Hand calculation

T_L T_R

i=0 i=1 i=2 i=3 i=4 i=5

$T_L = 1$ $T_R = 0$ $T_I = 0$

$N = 6$ lattice nodes

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \qquad \alpha = \frac{k}{\rho c_p} = 0.25$$

FDM

D1Q3

2 0 1

$c_2 = -1$ $c_0 = 0$ $c_1 = 1$

$w_2 = \frac{1}{6}$ $w_0 = 4/6$ $w_1 = 1/6$

$\Delta x = \Delta t = 1$

$$\boxed{T = \sum_{i=0}^{2} f_i}$$

$$\boxed{f_i^{eq} = w_i\, T(x,t)}$$

↳ Eqbrm distribution function

<u>Step 1</u> Educated guess of Distribution function ⟶ Compute Eqbrm Distribution function

$f_{dP\,Node}$ dP = 0, 1, 2

Node = 0, 1, 2, 3, 4, 5

	Digital particle (f)			f^{eq}			
Node	0	1	2	T(x,t)	0	1	2
0	1/3	1/3	1/3	1	4/6	1/6	1/6
1	0	1	-1	0	0	0	0
2	0	1	-1	0	0	0	0
3	0	1	-1	0	0	0	0
4	0	1	-1	0	0	0	0
5	0	1	-1	0	0	0	0

Step 2 : Collision

$$\boxed{f_i^*(x,t) = (1-\omega) f_i(x,t) + \omega f_i^{eq}(x,t)}$$

$$\boxed{\alpha = \tau - \frac{\Delta t}{2}} \qquad \boxed{\omega = \frac{\Delta t}{\tau}}$$

$$0.25 = \frac{1}{\omega} - \frac{1}{2} \Rightarrow \boxed{\omega = \frac{4}{3}}$$

~~Node~~	~~Particle 1~~ ~~$(1-\omega)f_i(x,t)+\omega f_i^{eq}(x,t)$~~
~~0~~	
~~1~~	
~~2~~	
~~3~~	
~~4~~	
~~5~~	

<u>Node 0</u>

$$f_{00}^* = (1-\omega) f_{00} + \omega f_{00}^{eq} = \left(-\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) + \tfrac{4}{3}\left(\tfrac{4}{6}\right) = -\tfrac{1}{9} + \tfrac{16}{18} = \tfrac{7}{9}$$

$$f_{10}^* = (1-\omega) f_{10} + \omega f_{10}^{eq} = \left(-\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) + \tfrac{4}{3}\left(\tfrac{1}{6}\right) = -\tfrac{1}{9} + \tfrac{4}{18} = \tfrac{1}{9}$$

$$f_{20}^* = (1-\omega) f_{20} + \omega f_{20}^{eq} = \left(-\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) + \tfrac{4}{3}\left(\tfrac{1}{6}\right) = -\tfrac{1}{9} + \tfrac{4}{18} = \tfrac{1}{9}$$

<u>Node 1</u>

$$f_{01}^* = (1-\omega) f_{01} + \omega f_{01}^{eq} = 0$$

$$f_{11}^* = (1-\omega) f_{11} + \omega f_{11}^{eq} = \left(-\tfrac{1}{3}\right)(1) + 0 = -\tfrac{1}{3}$$

$$f_{21}^* = (1-\omega) f_{21} + \omega f_{21}^{eq} = \left(-\tfrac{1}{3}\right)(-1) = \tfrac{1}{3}$$

<u>Node 2</u>

$$f_{02}^* = (1-\omega) f_{02} + \omega f_{02}^{eq} = 0$$

$$f_{12}^* = (1-\omega) f_{12} + \omega f_{12}^{eq} = \left(-\tfrac{1}{3}\right)(1) + 0 = -\tfrac{1}{3}$$

$$f_{22}^* = (1-\omega) f_{22} + \omega f_{22}^{eq} = \left(-\tfrac{1}{3}\right)(-1) = \tfrac{1}{3}$$

<u>Node 3</u>

$$f_{03}^* = 0$$

$$f_{13}^* = -\tfrac{1}{3}$$

Node 4

$f_{04}^* = 0$

$f_{14}^* = -\frac{1}{3}$

$f_{24}^* = \frac{1}{3}$

Node 5

$f_{05}^* = 0$

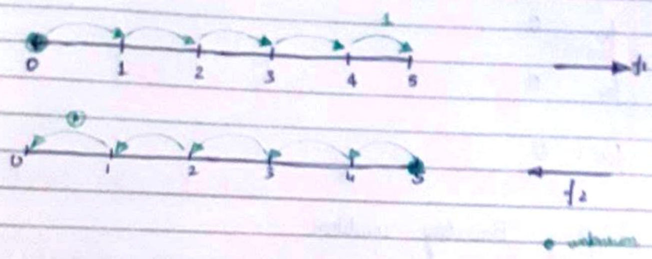
$f_{15}^* = -\frac{1}{3}$

$f_{25}^* = \frac{1}{3}$

Step 3: Streaming / Propagation

$$f_i(x + c_i\Delta t,\ t + \Delta t) = f_i^*(x, t)$$

f for Node



$f_{15} = f_{14}^* \Rightarrow f_{15} = -\frac{1}{3}$

$f_{14} = f_{13}^* \Rightarrow f_{14} = -\frac{1}{3}$

$f_{13} = f_{12}^* \Rightarrow f_{13} = -\frac{1}{3}$

$f_{12} = f_{11}^* \Rightarrow f_{12} = -\frac{1}{3}$

$f_{11} = f_{10}^* \Rightarrow f_{11} = \frac{1}{9}$

f_{10} is unknown

$f_{20} = f_{21}^* \Rightarrow f_{20} = \frac{1}{3}$

$f_{21} = f_{22}^* \Rightarrow f_{21} = \frac{1}{3}$

$f_{22} = f_{23}^* \Rightarrow f_{22} = \frac{1}{3}$

$f_{23} = f_{24}^* \Rightarrow f_{23} = \frac{1}{3}$

$f_{24} = f_{25}^* \Rightarrow f_{24} = \frac{1}{3}$

f_{25} is unknown

~~Applying Boundary condition~~

~~T(x,t)~~

$f_{00} = 7/9$

$f_{01} = 0$

$f_{02} = 0$

$f_{03} = 0$

$f_{04} = 0$

$f_{05} = 0$

Applying Boundary condition

$$T = \sum_{i=0}^{2} f_i$$

$$f_i^{eq} = w_i T(x,t)$$

$$\sum_i f_i^{eq} = \sum w_i T(x,t)$$

$$\sum_i f_i^{eq} = T(x,t) = \sum f_i$$

$$\boxed{\sum_{i=0}^{2} (f_i^{eq} - f_i) = 0}$$

+ Node 0

$f_{dP\,node}$

$$f_{00}^{eq} - f_{00} + f_{10}^{eq} - f_{10} + f_{20}^{eq} - f_{20} = 0$$

$$f_{00}^{eq} + f_{10}^{eq} + f_{20}^{eq} = f_{00} + f_{10} + f_{20} \implies \boxed{f_{10} = T(0) - f_{00} - f_{20}}$$

$$1 = \frac{7}{9} + f_{10} + \frac{1}{3}$$

$$f_{10} = 1 - \frac{7}{9} - \frac{1}{3}$$

At Node 5

$$f_{05}^{eq} + f_{15}^{eq} + f_{25}^{eq} = f_{15} + f_{25} + f_{05}$$

$$0 = -\tfrac{1}{3} + f_{25} + 0$$

$$\boxed{f_{25} = \tfrac{1}{3}} \qquad \boxed{f_{25} = T(L) - f_{15} - f_{05}}$$

Step 4 Compute Macroscopic

$$T(x,t) = \sum_{i=0}^{2} f_i$$

for each Node

$$T(0) = f_{00} + f_{10} + f_{20} = \tfrac{7}{9} - \tfrac{1}{9} + \tfrac{1}{3} = 1$$

$$T(1) = f_{01} + f_{11} + f_{21} = 0 + \tfrac{1}{9} + \tfrac{1}{3} = \tfrac{4}{9}$$

$$T(2) = f_{02} + f_{12} + f_{22} = 0 - \tfrac{1}{3} + \tfrac{1}{3} = 0$$

$$T(3) = f_{03} + f_{13} + f_{23} = 0 - \tfrac{1}{3} + \tfrac{1}{3} = 0$$

$$T(4) = f_{04} + f_{14} + f_{24} = 0 - \tfrac{1}{3} + \tfrac{1}{3} = 0$$

$$T(5) = f_{05} + f_{15} + f_{25} = 0 - \tfrac{1}{3} + \tfrac{1}{3} = 0$$

This completes 1 Iteration

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Algorithm

(1) Make educated guess of Distribution function using macroscopic property $\sum f_i = T$

(2) Compute Equilibrium Distribution function $f_i^{eq} = W_i T(x,t)$

(3) Perform Collision step

(4) Perform Streaming/Propogation step
Apply Boundary condition & Flux balance

(5) Compute Macroscopic property using Distribution function $T = \sum f_i$

(6) Repeat from step 2 till prediction for certain time