threshold 0.1878 leads to a better percentage correctly predicted of 92.17% instead of 89.71%.

The latter does not need to be the case in general.

Lecture Problems: Exercise 2(2)

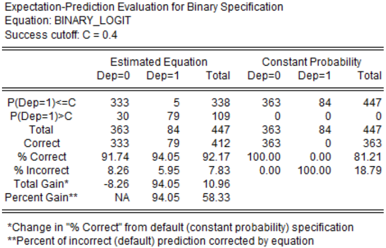
(2) Change the threshold from 0.5 *to* 0.4*. Compare the percentage correctly predicted between the binary probit and binary logit model.*



Expectation-Prediction Evaluation for Binary Specification
Equation: BINARY_PROBIT
Success cutoff: C = 0.4

	Estimated Equation			Constant Probability		
	Dep=0	Dep=1	Total	Dep=0	Dep=1	Total
P(Dep=1)<=C	357	40	397	363	84	447
P(Dep=1)>C	6	44	50	0	0	0
Total	363	84	447	363	84	447
Correct	357	44	401	363	0	363
% Correct	98.35	52.38	89.71	100.00	0.00	81.21
% Incorrect	1.65	47.62	10.29	0.00	100.00	18.79
Total Gain*	-1.65	52.38	8.50			
Percent Gain**	NA	52.38	45.24			

*Change in "% Correct" from default (constant probability) specification
**Percent of incorrect (default) prediction corrected by equation



Expectation-Prediction Evaluation for Binary Specification
Equation: BINARY_LOGIT
Success cutoff: C = 0.4

	Estimated Equation			Constant Probability		
	Dep=0	Dep=1	Total	Dep=0	Dep=1	Total
P(Dep=1)<=C	333	5	338	363	84	447
P(Dep=1)>C	30	79	109	0	0	0
Total	363	84	447	363	84	447
Correct	333	79	412	363	0	363
% Correct	91.74	94.05	92.17	100.00	0.00	81.21
% Incorrect	8.26	5.95	7.83	0.00	100.00	18.79
Total Gain*	-8.26	94.05	10.96			
Percent Gain**	NA	94.05	58.33			

*Change in "% Correct" from default (constant probability) specification
**Percent of incorrect (default) prediction corrected by equation

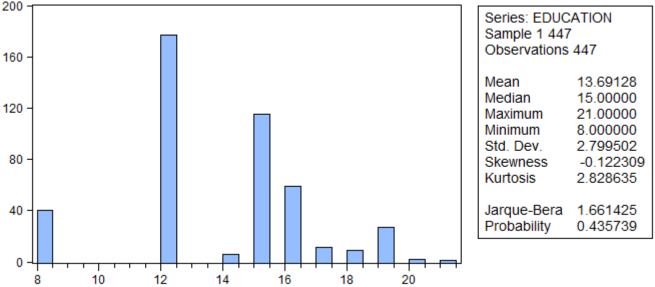
Percentage correctly predicted = 89.71% in binary probit model.
Percentage correctly predicted = 92.17% in binary logit model.

So, for this value of the threshold 0.4 the binary logit model has a better percentage correctly predicted than the binary probit model.

Lecture Problems: Exercise 2(3)

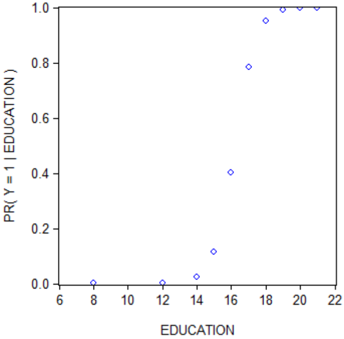
(3) Can you find a threshold so that $\sum_{i=1}^{n} \tilde{y}_i = \sum_{i=1}^{n} y_i$*? Motivate your answer.*

No. The explanatory variable *education* (the only explanatory variable in this model) takes a finite number of values, so that the estimated probability $\hat{\mathbb{P}}(y_i = 1|x_i)$ is exactly the same for groups of individuals that have exactly the same *education*.



```
Series: EDUCATION
Sample 1 447
Observations 447

Mean         13.69128
Median       15.00000
Maximum      21.00000
Minimum      8.000000
Std. Dev.    2.799502
Skewness    -0.122309
Kurtosis     2.828635

Jarque-Bera  1.661425
Probability  0.435739
```

We have $\sum_{i=1}^{n} y_i = 84$ observations with $y_i = 1$ in dataset.



There are 50 individuals with $education \geq 17$; these have $\hat{\mathbb{P}}(y_i = 1|x_i) > 0.5$ in the binary logit model.

There are 59 individuals with $education = 16$; these have $\hat{\mathbb{P}}(y_i = 1|x_i) = 0.4036$ in the binary logit model.

So:

- We have 50 predictions with $\tilde{y}_i = 1$, if we take a threshold like 0.5, so that each individual with $education \geq 17$ gets prediction $\tilde{y}_i = 1$ and so that each individual with $education \leq 16$ gets prediction $\tilde{y}_i = 0$.
- We have $50 + 59 = 109$ predictions with $\tilde{y}_i = 1$, if we take a threshold like 0.4, so that each individual with $education \geq 16$ gets prediction $\tilde{y}_i = 1$ and so that each individual with $education \leq 15$ gets prediction $\tilde{y}_i = 0$.
- We can not get exactly 84 observations with $\tilde{y}_i = 1$.

Lecture Problems: Exercise 2(4)

(4) Change the value of education of the last observation from 12 to 120 (to create an extreme outlier with enormous education and $y_i = 0$).

Note: first you may need to click the `Edit +-` *button above the spreadsheet with values. Re-estimate the binary probit and logit models. Compare the percentage correctly predicted between the binary probit and binary logit model. Can you explain the difference in quality between the probit and logit models?*

Note: Here we have a binary logit/probit model, where we have added an outlier by changing the *education* of the last observation to 120 (instead of 12), where $y_i = 0$ for this last observation.

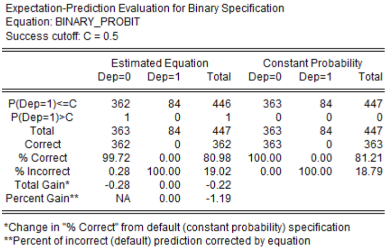
So, we created an extreme observation with an extremely high value of education and still $y_i = 0$.

The person of the last observation has an administrative job, not a management job.

Note: if the last observation would have $y_i = 1$, then this would not be an outlier!

Then, the extremely high value of education would simply 'match' with $y_i = 1$, so that changing 12 into 120 would hardly affect the parameter estimates.

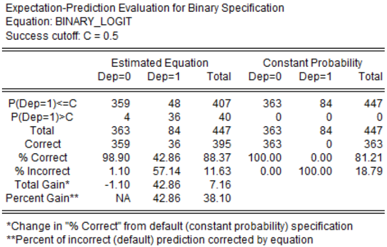
For threshold $= 0.5$ we obtain:



Expectation-Prediction Evaluation for Binary Specification
Equation: BINARY_PROBIT
Success cutoff: C = 0.5

	Estimated Equation			Constant Probability		
	Dep=0	Dep=1	Total	Dep=0	Dep=1	Total
P(Dep=1)<=C	362	84	446	363	84	447
P(Dep=1)>C	1	0	1	0	0	0
Total	363	84	447	363	84	447
Correct	362	0	362	363	0	363
% Correct	99.72	0.00	80.98	100.00	0.00	81.21
% Incorrect	0.28	100.00	19.02	0.00	100.00	18.79
Total Gain*	-0.28	0.00	-0.22			
Percent Gain**	NA	0.00	-1.19			

*Change in "% Correct" from default (constant probability) specification
**Percent of incorrect (default) prediction corrected by equation



Expectation-Prediction Evaluation for Binary Specification
Equation: BINARY_LOGIT
Success cutoff: C = 0.5

	Estimated Equation			Constant Probability		
	Dep=0	Dep=1	Total	Dep=0	Dep=1	Total
P(Dep=1)<=C	359	48	407	363	84	447
P(Dep=1)>C	4	36	40	0	0	0
Total	363	84	447	363	84	447
Correct	359	36	395	363	0	363
% Correct	98.90	42.86	88.37	100.00	0.00	81.21
% Incorrect	1.10	57.14	11.63	0.00	100.00	18.79
Total Gain*	-1.10	42.86	7.16			
Percent Gain**	NA	42.86	38.10			

*Change in "% Correct" from default (constant probability) specification
**Percent of incorrect (default) prediction corrected by equation

Now we see a substantial difference in percentage correctly predicted between the binary probit model and the binary logit model: 80.98% versus 88.37%.

(The binary probit model does not even beat the approach of simply predicting $\tilde{y}_i = 0$ for each observation, which has percentage correctly predicted of 81.21%.)

Explanation: the tails of the logistic distribution are fatter than the tails of the normal distribution!

Outliers can occur in the logistic distribution, so that the parameter estimates are relatively less affected by outliers.

In the normal distribution, the presence of one outlier can have a huge effect on the parameter estimates.

Roughly stated, in the binary probit model the parameter estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ are enormously changed in order to "keep the last observation with $(education = 120, y = 0)$ out of the extreme tail".

Exercises

W17/1 (i)

(i) For a binary response y, let $\bar{y}$ be the proportion of ones in the sample (which is equal to the sample average of the y_i).

Let $\hat{q}_0$ be the percent correctly predicted for the outcome $y = 0$ and let $\hat{q}_1$ be the percent correctly predicted for the outcome $y = 1$.

If $\hat{p}$ is the overall percent correctly predicted, show that $\hat{p}$ is a weighted average of $\hat{q}_0$ and $\hat{q}_1$:

$$\hat{p} = (1 - \bar{y})\hat{q}_0 + \bar{y}\hat{q}_1.$$

Let:
n_0 – the number of observations when $y_i = 0$,
n_1 – the number of observations when $y_i = 1$,
$n = n_0 + n_1$ the sample size,

m_0 – the number (not the percent!) correctly predicted when $y_i = 0$ (so the prediction is also zero),
m_1 – the number correctly predicted when $y_i = 1$.

Then, the proportion correctly predicted is

$$\frac{m_0 + m_1}{n}.$$

We can write this as follows:

$$\frac{m_0 + m_1}{n} = \frac{n_0}{n} \cdot \frac{m_0}{n_0} + \frac{n_1}{n} \cdot \frac{m_1}{n_1} = (1 - \bar{y})\frac{m_0}{n_0} + \bar{y}\frac{m_1}{n_1},$$

since:

$\bar{y} = \frac{n_1}{n}$ (the proportion of the sample with $y_i = 1$)

and

$1 - \bar{y} = \frac{n_0}{n}$ (the proportion of the sample with $y_i = 0$).

Next, notice that:
$\frac{m_0}{n_0}$ – the proportion correctly predicted when $y_i = 0$,
and
$\frac{m_1}{n_1}$ – the proportion correctly predicted when $y_i = 1$.

Hence:

$$\frac{m_0 + m_1}{n} = (1 - \bar{y})\frac{m_0}{n_0} + \bar{y}\frac{m_1}{n_1}.$$

Multiplying both sides by 100 yields

$$\hat{p} = (1 - \bar{y})\hat{q}_0 + \bar{y}\hat{q}_1, \tag{6}$$

where we use the fact that, by definition,

$$\hat{p} = 100 \cdot \frac{m_0 + m_1}{n}, \qquad \hat{q}_0 = 100 \cdot \frac{m_0}{n_0}, \qquad \hat{q}_1 = 100 \cdot \frac{m_1}{n_1}.$$

W17/1 (ii)

(ii) In a sample of 300*, suppose that* $\bar{y} = 0.70$*, so that there are* 210 *outcomes with* $y_1 = 1$ *and* 90 *with* $y_i = 0$*.*

Suppose that the percent correctly predicted when $y = 0$ *is* 80*, and the percent correctly predicted when* $y = 1$ *is* 40*.*

Find the overall percent correctly predicted.

We just use formula (6) from part *(i)*:

$$\hat{p} = 0.30 \cdot 80 + 0.70 \cdot 40 = 52.$$

Therefore, overall we correctly predict only 52% of the outcomes.

This is because, while 80% of the time we correctly predict $y = 0$, the observations where $y_i = 0$ account for only 30 percent of the outcomes.

More weight (i.e. 0.70) is given to the predictions when $y_i = 1$, and we do much less well predicting that outcome (getting it right only 40% of the time).

W17/2

Let grad be a dummy variable for whether a student-athlete at a large university graduates in five years. Let hsGPA and SAT be high school grade point average and SAT score, respectively. Let study be the number of hours spent per week in an organized study hall. Suppose that, using data on 420 *student-athletes, the following logit model is obtained:*

$$\hat{\mathbb{P}}(grad = 1|hsGPA, SAT, study) =$$
$$\Lambda(-1.17 + 0.24\, hsGPA + 0.00058\, SAT + 0.073\, study),$$

where $\Lambda(z) = \exp(z)/[1 + \exp(z)]$ *is the logit function. Holding hsGPA fixed at* 3.0 *and SAT fixed at* 1,200*, compute the estimated difference in the graduation probability for someone who spent* 10 *hours per week in study hall and someone who spent* 5 *hours per week.*

We first need to compute the estimated probability at $hsGPA = 3.0$, $SAT = 1,200$, and $study = 10$, second at $hsGPA = 3.0$, $SAT = 1,200$, and $study = 5$, and then subtract the former from the latter.

To obtain the first probability, we start by computing the linear function inside $\Lambda(\cdot)$:

$$-1.17 + 0.24 \cdot hsGPA + 0.00058 \cdot SAT + 0.073 \cdot study =$$
$$-1.17 + 0.24 \cdot 3.0 + 0.00058 \cdot 1,200 + 0.073 \cdot 10 = 0.9760.$$

Next, we plug this into the logit function:

$$\frac{\exp(0.9760)}{1 + \exp(0.9760)} \approx 0.7263.$$

This is the estimated probability that a student-athlete with the given characteristics graduates in five years.

For the student-athlete who attended study hall five hours a week, we compute:

$$-1.17 + 0.24 \cdot 3.0 + 0.00058 \cdot 1,200 + 0.073 \cdot 5 = 0.6110.$$

Evaluating the logit function at this value gives

$$\frac{\exp(0.6110)}{1 + \exp(0.6110)} \approx 0.6482.$$

Therefore, the difference in estimated probabilities is

$$0.7263 - 0.6482 = 0.0781,$$

which is under 0.10.

Note how far off the calculation would be if we simply use the coefficient on study (in the linear function inside Λ) to conclude that the difference in probabilities is

$$0.073 \cdot (10\text{–}5) = 0.365.$$

Computer Exercises

W17/C1

Use the data in `pntsprd.wf1` *for this exercise.*

$N = 553$, *cross-sectional gambling point spread data for the 1994–1995 men's college basketball seasons. The spread is for the day before the game was played.*

Spread?

The number of points added to the score of the "underdog".

So if team A is the favourite and team B the underdog, where the betting office quotes a spread of 10, and if the result is 70 – 65 (e.g., in basketball), then team B has "won" if we take into account the spread (since $70 < 65 + 10$).

So, betting with spread 10 is the same as betting that the score of team A is higher or lower than the score of team B plus 10.

The larger the spread, the larger the expected difference in strength between the teams. It is a "trick" to make betting interesting even on matches between teams with very different levels, because it is sometimes almost sure that the favourite team wins, but it is then not sure if they win with more or less than e.g. 20 points difference.

W17/C1 (i)

(i) The variable favwin is a binary variable if the team favoured by the Las Vegas point spread wins.

A linear probability model to estimate the probability that the favoured team wins is

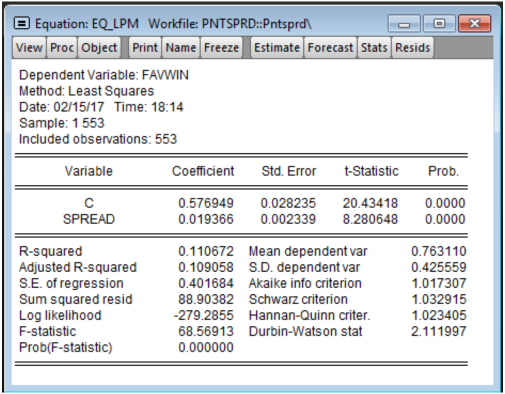
$$\mathbb{P}(favin = 1|spread) = \beta_0 + \beta_1 spread.$$

Explain why, if the spread incorporates all relevant information, we expect $\beta_0 = 0.5$.

If *spread* is zero, there is no favourite, and the probability that the team we (arbitrarily) label the favourite should have a 50% chance of winning.

W17/C1 (ii)

(ii) Estimate the model from part (i) by OLS. Test $H_0 : \beta_0 = 0.5$ *against a two-sided alternative.*



Equation: EQ_LPM Workfile: PNTSPRD::Pntsprd\
View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Dependent Variable: FAVWIN
Method: Least Squares
Date: 02/15/17 Time: 18:14
Sample: 1 553
Included observations: 553

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.576949	0.028235	20.43418	0.0000
SPREAD	0.019366	0.002339	8.280648	0.0000

R-squared	0.110672	Mean dependent var	0.763110
Adjusted R-squared	0.109058	S.D. dependent var	0.425559
S.E. of regression	0.401684	Akaike info criterion	1.017307
Sum squared resid	88.90382	Schwarz criterion	1.032915
Log likelihood	-279.2855	Hannan-Quinn criter.	1.023405
F-statistic	68.56913	Durbin-Watson stat	2.111997
Prob(F-statistic)	0.000000		

The linear probability model estimated by OLS gives

$$\widehat{favwin} = \underset{(0.028)}{0.577} + \underset{(0.0023)}{0.0194}\, spread$$

with $n = 553$ and $R^2 = 0.111$, where the usual standard errors are in parentheses.

The t–statistic for $H_0 : \beta_0 = 0.5$ is

$$\frac{0.577 - 0.500}{0.028} = 2.75,$$

which leads to rejecting H_0 against a two-sided alternative at the 1% level (critical value $t_{553-2} \approx 2.5848$).

W17/C1 (iii)

(iii) Is spread statistically significant? What is the estimated probability that the favoured team wins when $spread = 10$*?*

The t–statistic for $H_0 : \beta_1 = 0.0$ is

$$\frac{0.0194 - 0}{0.0023} = 8.4348,$$

so as we expect, *spread* is very statistically significant.

If $spread = 10$ the estimated probability that the favoured team wins is

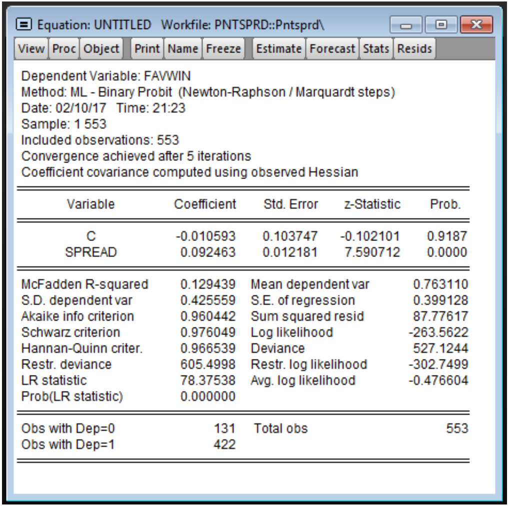
$$0.577 + 0.0194 \cdot 10 = 0.771.$$

W17/C1 (iii)

(iii) Now, estimate a probit model for $P(favwin = 1|spread)$.

Interpret and test the null hypothesis that the intercept is zero.

[Hint: Remember that $\Phi(0) = 0.5$.*]*



```
Dependent Variable: FAVWIN
Method: ML - Binary Probit (Newton-Raphson / Marquardt steps)
Date: 02/10/17   Time: 21:23
Sample: 1 553
Included observations: 553
Convergence achieved after 5 iterations
Coefficient covariance computed using observed Hessian

        Variable          Coefficient    Std. Error    z-Statistic    Prob.

           C               -0.010593     0.103747     -0.102101     0.9187
        SPREAD              0.092463     0.012181      7.590712     0.0000

McFadden R-squared          0.129439   Mean dependent var        0.763110
S.D. dependent var          0.425559   S.E. of regression        0.399128
Akaike info criterion       0.960442   Sum squared resid         87.77617
Schwarz criterion           0.976049   Log likelihood           -263.5622
Hannan-Quinn criter.        0.966539   Deviance                  527.1244
Restr. deviance             605.4998   Restr. log likelihood    -302.7499
LR statistic                78.37538   Avg. log likelihood      -0.476604
Prob(LR statistic)          0.000000

Obs with Dep=0                   131    Total obs                     553
Obs with Dep=1                   422
```

In the Probit model

$$\mathbb{P}(favwin = 1|spread) = \Phi(\beta_0 + \beta_1 spread),$$

where $\Phi(\cdot)$ denotes the standard normal cdf. If $\beta_0 = 0$, then

$$\mathbb{P}(favwin = 1|spread) = \Phi(\beta_1 spread)$$

and, in particular,

$$\mathbb{P}(favwin = 1|spread = 0) = \Phi(0) = 0.5.$$

This is the analog of testing whether the intercept is 0.5 in the LPM. From the EViews output, the t statistic (or, actually, the z statistic, only valid asymptotically) for testing $H_0 : \beta_0 = 0$ is only about -0.102 so there are no grounds to reject H_0.

W17/C1 (iv)

(iv) Use the probit model to estimate the probability that the favoured team wins when $spread = 10$*.*

Compare this with the LPM estimate from part (iii).

When *spread* $= 10$ the predicted response probability from the estimated probit model is

$$\Phi(-0.0106 + 0.0925 \cdot 10) = \Phi(0.9144) \approx 0.820.$$

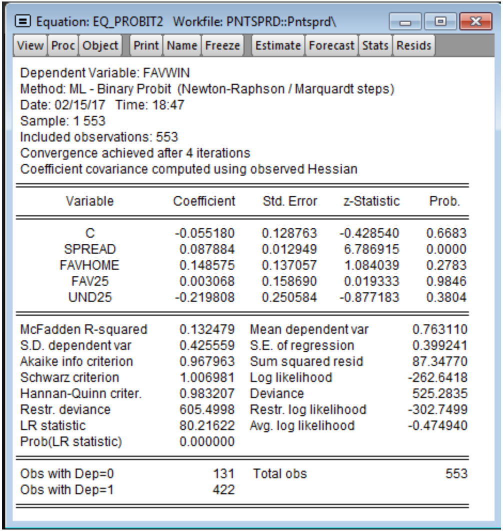
This is somewhat above the estimate for the LPM.

W17/C1 (v)

Add the variables $favhome$, $fav25$, and $und25$ to the probit model and test joint significance of these variables using the likelihood ratio test.

(How many df are in the χ^2 distribution?)

Interpret this result, focusing on the question of whether the spread incorporates all observable information prior to a game.



Equation: EQ_PROBIT2 Workfile: PNTSPRD::Pntsprd\
View Proc Object Print Name Freeze Estimate Forecast Stats Resids

```
Dependent Variable: FAVWIN
Method: ML - Binary Probit (Newton-Raphson / Marquardt steps)
Date: 02/15/17   Time: 18:47
Sample: 1 553
Included observations: 553
Convergence achieved after 4 iterations
Coefficient covariance computed using observed Hessian
==========================================================================
        Variable          Coefficient    Std. Error    z-Statistic    Prob.
==========================================================================
           C               -0.055180     0.128763     -0.428540     0.6683
        SPREAD              0.087884     0.012949      6.786915     0.0000
        FAVHOME             0.148575     0.137057      1.084039     0.2783
         FAV25              0.003068     0.158690      0.019333     0.9846
         UND25             -0.219808     0.250584     -0.877183     0.3804
==========================================================================
McFadden R-squared          0.132479   Mean dependent var       0.763110
S.D. dependent var          0.425559   S.E. of regression       0.399241
Akaike info criterion       0.967963   Sum squared resid        87.34770
Schwarz criterion           1.006981   Log likelihood          -262.6418
Hannan-Quinn criter.        0.983207   Deviance                 525.2835
Restr. deviance             605.4998   Restr. log likelihood   -302.7499
LR statistic                80.21622   Avg. log likelihood     -0.474940
Prob(LR statistic)          0.000000
==========================================================================
Obs with Dep=0                   131    Total obs                    553
Obs with Dep=1                   422
==========================================================================
```

When $favhome$, $fav25$, and $und25$ are added to the probit model, the value of the loglikelihood becomes –262.64, while it used to be -263.56.

Therefore, the likelihood ratio statistic is

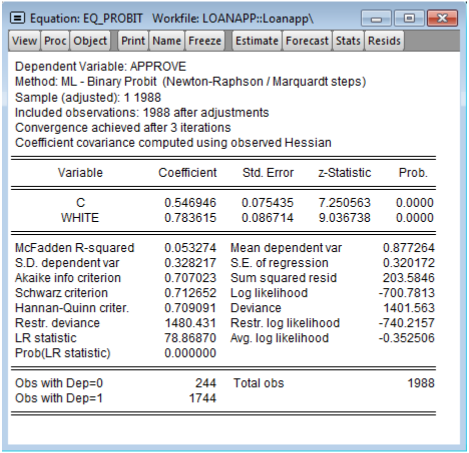
$$2 \cdot [-262.64–(-263.56)] = 2 \cdot (263.56–262.64) = 1.84.$$

The p-value from the χ^2_3 ($df = 3$ because we add 3 variables) distribution is about 0.61, so $favhome$, $fav25$, and $und25$ are jointly very insignificant. Once spread is controlled for, these other factors have no additional power for predicting the outcome.

W17/C2 (i)

Use the data in `loanapp.wf1` *for this exercise.*

(i) Estimate a probit model of approve on white. Find the estimated probability of loan approval for both whites and nonwhites. How do these compare with the linear probability estimates?



```
Dependent Variable: APPROVE
Method: ML - Binary Probit (Newton-Raphson / Marquardt steps)
Sample (adjusted): 1 1988
Included observations: 1988 after adjustments
Convergence achieved after 3 iterations
Coefficient covariance computed using observed Hessian

        Variable        Coefficient    Std. Error    z-Statistic    Prob.

           C             0.546946      0.075435      7.250563      0.0000
         WHITE           0.783615      0.086714      9.036738      0.0000

McFadden R-squared       0.053274   Mean dependent var      0.877264
S.D. dependent var       0.328217   S.E. of regression      0.320172
Akaike info criterion    0.707023   Sum squared resid       203.5846
Schwarz criterion        0.712652   Log likelihood         -700.7813
Hannan-Quinn criter.     0.709091   Deviance                1401.563
Restr. deviance          1480.431   Restr. log likelihood  -740.2157
LR statistic             78.86870   Avg. log likelihood    -0.352506
Prob(LR statistic)       0.000000

Obs with Dep=0                244    Total obs                  1988
Obs with Dep=1               1744
```

As there is only one explanatory variable that takes on just two values, there are only two different predicted values: the estimated probabilities of loan approval for white and nonwhite applicants. Rounded to three decimal places these are:

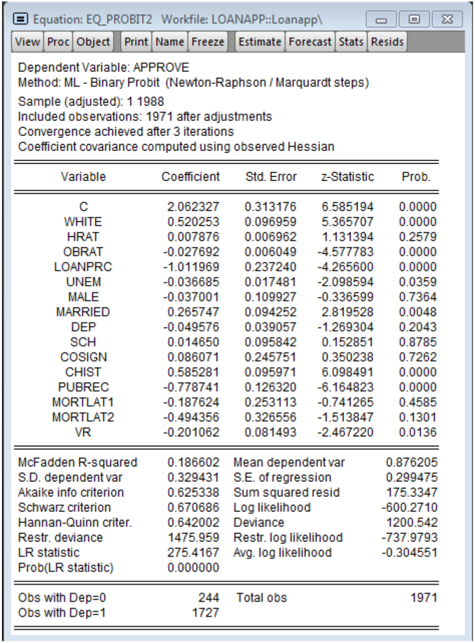
$$\begin{aligned}\mathbb{P}(approve = 1|white = 0) &= \Phi(\beta_0 + \beta_1 \cdot 0)\\ &= \Phi(0.547) = 0.708,\\ \mathbb{P}(approve = 1|white = 1) &= \Phi(\beta_0 + \beta_1 \cdot 1)\\ &= \Phi(0.547 + 0.784) = 0.908,\end{aligned}$$

for nonwhites and whites, respectively.

(In other words, 0.708 is the proportion of loans approved for nonwhites and 0.908 is the proportion approved for whites.)

W17/C2 (ii)

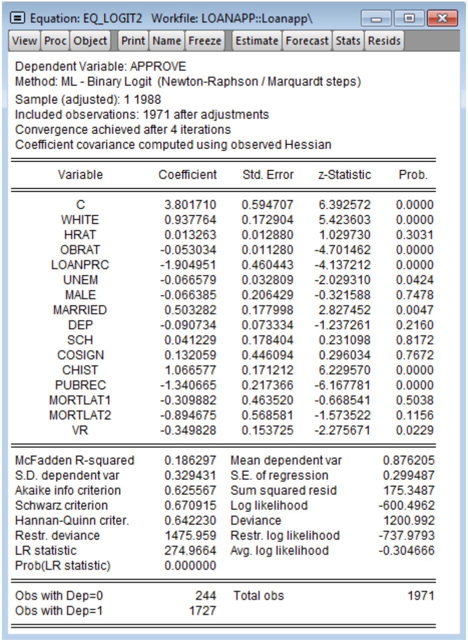
*(ii) Now, add the variables hrat, obrat, loanprc, unem, male, married, dep, sch, cosign, chist, pubrec, mortlat*1*, mortlat*2*, and vr to the probit model. Is there statistically significant evidence of discrimination against nonwhites?*



```
Dependent Variable: APPROVE
Method: ML - Binary Probit (Newton-Raphson / Marquardt steps)
Sample (adjusted): 1 1988
Included observations: 1971 after adjustments
Convergence achieved after 3 iterations
Coefficient covariance computed using observed Hessian
==========================================================================
     Variable        Coefficient    Std. Error    z-Statistic    Prob.
==========================================================================
        C              2.062327      0.313176      6.585194     0.0000
      WHITE            0.520253      0.096959      5.365707     0.0000
       HRAT            0.007876      0.006962      1.131394     0.2579
      OBRAT           -0.027692      0.006049     -4.577783     0.0000
     LOANPRC          -1.011969      0.237240     -4.265600     0.0000
       UNEM           -0.036685      0.017481     -2.098594     0.0359
       MALE           -0.037001      0.109927     -0.336599     0.7364
     MARRIED           0.265747      0.094252      2.819528     0.0048
       DEP            -0.049576      0.039057     -1.269304     0.2043
       SCH             0.014650      0.095842      0.152851     0.8785
      COSIGN           0.086071      0.245751      0.350238     0.7262
      CHIST            0.585281      0.095971      6.098491     0.0000
      PUBREC          -0.778741      0.126320     -6.164823     0.0000
     MORTLAT1         -0.187624      0.253113     -0.741265     0.4585
     MORTLAT2         -0.494356      0.326556     -1.513847     0.1301
        VR            -0.201062      0.081493     -2.467220     0.0136
==========================================================================
McFadden R-squared      0.186602   Mean dependent var        0.876205
S.D. dependent var      0.329431   S.E. of regression        0.299475
Akaike info criterion   0.625338   Sum squared resid         175.3347
Schwarz criterion       0.670686   Log likelihood           -600.2710
Hannan-Quinn criter.    0.642002   Deviance                  1200.542
Restr. deviance         1475.959   Restr. log likelihood    -737.9793
LR statistic            275.4167   Avg. log likelihood      -0.304551
Prob(LR statistic)      0.000000
==========================================================================
Obs with Dep=0               244    Total obs                    1971
Obs with Dep=1              1727
==========================================================================
```

W17/C2 (iii)

(iii) Estimate the model from part (ii) by logit. Compare the coefficient on white to the probit estimate.



```
Dependent Variable: APPROVE
Method: ML - Binary Logit (Newton-Raphson / Marquardt steps)
Sample (adjusted): 1 1988
Included observations: 1971 after adjustments
Convergence achieved after 4 iterations
Coefficient covariance computed using observed Hessian
=========================================================================
      Variable        Coefficient    Std. Error    z-Statistic    Prob.
=========================================================================
         C             3.801710      0.594707      6.392572      0.0000
       WHITE           0.937764      0.172904      5.423603      0.0000
       HRAT            0.013263      0.012880      1.029730      0.3031
       OBRAT          -0.053034      0.011280     -4.701462      0.0000
      LOANPRC         -1.904951      0.460443     -4.137212      0.0000
       UNEM           -0.066579      0.032809     -2.029310      0.0424
       MALE           -0.066385      0.206429     -0.321588      0.7478
      MARRIED          0.503282      0.177998      2.827452      0.0047
        DEP           -0.090734      0.073334     -1.237261      0.2160
        SCH            0.041229      0.178404      0.231098      0.8172
      COSIGN           0.132059      0.446094      0.296034      0.7672
       CHIST           1.066577      0.171212      6.229570      0.0000
      PUBREC          -1.340665      0.217366     -6.167781      0.0000
     MORTLAT1         -0.309882      0.463520     -0.668541      0.5038
     MORTLAT2         -0.894675      0.568581     -1.573522      0.1156
        VR            -0.349828      0.153725     -2.275671      0.0229
=========================================================================
McFadden R-squared      0.186297    Mean dependent var        0.876205
S.D. dependent var      0.329431    S.E. of regression        0.299487
Akaike info criterion   0.625567    Sum squared resid         175.3487
Schwarz criterion       0.670915    Log likelihood           -600.4962
Hannan-Quinn criter.    0.642230    Deviance                  1200.992
Restr. deviance         1475.959    Restr. log likelihood    -737.9793
LR statistic            274.9664    Avg. log likelihood      -0.304666
Prob(LR statistic)      0.000000
=========================================================================
Obs with Dep=0               244     Total obs                    1971
Obs with Dep=1              1727
=========================================================================
```

W17/C1 (iv)

(iv) Use equation

$$n^{-1}\sum_{i=1}^{n}\Big\{G\big[\hat{\beta}_0+\hat{\beta}_1 x_{i1}+\cdots+\hat{\beta}_{k-1}x_{ik-1}+\hat{\beta}_k(c_k+1)\big] \\ -G\big[\hat{\beta}_0+\hat{\beta}_1 x_{i1}+\cdots+\hat{\beta}_{k-1}x_{ik-1}+\hat{\beta}_k c_k\big]\Big\} \quad (17.17)$$

to estimate the sizes of the discrimination effects for probit and logit.

Note that (17.17) is the average partial effect for a discrete explanatory variable.

We consider all the variables but *white*. Instead, for each individual we consider two counterfactual scenarios: as if he or she was white and otherwise (new generated variables `white1` and `white0`), which we use to create two `groups` (`variables_white1` and `variables_white0`).

Then, we use the coefficients from two estimations (`coef_probit` and `coef_logit`) to sum all the variables multiplied by their respective coefficient.

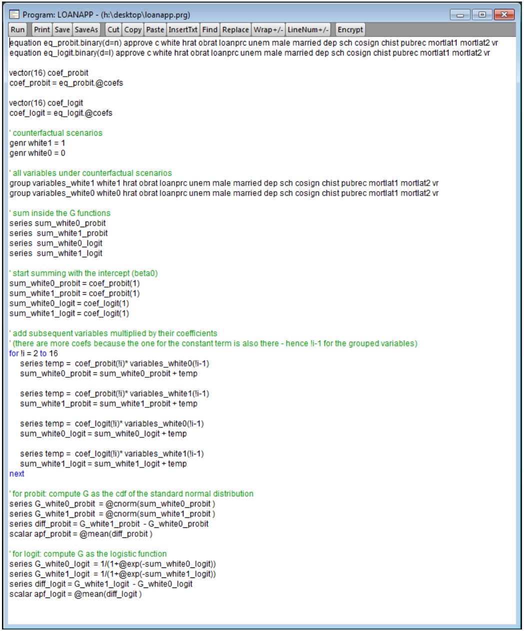
This gives us the arguments inside $G(\cdot)$ in (17.17).

To evaluate $G(\cdot)$ we need to apply the appropriate function for each model.

For probit, it is $\Phi(z)$, the cdf of the standard normal distribution;
for logit, it is $\frac{1}{1+\exp(-z)}$.

Finally, we subtract the vector with $G(\cdot)$ applied to the sum under the "nonwhites scenario" from that under the "whites scenario" and average out.

The obtained values are $APE_{probit} = 0.1042$ and $APE_{logit} = 0.1009$, hence quite similar.



```
equation eq_probit.binary(d=n) approve c white hrat obrat loanprc unem male married dep sch cosign chist pubrec mortlat1 mortlat2 vr
equation eq_logit.binary(d=l) approve c white hrat obrat loanprc unem male married dep sch cosign chist pubrec mortlat1 mortlat2 vr

vector(16) coef_probit
coef_probit = eq_probit.@coefs

vector(16) coef_logit
coef_logit = eq_logit.@coefs

' counterfactual scenarios
genr white1 = 1
genr white0 = 0

' all variables under counterfactual scenarios
group variables_white1 white1 hrat obrat loanprc unem male married dep sch cosign chist pubrec mortlat1 mortlat2 vr
group variables_white0 white0 hrat obrat loanprc unem male married dep sch cosign chist pubrec mortlat1 mortlat2 vr

' sum inside the G functions
series sum_white0_probit
series  sum_white1_probit
series  sum_white0_logit
series  sum_white1_logit

' start summing with the intercept (beta0)
sum_white0_probit = coef_probit(1)
sum_white1_probit = coef_probit(1)
sum_white0_logit = coef_logit(1)
sum_white1_logit = coef_logit(1)

' add subsequent variables multiplied by their coefficients
' (there are more coefs because the one for the constant term is also there - hence !i-1 for the grouped variables)
for !i = 2 to 16
    series temp =  coef_probit(!i)* variables_white0(!i-1)
    sum_white0_probit = sum_white0_probit + temp

    series temp =  coef_probit(!i)* variables_white1(!i-1)
    sum_white1_probit = sum_white1_probit + temp

    series temp =  coef_logit(!i)* variables_white0(!i-1)
    sum_white0_logit = sum_white0_logit + temp

    series temp =  coef_logit(!i)* variables_white1(!i-1)
    sum_white1_logit = sum_white1_logit + temp
next

' for probit: compute G as the cdf of the standard normal distribution
series G_white0_probit  = @cnorm(sum_white0_probit )
series G_white1_probit  = @cnorm(sum_white1_probit )
series diff_probit = G_white1_probit  - G_white0_probit
scalar apf_probit = @mean(diff_probit )

' for logit: compute G as the logistic function
series G_white0_logit  = 1/(1+@exp(-sum_white0_logit))
series G_white1_logit  = 1/(1+@exp(-sum_white1_logit))
series diff_logit = G_white1_logit  - G_white0_logit
scalar apf_logit = @mean(diff_logit )
```