D3Q15_General_diffusion_solver_Dirichlet

December 5, 2023

[2]:
```
import numpy as np
import matplotlib.pyplot as plt
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[4]:
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##physical dimension of the cube
Lx=1
Ly=1
Lz=1
##Number of grid points in z -direction
Nz=50
##Number of grid points in x-direction
Nx=50
##Number of grid points in y-direction
Ny=50
##grid spacing
dx=Lx/(Nx-1)
dy=Ly/(Ny-1)
dz=Lz/(Nz-1)
##Creating grids
x=np.linspace(0,Lx,Nx)
y=np.linspace(0,Ly,Ny)
z=np.linspace(0,Lz,Nz)
[Z,X,Y]=np.meshgrid(z,x,y)
##Weights of digital particles
##15 digital particles
w=np.zeros(15)
##For rest digital particle
w[0]=16/72
##For digital particle on center of faces
for i in np.arange(1,7):
    w[i]=8/72
##For digital particle on corners
for i in np.arange(7,15):
    w[i]=1/72
##Macroscopic property temperature which is to be predicted
T=np.zeros((Nz,Ny,Nx))
##Boundary conditions
##Top face boundary
##Temperature at top boundary
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```
T_top=0
T[Nz-1,:,:]=T_top
##Bottom face boundary
##Temperature at bottom boundary
T_bot=1
T[0,:,:]=T_bot
##North face boundary
##Temperature at north boundary
Tn=0
T[1:Nz-1,0,:]=Tn
##South face boundary
##Temperature at south boundary
Ts=0
T[1:Nz-1,Ny-1,:]=Ts
##West face boundary
##Temperature at west boundary
Tw=0
T[1:Nz-1,1:Ny-1,0]=Tw
##East face boundary
##Temperature at east boundary
Te=0
T[1:Nz-1,1:Ny-1,Nx-1]=Te
##Initial condition
##Temperature specified at internal of domain
##Initial temperature of domain i.e at t=0
Ti=0
T[1:Nz-1,1:Nx-1,1:Nx-1]=Ti
##Initialise equilibrium particle distribution function
f0eq=np.zeros((Nz,Ny,Nx))
f1eq=np.zeros((Nz,Ny,Nx))
f2eq=np.zeros((Nz,Ny,Nx))
f3eq=np.zeros((Nz,Ny,Nx))
f4eq=np.zeros((Nz,Ny,Nx))
f5eq=np.zeros((Nz,Ny,Nx))
f6eq=np.zeros((Nz,Ny,Nx))
f7eq=np.zeros((Nz,Ny,Nx))
f8eq=np.zeros((Nz,Ny,Nx))
f9eq=np.zeros((Nz,Ny,Nx))
f10eq=np.zeros((Nz,Ny,Nx))
f11eq=np.zeros((Nz,Ny,Nx))
f12eq=np.zeros((Nz,Ny,Nx))
f13eq=np.zeros((Nz,Ny,Nx))
f14eq=np.zeros((Nz,Ny,Nx))
##Computing equilibrium distribution function
f0eq=w[0]*T
f1eq=w[1]*T
f2eq=w[2]*T
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```
f3eq=w[3]*T
f4eq=w[4]*T
f5eq=w[5]*T
f6eq=w[6]*T
f7eq=w[7]*T
f8eq=w[8]*T
f9eq=w[9]*T
f10eq=w[10]*T
f11eq=w[11]*T
f12eq=w[12]*T
f13eq=w[13]*T
f14eq=w[14]*T
##Initiliase particle distribution function
##D3Q15
f0=np.zeros((Nz,Ny,Nx))
f1=np.zeros((Nz,Ny,Nx))
f2=np.zeros((Nz,Ny,Nx))
f3=np.zeros((Nz,Ny,Nx))
f4=np.zeros((Nz,Ny,Nx))
f5=np.zeros((Nz,Ny,Nx))
f6=np.zeros((Nz,Ny,Nx))
f7=np.zeros((Nz,Ny,Nx))
f8=np.zeros((Nz,Ny,Nx))
f9=np.zeros((Nz,Ny,Nx))
f10=np.zeros((Nz,Ny,Nx))
f11=np.zeros((Nz,Ny,Nx))
f12=np.zeros((Nz,Ny,Nx))
f13=np.zeros((Nz,Ny,Nx))
f14=np.zeros((Nz,Ny,Nx))
##Initial value of particle distribution function
f0=f0eq
f1=f1eq
f2=f2eq
f3=f3eq
f4=f4eq
f5=f5eq
f6=f6eq
f7=f7eq
f8=f8eq
f9=f9eq
f10=f10eq
f11=f11eq
f12=f12eq
f13=f13eq
f14=f14eq
##Collision of digital particles
##collision parameter
```

```
omega=1.2
##Initiliase post collision values of digital particles
f0c=np.zeros((Nz,Ny,Nx))
f1c=np.zeros((Nz,Ny,Nx))
f2c=np.zeros((Nz,Ny,Nx))
f3c=np.zeros((Nz,Ny,Nx))
f4c=np.zeros((Nz,Ny,Nx))
f5c=np.zeros((Nz,Ny,Nx))
f6c=np.zeros((Nz,Ny,Nx))
f7c=np.zeros((Nz,Ny,Nx))
f8c=np.zeros((Nz,Ny,Nx))
f9c=np.zeros((Nz,Ny,Nx))
f10c=np.zeros((Nz,Ny,Nx))
f11c=np.zeros((Nz,Ny,Nx))
f12c=np.zeros((Nz,Ny,Nx))
f13c=np.zeros((Nz,Ny,Nx))
f14c=np.zeros((Nz,Ny,Nx))
##Collision of digital particles
f0c=(1-omega)*f0+(omega*f0eq)
f1c=(1-omega)*f1+(omega*f1eq)
f2c=(1-omega)*f2+(omega*f2eq)
f3c=(1-omega)*f3+(omega*f3eq)
f4c=(1-omega)*f4+(omega*f4eq)
f5c=(1-omega)*f5+(omega*f5eq)
f6c=(1-omega)*f6+(omega*f6eq)
f7c=(1-omega)*f7+(omega*f7eq)
f8c=(1-omega)*f8+(omega*f8eq)
f9c=(1-omega)*f9+(omega*f9eq)
f10c=(1-omega)*f10+(omega*f10eq)
f11c=(1-omega)*f11+(omega*f11eq)
f12c=(1-omega)*f12+(omega*f12eq)
f13c=(1-omega)*f13+(omega*f13eq)
f14c=(1-omega)*f14+(omega*f14eq)
##Streaming of digital particles

##particle number 0 is at rest
f0=f0c
##digital particle numbered from 1,2,3,...6 located on faces

##particle number 3 which is moving upwards towards top boundary
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny):
        for j in np.arange(0,Nx):
            f3[k+1,i,j]=f3c[k,i,j]

##particle number 4 which is moving downwards towards bottom boundary
for k in np.arange(0,Nz-1):
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```
    for i in np.arange(0,Ny):
        for j in np.arange(0,Nx):
            f4[k,i,j]=f4c[k+1,i,j]

##digital particle 1 which moves on x-y plane with unit velocity in x-direction
for k in np.arange(0,Nz):
    for i in np.arange(0,Ny):
        for j in np.arange(0,Nx-1):
            f1[k,i,j+1]=f1c[k,i,j]

##digital particle 2 moves on x-y plane in negative x-direction
for k in np.arange(0,Nz):
    for i in np.arange(0,Ny):
        for j in np.arange(0,Nx-1):
            f2[k,i,j]=f2c[k,i,j+1]

##digital particle 6 which moves on x-y plane upwards in positive y -direction
for k in np.arange(0,Nz):
    for j in np.arange(0,Nx):
        for i in np.arange(0,Ny-1):
            f6[k,i,j]=f6c[k,i+1,j]

##digital particle 5 which moves on x-y plane upwards in negative y -direction
for k in np.arange(0,Nz):
    for j in np.arange(0,Nx):
        for i in np.arange(0,Ny-1):
            f5[k,i+1,j]=f5c[k,i,j]

##digital particle 7
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f7[k+1,i+1,j+1]=f7c[k,i,j]

##digital particle 12
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f12[k+1,i,j]=f12c[k,i+1,j+1]


##digital particle 9
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f9[k+1,i,j+1]=f9c[k,i+1,j]
```

```
##digital particle 14
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f14[k+1,i+1,j]=f14c[k,i,j+1]

##digital particle 11
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f11[k,i+1,j+1]=f11c[k+1,i,j]

##Digital particle 8
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f8[k,i,j]=f8c[k+1,i+1,j+1]

##digital particle 13
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f13[k,i,j+1]=f13c[k+1,i+1,j]

##digital particle 10
for k in np.arange(0,Nz-1):
    for i in np.arange(0,Ny-1):
        for j in np.arange(0,Nx-1):
            f10[k,i+1,j]=f10c[k+1,i,j+1]


##Boundary conditions

##West boundaries
##unknowns are f1 f7 f9 f11 f13
for k in np.arange(1,Nz-1):
    j=0
    for i in np.arange(1,Ny-1):
        f1[k,i,j]=w[1]*Tw+ w[2]*Tw-f2[k,i,j]
        f7[k,i,j]=w[7]*Tw+w[8]*Tw-f8[k,i,j]
        f9[k,i,j]=w[9]*Tw+w[10]*Tw-f10[k,i,j]
        f11[k,i,j]=w[11]*Tw+w[12]*Tw-f12[k,i,j]
      ␣
↪f13[k,i,j]=Tw*(1-w[1]-w[2]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f3[k,i,j]+f4[k,i,j]


##East boundary
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##unkown df are f2,f8,f10,f12,f14
for k in np.arange(1,Nz-1):
    j=Nx-1
    for i in np.arange(1,Ny-1):
        f2[k,i,j]=w[1]*Te+ w[2]*Te-f1[k,i,j]
        f8[k,i,j]=w[7]*Te+w[8]*Te-f7[k,i,j]
        f10[k,i,j]=w[9]*Te+w[10]*Te-f9[k,i,j]
        f12[k,i,j]=w[11]*Te+w[12]*Te-f11[k,i,j]
       ␣
 ↪f14[k,i,j]=Te*(1-w[1]-w[2]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f3[k,i,j]+f4[k,i,j]

##North boundary

##Interior lattice nodes
##unknown df are f5 ,f7 ,f10 , f11 , f14
for k in np.arange(1,Nz-1):
    i=0
    for j in np.arange(1,Nx-1):
        f5[k,i,j]=w[5]*Tn +w[6]*Tn-f6[k,i,j]
        f7[k,i,j]=w[7]*Tn+w[8]*Tn-f8[k,i,j]
        f10[k,i,j]=w[9]*Tn+w[10]*Tn-f9[k,i,j]
        f11[k,i,j]=w[11]*Tn+w[12]*Tn-f12[k,i,j]
       ␣
 ↪f14[k,i,j]=Tn*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i,j]

##corner lattice nodes
##Left corner lattice node
##f1,f5,f7,f9,f10,f11,f13,f14
for k in np.arange(1,Nz-1):
    i=0
    j=0
    f1[k,i,j]=w[1]*Tn+w[2]*Tn-f2[k,i,j]
    f5[k,i,j]=w[5]*Tn +w[6]*Tn-f6[k,i,j]
    f7[k,i,j]=w[7]*Tn+w[8]*Tn-f8[k,i,j]
    f11[k,i,j]=w[11]*Tn+w[12]*Tn-f12[k,i,j]
    f9[k,i,j]=0.5*((18/72)*Tn-f0[k,i,j]-f3[k,i,j])
    f10[k,i,j]=f9[k,i,j]
    f13[k,i,j]=0.5*((18/72)*Tn-f4[k,i,j])
    f14[k,i,j]=f13[k,i,j]
##Right corner lattice node
##f2,f5,f7,f8,f10,f11,f12,f14
for k in np.arange(1,Nz-1):
    i=0
    j=Nx-1
    f2[k,i,j]=w[1]*Tn+w[2]*Tn-f1[k,i,j]
    f5[k,i,j]=w[5]*Tn +w[6]*Tn-f6[k,i,j]
    f10[k,i,j]=w[9]*Tn +w[10]*Tn-f9[k,i,j]
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    f14[k,i,j]=w[13]*Tn+w[14]*Tn-f13[k,i,j]
    f7[k,i,j]=0.5*((18/72)*Tn-f0[k,i,j]-f3[k,i,j])
    f8[k,i,j]=f7[1,0,5]
    f11[k,i,j]=0.5*((18/72)*Tn-f4[k,i,j])
    f12[k,i,j]=f11[k,i,j]


##South boundary

##Interior lattice nodes
##unknown vdf f6,f8,f9,f12,f13
for k in np.arange(1,Nz-1):
    i=Ny-1
    for j in np.arange(1,Nx-1):
        f6[k,i,j]=w[5]*Ts +w[6]*Ts-f5[k,i,j]
        f8[k,i,j]=w[7]*Ts+w[8]*Ts-f7[k,i,j]
        f9[k,i,j]=w[9]*Ts+w[10]*Ts-f10[k,i,j]
        f12[k,i,j]=w[11]*Ts+w[12]*Ts-f11[k,i,j]
        ␣
 ↪f13[k,i,j]=Ts*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i,j]

##corner lattice nodes
##Left corner node
##f1,f6,f7,f8,f9,f11,f12,f13
for k in np.arange(1,Nz-1):
    i=Ny-1
    j=0
    f1[k,i,j]=w[1]*Ts+w[2]*Ts-f2[k,i,j]
    f6[k,i,j]=w[5]*Ts +w[6]*Ts-f5[k,i,j]
    f9[k,i,j]=w[9]*Ts +w[10]*Ts-f10[k,i,j]
    f13[k,i,j]=w[13]*Ts+w[14]*Ts-f14[k,i,j]
    f7[k,i,j]=0.5*((18/72)*Ts-f0[k,i,j]-f3[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f11[k,i,j]=0.5*((18/72)*Ts-f4[k,i,j])
    f12[k,i,j]=f11[k,i,j]

##Right corner node
##f2,f6,f8,f9,f10,f12,f13,f14
for k in np.arange(1,Nz-1):
    i=Ny-1
    j=Nx-1
    f2[k,i,j]=w[1]*Ts+w[2]*Ts-f1[k,i,j]
    f6[k,i,j]=w[5]*Ts +w[6]*Ts-f5[k,i,j]
    f8[k,i,j]=w[7]*Ts+w[8]*Ts-f7[k,i,j]
    f12[k,i,j]=w[11]*Ts+w[12]*Ts-f11[k,i,j]
    f9[k,i,j]=0.5*((18/72)*Ts-f0[k,i,j]-f3[k,i,j])
    f10[k,i,j]=f9[k,i,j]
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    f13[k,i,j]=0.5*((18/72)*Ts-f4[k,i,j])
    f14[k,i,j]=f13[k,i,j]

##Top boundary

##Inner lattice nodes
##Unknown distribution functions are f4,f8,f10,f11,,f13
k=Nz-1
for i in np.arange(1,Ny-1):
    for j in np.arange(1,Nx-1):
        f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
        f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
        f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
        f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
      ␣
 ↪f13[k,i,j]=T_top*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i



##Top north
##Interior
##unknownf4,f5,f7,f8,f10,f11,f13,f14
k=Nz-1
i=0
for j in np.arange(1,Nx-1):
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f5[k,i,j]=w[5]*T_top +w[6]*T_top-f6[k,i,j]
    f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
    f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
    f7[k,i,j]=0.5*((18/72)*T_top-f0[k,0,1]-f1[k,i,j])
    f8[k,i,j]=f7[5,0,1]
    f13[k,i,j]=0.5*((18/72)*T_top-f2[k,i,j])
    f14[k,i,j]=f13[k,i,j]

##Left corner north lattice node
##unknown df are f1,f4,f5,f7,f8,f9,f10,f11,f13,f14
k=Nz-1
i=0
j=0
f1[k,i,j]=w[1]*T_top+w[2]*T_top-f2[k,i,j]
f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
f5[k,i,j]=w[5]*T_top +w[6]*T_top-f6[k,i,j]
f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_top-f0[k,i,j])
f8[k,i,j]=f7[k,i,j]
f9[k,i,j]=0.5*((6/72)*T_top)
f10[k,i,j]=f9[k,i,j]
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f13[k,i,j]=0.5*((6/72)*T_top)
f14[k,i,j]=f13[k,i,j]

##Right corner north lattice node
##unknown df are f2,f4,f5,f7,f8,f10,f11,f13,f14
k=Nz-1
i=0
j=5
f2[k,i,j]=w[1]*T_top+w[2]*T_top-f1[k,i,j]
f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
f5[k,i,j]=w[5]*T_top +w[6]*T_top-f6[k,i,j]
f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_top-f0[k,i,j])
f8[k,i,j]=f7[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_top)
f12[k,i,j]=f11[k,i,j]
f13[k,i,j]=0.5*((6/72)*T_top)
f14[k,i,j]=f13[k,i,j]


##Top south
##unknowns f4,f6,f8,f9,f10,f11,f12,f13
k=Nz-1
i=Ny-1
for j in np.arange(1,Nx-1):
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f6[k,i,j]=w[5]*T_top +w[6]*T_top-f5[k,i,j]
    f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
    f13[k,i,j]=w[14]*T_top+w[13]*T_top-f14[k,i,j]
    f9[k,i,j]=0.5*((18/72)*T_top-f0[k,i,j]-f1[k,i,j])
    f10[k,i,j]=f9[k,i,j]
    f11[k,i,j]=0.5*((18/72)*T_top-f2[k,i,j])
    f12[k,i,j]=f11[k,i,j]

##Left corner lattice node
##unknown df are f1,f4,f6,f7,f8,f9,f10,f11,f12,f13
k=Nz-1
i=Ny-1
j=0
f1[k,i,j]=w[1]*T_top+w[2]*T_top-f2[k,i,j]
f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
f6[k,i,j]=w[5]*T_top +w[6]*T_top-f5[k,i,j]
f13[k,i,j]=w[14]*T_top+w[13]*T_top-f14[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_top-f0[k,5,0])
f8[k,i,j]=f7[k,i,j]
f9[k,i,j]=0.5*((6/72)*T_top)
f10[k,i,j]=f9[k,i,j]
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f11[k,i,j]=0.5*((6/72)*T_top)
f12[k,i,j]=f11[k,i,j]

##Right corner lattice node
##unknown df are f2,f4,f6,f8,f9,f10,f11,f12,f13,f14
k=Nz-1
i=Ny-1
j=Nx-1
f2[k,i,j]=w[1]*T_top+w[2]*T_top-f1[k,i,j]
f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
f6[k,i,j]=w[5]*T_top +w[6]*T_top-f5[k,i,j]
f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
f9[k,i,j]=0.5*((10/72)*T_top-f0[k,i,j])
f10[k,i,j]=f9[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_top)
f12[k,i,j]=f11[k,i,j]
f13[k,i,j]=0.5*((6/72)*T_top)
f14[k,i,j]=f13[k,i,j]

##Top west
##unknown distribution functions
##f1,f4,f7,f8,f9,f10,f11,f13
k=Nz-1
j=0
for i in np.arange(1,Ny-1):
    f1[k,i,j]=w[1]*T_top+w[2]*T_top-f2[k,i,j]
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
    f13[k,i,j]=w[14]*T_top+w[13]*T_top-f14[k,i,j]
    f7[k,i,j]=0.5*((18/72)*T_top-f0[k,i,j]-f5[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f9[k,i,j]=0.5*((18/72)*T_top-f6[5,1,0])
    f10[k,i,j]=f9[k,i,j]


##Top East
##unknown df f2,f4,f8,f10,f11,f12,f13,f14
k=Nz-1
j=Ny-1
for i in np.arange(1,Nx-1):
    f2[k,i,j]=w[1]*T_top+w[2]*T_top-f1[k,i,j]
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
    f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
    f11[k,i,j]=0.5*((18/72)*T_top-f0[k,i,j]-f5[k,i,j])
    f12[k,i,j]=f11[5,1,5]
    f13[k,i,j]=0.5*((18/72)*T_top-f6[k,i,j])
```

```
    f14[k,i,j]=f13[k,i,j]



##Bottom boundary

##Inner lattice nodes
##unknown distribution functions are f3,f7,f9,f12,f14
k=0
for i in np.arange(1,Ny-1):
    for j in np.arange(1,Nx-1):
        f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
        f7[k,i,j]=w[7]*T_bot+w[8]*T_bot-f8[k,i,j]
        f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
        f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
        ␣
 ↪f14[k,i,j]=T_bot*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i


##Bottom north
##Interior lattice nods
##unknown distribution functions are f3,f5,f7,f9,f10,f11,f12,f14
k=0
i=0
for j in np.arange(1,Nx-1):
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f5[k,i,j]=w[5]*T_bot +w[6]*T_bot-f6[k,i,j]
    f7[k,i,j]=w[7]*T_bot+ w[8]*T_bot-f8[k,i,j]
    f14[k,i,j]=w[14]*T_bot+w[13]*T_bot-f13[k,i,j]
    f9[k,i,j]=0.5*((18/72)*T_bot-f0[k,i,j]-f1[k,i,j])
    f10[k,i,j]=f9[k,i,j]
    f11[k,i,j]=0.5*((18/72)*T_bot-f2[k,i,j])
    f12[k,i,j]=f11[k,i,j]

##left corner lattice node
##unknown df is f1, f3,f5,f7,f9,f10,f11,f12,f13,f14
k=0
i=0
j=0
f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
f5[k,i,j]=w[5]*T_bot +w[6]*T_bot-f6[k,i,j]
f7[k,i,j]=w[7]*T_bot+w[8]*T_bot-f8[k,i,j]
f9[k,i,j]=0.5*((10/72)*T_bot-f0[k,i,j])
f10[k,i,j]=f9[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_bot)
f12[k,i,j]=f11[k,i,j]
```

```
f13[k,i,j]=0.5*((6/72)*T_bot)
f14[k,i,j]=f13[k,i,j]

##right corner lattice node
##unknown df is f2,f3,f5,f7,f8,f9,f10,f11,f12,f14
k=0
i=0
j=Nx-1
f2[k,i,j]=w[1]*T_bot+w[2]*T_bot-f1[k,i,j]
f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
f5[k,i,j]=w[5]*T_bot +w[6]*T_bot-f6[k,i,j]
f14[k,i,j]=w[11]*T_bot+w[12]*T_bot-f13[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_bot-f0[k,i,j])
f8[k,i,j]=f7[k,i,j]
f9[k,i,j]=0.5*((6/72)*T_bot)
f10[k,i,j]=f9[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_bot)
f12[k,i,j]=f11[k,i,j]


##Bottom south

##Interior lattice nodes
##f3,f6,f7,f8,f9,f12,f13,f14
k=0
i=Ny-1
for j in np.arange(1,Nx-1):
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f6[k,i,j]=w[5]*T_bot +w[6]*T_bot-f5[k,i,j]
    f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
    f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
    f7[k,i,j]=0.5*((18/72)*T_bot-f0[0,5,1]-f1[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f13[k,i,j]=0.5*((18/72)*T_bot-f2[k,i,j])
    f14[k,i,j]=f13[k,i,j]


##Bottom south left corner lattice node
##unknown vdf is f1,f3,f6,f7,f8,f9,f11,f12,f13,f14
k=0
i=Ny-1
j=0
f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
f6[k,i,j]=w[5]*T_bot +w[6]*T_bot-f5[k,i,j]
f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_bot-f0[k,5,0])
```

```
f8[k,i,j]=f7[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_bot)
f12[k,i,j]=f11[k,i,j]
f13[k,i,j]=0.5*((6/72)*T_bot)
f14[k,i,j]=f13[k,i,j]
##Bottom south right corner lattice node
##unknown df are f2,f3,f6,f7,f8,f9,f10,f12,f13,f14
k=0
i=Ny-1
j=Nx-1
f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
f6[k,i,j]=w[5]*T_bot +w[6]*T_bot-f5[k,i,j]
f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_bot-f0[k,i,j])
f8[k,i,j]=f7[k,i,j]
f9[k,i,j]=0.5*((6/72)*T_bot)
f10[k,i,j]=f9[k,i,j]
f13[k,i,j]=0.5*((6/72)*T_bot)
f14[k,i,j]=f13[k,i,j]

##Bottom west
##f1,f3,f7,f9,f11,f12,f13,f14
k=0
j=0
for i in np.arange(1,Ny-1):
    f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f7[k,i,j]=w[7]*T_bot+w[8]*T_bot-f8[k,i,j]
    f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
    f11[k,i,j]=0.5*((18/72)*T_bot-f0[k,i,j]-f5[k,i,j])
    f12[k,i,j]=f11[k,i,j]
    f13[k,i,j]=0.5*((18/72)*T_bot-f6[k,i,j])
    f14[k,i,j]=f13[k,i,j]



##Bottom east
#f2,f3,f7,f8,f9,f10,f12,f14
k=0
j=Nx-1
for i in np.arange(1,Ny-1):
    f2[k,i,j]=w[1]*T_bot+w[2]*T_bot-f1[k,i,j]
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
    f14[k,i,j]=w[13]*T_bot+w[14]*T_bot-f13[k,i,j]
    f7[k,i,j]=0.5*((18/72)*T_bot-f0[k,1,5]-f5[k,i,j])
```

```
    f8[k,i,j]=f7[k,i,j]
    f9[k,i,j]=0.5*((18/72)*T_bot-f6[0,1,5])
    f10[k,i,j]=f9[k,i,j]



##Compute macroscopic property
T=f0+f1+f2+f3+f4+f5+f6+f7+f8+f9+f10+f11+f12+f13+f14
```

[37]:

```
print(T)
```

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   0.00000000e+00  0.00000000e+00]]]
```

[5]:
```
iter=10000
while(iter>=1):
    ##Computing equilibrium distribution function
    f0eq=w[0]*T
    f1eq=w[1]*T
    f2eq=w[2]*T
    f3eq=w[3]*T
    f4eq=w[4]*T
    f5eq=w[5]*T
    f6eq=w[6]*T
    f7eq=w[7]*T
    f8eq=w[8]*T
    f9eq=w[9]*T
```

```
    f10eq=w[10]*T
    f11eq=w[11]*T
    f12eq=w[12]*T
    f13eq=w[13]*T
    f14eq=w[14]*T
    ##Collision of digital particles
    f0c=(1-omega)*f0+(omega*f0eq)
    f1c=(1-omega)*f1+(omega*f1eq)
    f2c=(1-omega)*f2+(omega*f2eq)
    f3c=(1-omega)*f3+(omega*f3eq)
    f4c=(1-omega)*f4+(omega*f4eq)
    f5c=(1-omega)*f5+(omega*f5eq)
    f6c=(1-omega)*f6+(omega*f6eq)
    f7c=(1-omega)*f7+(omega*f7eq)
    f8c=(1-omega)*f8+(omega*f8eq)
    f9c=(1-omega)*f9+(omega*f9eq)
    f10c=(1-omega)*f10+(omega*f10eq)
    f11c=(1-omega)*f11+(omega*f11eq)
    f12c=(1-omega)*f12+(omega*f12eq)
    f13c=(1-omega)*f13+(omega*f13eq)
    f14c=(1-omega)*f14+(omega*f14eq)
    ##Streaming of digital particles

    ##particle number 0 is at rest
    f0=f0c
    ##digital particle numbered from 1,2,3,...6 located on faces

    ##particle number 3 which is moving upwards towards top boundary
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny):
             for j in np.arange(0,Nx):
                f3[k+1,i,j]=f3c[k,i,j]

    ##particle number 4 which is moving downwards towards bottom boundary
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny):
            for j in np.arange(0,Nx):
                f4[k,i,j]=f4c[k+1,i,j]

    ##digital particle 1 which moves on x-y plane with unit velocity in␣
↪x-direction
    for k in np.arange(0,Nz):
        for i in np.arange(0,Ny):
            for j in np.arange(0,Nx-1):
                f1[k,i,j+1]=f1c[k,i,j]

    ##digital particle 2 moves on x-y plane in negative x-direction
```

```
    for k in np.arange(0,Nz):
        for i in np.arange(0,Ny):
            for j in np.arange(0,Nx-1):
                f2[k,i,j]=f2c[k,i,j+1]

    ##digital particle 6 which moves on x-y plane upwards in positive y-direction
    for k in np.arange(0,Nz):
        for j in np.arange(0,Nx):
            for i in np.arange(0,Ny-1):
                f6[k,i,j]=f6c[k,i+1,j]

    ##digital particle 5 which moves on x-y plane upwards in negative y-direction
    for k in np.arange(0,Nz):
        for j in np.arange(0,Nx):
            for i in np.arange(0,Ny-1):
                f5[k,i+1,j]=f5c[k,i,j]

    ##digital particle 7
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f7[k+1,i+1,j+1]=f7c[k,i,j]

    ##digital particle 12
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f12[k+1,i,j]=f12c[k,i+1,j+1]


    ##digital particle 9
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f9[k+1,i,j+1]=f9c[k,i+1,j]

    ##digital particle 14
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f14[k+1,i+1,j]=f14c[k,i,j+1]

    ##digital particle 11
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
```

```
            for j in np.arange(0,Nx-1):
                f11[k,i+1,j+1]=f11c[k+1,i,j]

    ##Digital particle 8
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f8[k,i,j]=f8c[k+1,i+1,j+1]

    ##digital particle 13
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f13[k,i,j+1]=f13c[k+1,i+1,j]

    ##digital particle 10
    for k in np.arange(0,Nz-1):
        for i in np.arange(0,Ny-1):
            for j in np.arange(0,Nx-1):
                f10[k,i+1,j]=f10c[k+1,i,j+1]


    ##Boundary conditions

    ##West boundaries
    ##unknowns are f1 f7 f9 f11 f13
    for k in np.arange(1,Nz-1):
        j=0
        for i in np.arange(1,Ny-1):
            f1[k,i,j]=w[1]*Tw+ w[2]*Tw-f2[k,i,j]
            f7[k,i,j]=w[7]*Tw+w[8]*Tw-f8[k,i,j]
            f9[k,i,j]=w[9]*Tw+w[10]*Tw-f10[k,i,j]
            f11[k,i,j]=w[11]*Tw+w[12]*Tw-f12[k,i,j]
            ␣
↪f13[k,i,j]=Tw*(1-w[1]-w[2]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f3[k,i,j]+f4[k,i,j]


    ##East boundary
    ##unkown df are f2,f8,f10,f12,f14
    for k in np.arange(1,Nz-1):
        j=Nx-1
        for i in np.arange(1,Ny-1):
            f2[k,i,j]=w[1]*Te+ w[2]*Te-f1[k,i,j]
            f8[k,i,j]=w[7]*Te+w[8]*Te-f7[k,i,j]
            f10[k,i,j]=w[9]*Te+w[10]*Te-f9[k,i,j]
            f12[k,i,j]=w[11]*Te+w[12]*Te-f11[k,i,j]
```

```
        ␣
↪f14[k,i,j]=Te*(1-w[1]-w[2]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f3[k,i,j]+f4[k,i,j]

    ##North boundary

    ##Interior lattice nodes
    ##unknown df are f5 ,f7 ,f10 , f11 , f14
    for k in np.arange(1,Nz-1):
        i=0
        for j in np.arange(1,Nx-1):
            f5[k,i,j]=w[5]*Tn +w[6]*Tn-f6[k,i,j]
            f7[k,i,j]=w[7]*Tn+w[8]*Tn-f8[k,i,j]
            f10[k,i,j]=w[9]*Tn+w[10]*Tn-f9[k,i,j]
            f11[k,i,j]=w[11]*Tn+w[12]*Tn-f12[k,i,j]
        ␣
↪f14[k,i,j]=Tn*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i,j]

    ##corner lattice nodes
    ##Left corner lattice node
    ##f1,f5,f7,f9,f10,f11,f13,f14
    for k in np.arange(1,Nz-1):
        i=0
        j=0
        f1[k,i,j]=w[1]*Tn+w[2]*Tn-f2[k,i,j]
        f5[k,i,j]=w[5]*Tn +w[6]*Tn-f6[k,i,j]
        f7[k,i,j]=w[7]*Tn+w[8]*Tn-f8[k,i,j]
        f11[k,i,j]=w[11]*Tn+w[12]*Tn-f12[k,i,j]
        f9[k,i,j]=0.5*((18/72)*Tn-f0[k,i,j]-f3[k,i,j])
        f10[k,i,j]=f9[k,i,j]
        f13[k,i,j]=0.5*((18/72)*Tn-f4[k,i,j])
        f14[k,i,j]=f13[k,i,j]
    ##Right corner lattice node
    ##f2,f5,f7,f8,f10,f11,f12,f14
    for k in np.arange(1,Nz-1):
        i=0
        j=Nx-1
        f2[k,i,j]=w[1]*Tn+w[2]*Tn-f1[k,i,j]
        f5[k,i,j]=w[5]*Tn +w[6]*Tn-f6[k,i,j]
        f10[k,i,j]=w[9]*Tn +w[10]*Tn-f9[k,i,j]
        f14[k,i,j]=w[13]*Tn+w[14]*Tn-f13[k,i,j]
        f7[k,i,j]=0.5*((18/72)*Tn-f0[k,i,j]-f3[k,i,j])
        f8[k,i,j]=f7[1,0,5]
        f11[k,i,j]=0.5*((18/72)*Tn-f4[k,i,j])
        f12[k,i,j]=f11[k,i,j]


    ##South boundary
```

```
    ##Interior lattice nodes
    ##unknown vdf f6,f8,f9,f12,f13
    for k in np.arange(1,Nz-1):
        i=Ny-1
        for j in np.arange(1,Nx-1):
            f6[k,i,j]=w[5]*Ts +w[6]*Ts-f5[k,i,j]
            f8[k,i,j]=w[7]*Ts+w[8]*Ts-f7[k,i,j]
            f9[k,i,j]=w[9]*Ts+w[10]*Ts-f10[k,i,j]
            f12[k,i,j]=w[11]*Ts+w[12]*Ts-f11[k,i,j]
            ␣
↪f13[k,i,j]=Ts*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i,j]

    ##corner lattice nodes
    ##Left corner node
    ##f1,f6,f7,f8,f9,f11,f12,f13
    for k in np.arange(1,Nz-1):
        i=Ny-1
        j=0
        f1[k,i,j]=w[1]*Ts+w[2]*Ts-f2[k,i,j]
        f6[k,i,j]=w[5]*Ts +w[6]*Ts-f5[k,i,j]
        f9[k,i,j]=w[9]*Ts +w[10]*Ts-f10[k,i,j]
        f13[k,i,j]=w[13]*Ts+w[14]*Ts-f14[k,i,j]
        f7[k,i,j]=0.5*((18/72)*Ts-f0[k,i,j]-f3[k,i,j])
        f8[k,i,j]=f7[k,i,j]
        f11[k,i,j]=0.5*((18/72)*Ts-f4[k,i,j])
        f12[k,i,j]=f11[k,i,j]

    ##Right corner node
    ##f2,f6,f8,f9,f10,f12,f13,f14
    for k in np.arange(1,Nz-1):
        i=Ny-1
        j=Nx-1
        f2[k,i,j]=w[1]*Ts+w[2]*Ts-f1[k,i,j]
        f6[k,i,j]=w[5]*Ts +w[6]*Ts-f5[k,i,j]
        f8[k,i,j]=w[7]*Ts+w[8]*Ts-f7[k,i,j]
        f12[k,i,j]=w[11]*Ts+w[12]*Ts-f11[k,i,j]
        f9[k,i,j]=0.5*((18/72)*Ts-f0[k,i,j]-f3[k,i,j])
        f10[k,i,j]=f9[k,i,j]
        f13[k,i,j]=0.5*((18/72)*Ts-f4[k,i,j])
        f14[k,i,j]=f13[k,i,j]

    ##Top boundary

    ##Inner lattice nodes
    ##Unknown distribution functions are f4,f8,f10,f11,,f13
    k=Nz-1
```

```
    for i in np.arange(1,Ny-1):
        for j in np.arange(1,Nx-1):
            f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
            f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
            f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
            f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
            ␣
↪f13[k,i,j]=T_top*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i



    ##Top north
    ##Interior
    ##unknownf4,f5,f7,f8,f10,f11,f13,f14
    k=Nz-1
    i=0
    for j in np.arange(1,Nx-1):
        f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
        f5[k,i,j]=w[5]*T_top +w[6]*T_top-f6[k,i,j]
        f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
        f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
        f7[k,i,j]=0.5*((18/72)*T_top-f0[k,0,1]-f1[k,i,j])
        f8[k,i,j]=f7[5,0,1]
        f13[k,i,j]=0.5*((18/72)*T_top-f2[k,i,j])
        f14[k,i,j]=f13[k,i,j]

    ##Left corner north lattice node
    ##unknown df are f1,f4,f5,f7,f8,f9,f10,f11,f13,f14
    k=Nz-1
    i=0
    j=0
    f1[k,i,j]=w[1]*T_top+w[2]*T_top-f2[k,i,j]
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f5[k,i,j]=w[5]*T_top +w[6]*T_top-f6[k,i,j]
    f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
    f7[k,i,j]=0.5*((10/72)*T_top-f0[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f9[k,i,j]=0.5*((6/72)*T_top)
    f10[k,i,j]=f9[k,i,j]
    f13[k,i,j]=0.5*((6/72)*T_top)
    f14[k,i,j]=f13[k,i,j]

    ##Right corner north lattice node
    ##unknown df are f2,f4,f5,f7,f8,f10,f11,f13,f14
    k=Nz-1
    i=0
    j=5
```

```
    f2[k,i,j]=w[1]*T_top+w[2]*T_top-f1[k,i,j]
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f5[k,i,j]=w[5]*T_top +w[6]*T_top-f6[k,i,j]
    f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
    f7[k,i,j]=0.5*((10/72)*T_top-f0[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f11[k,i,j]=0.5*((6/72)*T_top)
    f12[k,i,j]=f11[k,i,j]
    f13[k,i,j]=0.5*((6/72)*T_top)
    f14[k,i,j]=f13[k,i,j]


    ##Top south
    ##unknowns f4,f6,f8,f9,f10,f11,f12,f13
    k=Nz-1
    i=Ny-1
    for j in np.arange(1,Nx-1):
        f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
        f6[k,i,j]=w[5]*T_top +w[6]*T_top-f5[k,i,j]
        f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
        f13[k,i,j]=w[14]*T_top+w[13]*T_top-f14[k,i,j]
        f9[k,i,j]=0.5*((18/72)*T_top-f0[k,i,j]-f1[k,i,j])
        f10[k,i,j]=f9[k,i,j]
        f11[k,i,j]=0.5*((18/72)*T_top-f2[k,i,j])
        f12[k,i,j]=f11[k,i,j]

    ##Left corner lattice node
    ##unknown df are f1,f4,f6,f7,f8,f9,f10,f11,f12,f13
    k=Nz-1
    i=Ny-1
    j=0
    f1[k,i,j]=w[1]*T_top+w[2]*T_top-f2[k,i,j]
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f6[k,i,j]=w[5]*T_top +w[6]*T_top-f5[k,i,j]
    f13[k,i,j]=w[14]*T_top+w[13]*T_top-f14[k,i,j]
    f7[k,i,j]=0.5*((10/72)*T_top-f0[k,5,0])
    f8[k,i,j]=f7[k,i,j]
    f9[k,i,j]=0.5*((6/72)*T_top)
    f10[k,i,j]=f9[k,i,j]
    f11[k,i,j]=0.5*((6/72)*T_top)
    f12[k,i,j]=f11[k,i,j]

    ##Right corner lattice node
    ##unknown df are f2,f4,f6,f8,f9,f10,f11,f12,f13,f14
    k=Nz-1
    i=Ny-1
    j=Nx-1
```

```
    f2[k,i,j]=w[1]*T_top+w[2]*T_top-f1[k,i,j]
    f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
    f6[k,i,j]=w[5]*T_top +w[6]*T_top-f5[k,i,j]
    f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
    f9[k,i,j]=0.5*((10/72)*T_top-f0[k,i,j])
    f10[k,i,j]=f9[k,i,j]
    f11[k,i,j]=0.5*((6/72)*T_top)
    f12[k,i,j]=f11[k,i,j]
    f13[k,i,j]=0.5*((6/72)*T_top)
    f14[k,i,j]=f13[k,i,j]

    ##Top west
    ##unknown distribution functions
    ##f1,f4,f7,f8,f9,f10,f11,f13
    k=Nz-1
    j=0
    for i in np.arange(1,Ny-1):
        f1[k,i,j]=w[1]*T_top+w[2]*T_top-f2[k,i,j]
        f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
        f11[k,i,j]=w[11]*T_top+w[12]*T_top-f12[k,i,j]
        f13[k,i,j]=w[14]*T_top+w[13]*T_top-f14[k,i,j]
        f7[k,i,j]=0.5*((18/72)*T_top-f0[k,i,j]-f5[k,i,j])
        f8[k,i,j]=f7[k,i,j]
        f9[k,i,j]=0.5*((18/72)*T_top-f6[5,1,0])
        f10[k,i,j]=f9[k,i,j]


    ##Top East
    ##unknown df f2,f4,f8,f10,f11,f12,f13,f14
    k=Nz-1
    j=Ny-1
    for i in np.arange(1,Nx-1):
        f2[k,i,j]=w[1]*T_top+w[2]*T_top-f1[k,i,j]
        f4[k,i,j]=w[4]*T_top +w[3]*T_top-f3[k,i,j]
        f8[k,i,j]=w[7]*T_top+w[8]*T_top-f7[k,i,j]
        f10[k,i,j]=w[9]*T_top+w[10]*T_top-f9[k,i,j]
        f11[k,i,j]=0.5*((18/72)*T_top-f0[k,i,j]-f5[k,i,j])
        f12[k,i,j]=f11[5,1,5]
        f13[k,i,j]=0.5*((18/72)*T_top-f6[k,i,j])
        f14[k,i,j]=f13[k,i,j]



    ##Bottom boundary

    ##Inner lattice nodes
    ##unknown distribution functions are f3,f7,f9,f12,f14
```

```
    k=0
    for i in np.arange(1,Ny-1):
        for j in np.arange(1,Nx-1):
            f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
            f7[k,i,j]=w[7]*T_bot+w[8]*T_bot-f8[k,i,j]
            f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
            f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
            ␣
↪f14[k,i,j]=T_bot*(1-w[5]-w[6]-w[7]-w[8]-w[9]-w[10]-w[11]-w[12])-(f0[k,i,j]+f1[k,i,j]+f2[k,i


    ##Bottom north
    ##Interior lattice nods
    ##unknown distribution functions are f3,f5,f7,f9,f10,f11,f12,f14
    k=0
    i=0
    for j in np.arange(1,Nx-1):
        f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
        f5[k,i,j]=w[5]*T_bot +w[6]*T_bot-f6[k,i,j]
        f7[k,i,j]=w[7]*T_bot+ w[8]*T_bot-f8[k,i,j]
        f14[k,i,j]=w[14]*T_bot+w[13]*T_bot-f13[k,i,j]
        f9[k,i,j]=0.5*((18/72)*T_bot-f0[k,i,j]-f1[k,i,j])
        f10[k,i,j]=f9[k,i,j]
        f11[k,i,j]=0.5*((18/72)*T_bot-f2[k,i,j])
        f12[k,i,j]=f11[k,i,j]

    ##left corner lattice node
    ##unknown df is f1, f3,f5,f7,f9,f10,f11,f12,f13,f14
    k=0
    i=0
    j=0
    f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f5[k,i,j]=w[5]*T_bot +w[6]*T_bot-f6[k,i,j]
    f7[k,i,j]=w[7]*T_bot+w[8]*T_bot-f8[k,i,j]
    f9[k,i,j]=0.5*((10/72)*T_bot-f0[k,i,j])
    f10[k,i,j]=f9[k,i,j]
    f11[k,i,j]=0.5*((6/72)*T_bot)
    f12[k,i,j]=f11[k,i,j]
    f13[k,i,j]=0.5*((6/72)*T_bot)
    f14[k,i,j]=f13[k,i,j]

    ##right corner lattice node
    ##unknown df is f2,f3,f5,f7,f8,f9,f10,f11,f12,f14
    k=0
    i=0
    j=Nx-1
```

```
f2[k,i,j]=w[1]*T_bot+w[2]*T_bot-f1[k,i,j]
f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
f5[k,i,j]=w[5]*T_bot +w[6]*T_bot-f6[k,i,j]
f14[k,i,j]=w[11]*T_bot+w[12]*T_bot-f13[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_bot-f0[k,i,j])
f8[k,i,j]=f7[k,i,j]
f9[k,i,j]=0.5*((6/72)*T_bot)
f10[k,i,j]=f9[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_bot)
f12[k,i,j]=f11[k,i,j]


##Bottom south

##Interior lattice nodes
##f3,f6,f7,f8,f9,f12,f13,f14
k=0
i=Ny-1
for j in np.arange(1,Nx-1):
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f6[k,i,j]=w[5]*T_bot +w[6]*T_bot-f5[k,i,j]
    f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
    f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
    f7[k,i,j]=0.5*((18/72)*T_bot-f0[0,5,1]-f1[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f13[k,i,j]=0.5*((18/72)*T_bot-f2[k,i,j])
    f14[k,i,j]=f13[k,i,j]


##Bottom south left corner lattice node
##unknown vdf is f1,f3,f6,f7,f8,f9,f11,f12,f13,f14
k=0
i=Ny-1
j=0
f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
f6[k,i,j]=w[5]*T_bot +w[6]*T_bot-f5[k,i,j]
f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
f7[k,i,j]=0.5*((10/72)*T_bot-f0[k,5,0])
f8[k,i,j]=f7[k,i,j]
f11[k,i,j]=0.5*((6/72)*T_bot)
f12[k,i,j]=f11[k,i,j]
f13[k,i,j]=0.5*((6/72)*T_bot)
f14[k,i,j]=f13[k,i,j]
##Bottom south right corner lattice node
##unknown df are f2,f3,f6,f7,f8,f9,f10,f12,f13,f14
k=0
```

```
    i=Ny-1
    j=Nx-1
    f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
    f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
    f6[k,i,j]=w[5]*T_bot +w[6]*T_bot-f5[k,i,j]
    f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
    f7[k,i,j]=0.5*((10/72)*T_bot-f0[k,i,j])
    f8[k,i,j]=f7[k,i,j]
    f9[k,i,j]=0.5*((6/72)*T_bot)
    f10[k,i,j]=f9[k,i,j]
    f13[k,i,j]=0.5*((6/72)*T_bot)
    f14[k,i,j]=f13[k,i,j]

    ##Bottom west
    ##f1,f3,f7,f9,f11,f12,f13,f14
    k=0
    j=0
    for i in np.arange(1,Ny-1):
        f1[k,i,j]=w[1]*T_bot+w[2]*T_bot-f2[k,i,j]
        f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
        f7[k,i,j]=w[7]*T_bot+w[8]*T_bot-f8[k,i,j]
        f9[k,i,j]=w[9]*T_bot+w[10]*T_bot-f10[k,i,j]
        f11[k,i,j]=0.5*((18/72)*T_bot-f0[k,i,j]-f5[k,i,j])
        f12[k,i,j]=f11[k,i,j]
        f13[k,i,j]=0.5*((18/72)*T_bot-f6[k,i,j])
        f14[k,i,j]=f13[k,i,j]



    ##Bottom east
    #f2,f3,f7,f8,f9,f10,f12,f14
    k=0
    j=Nx-1
    for i in np.arange(1,Ny-1):
        f2[k,i,j]=w[1]*T_bot+w[2]*T_bot-f1[k,i,j]
        f3[k,i,j]=w[4]*T_bot +w[3]*T_bot-f4[k,i,j]
        f12[k,i,j]=w[11]*T_bot+w[12]*T_bot-f11[k,i,j]
        f14[k,i,j]=w[13]*T_bot+w[14]*T_bot-f13[k,i,j]
        f7[k,i,j]=0.5*((18/72)*T_bot-f0[k,1,5]-f5[k,i,j])
        f8[k,i,j]=f7[k,i,j]
        f9[k,i,j]=0.5*((18/72)*T_bot-f6[0,1,5])
        f10[k,i,j]=f9[k,i,j]


    ##Compute macroscopic property
    T=f0+f1+f2+f3+f4+f5+f6+f7+f8+f9+f10+f11+f12+f13+f14
```

```
    print(iter)
    iter=iter-1
```

```
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7
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5
4
3
2
1
```
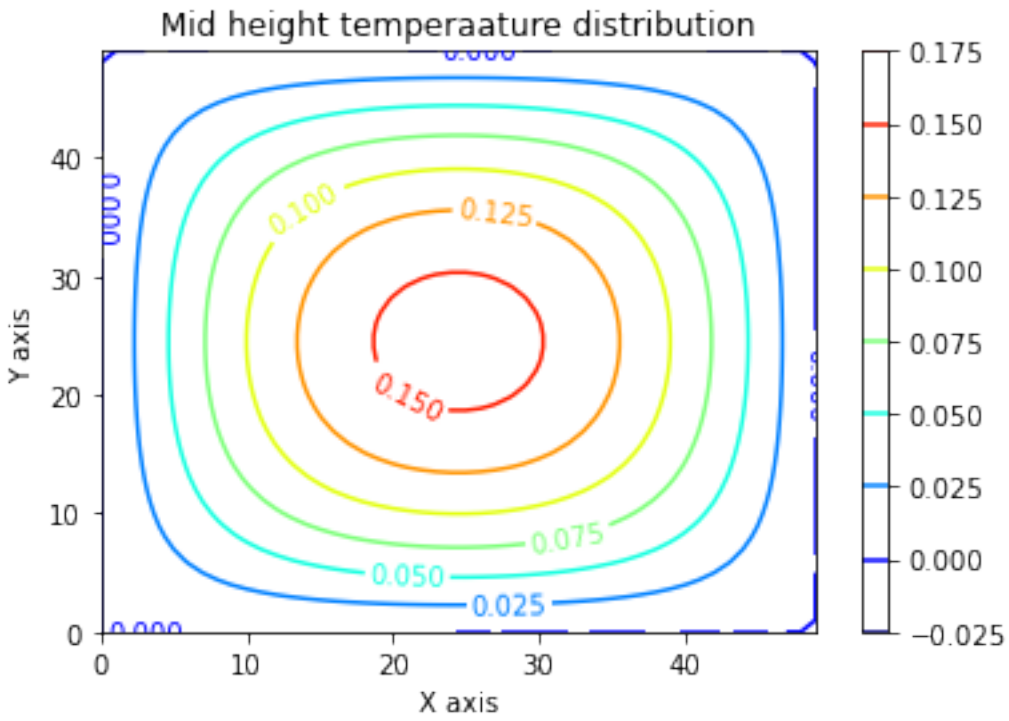
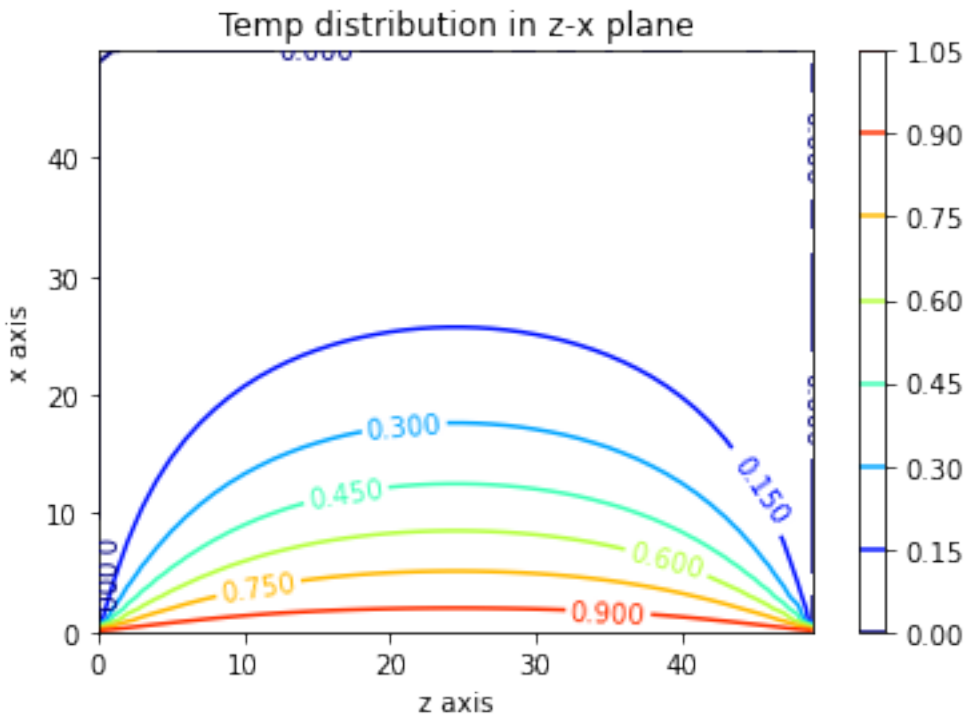

print(T)

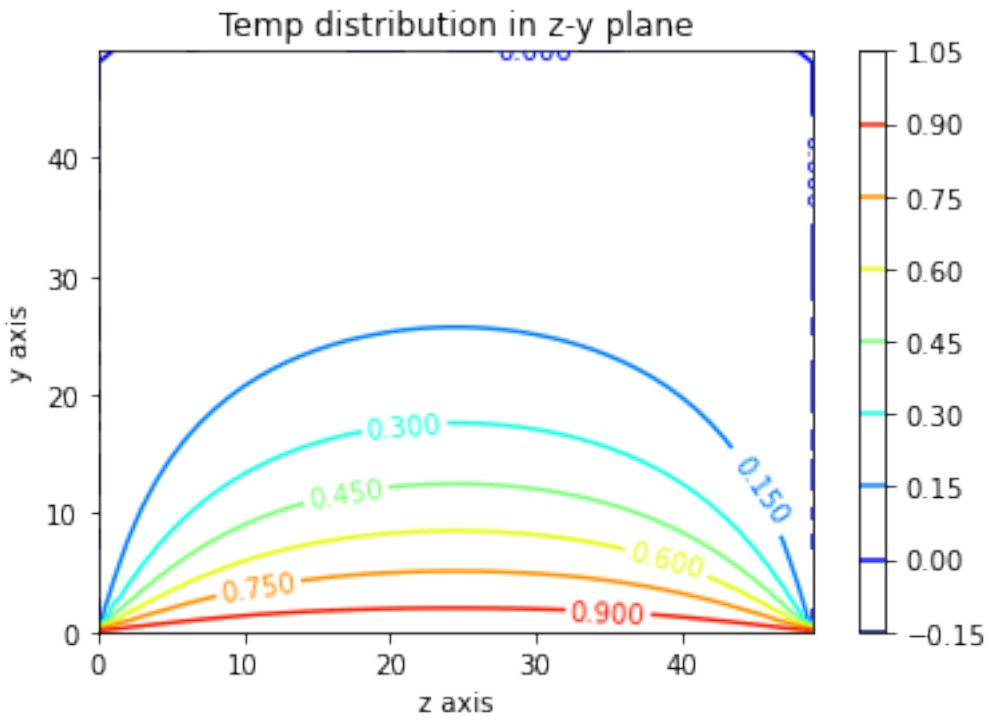
[6]:
```
cs=plt.contour(T[25,:,:],cmap='jet')
plt.clabel(cs, inline=1, fontsize=10)
plt.colorbar()
plt.xlabel('X axis')
plt.ylabel('Y axis')
plt.title('Mid height temperaature distribution')
plt.show()
```



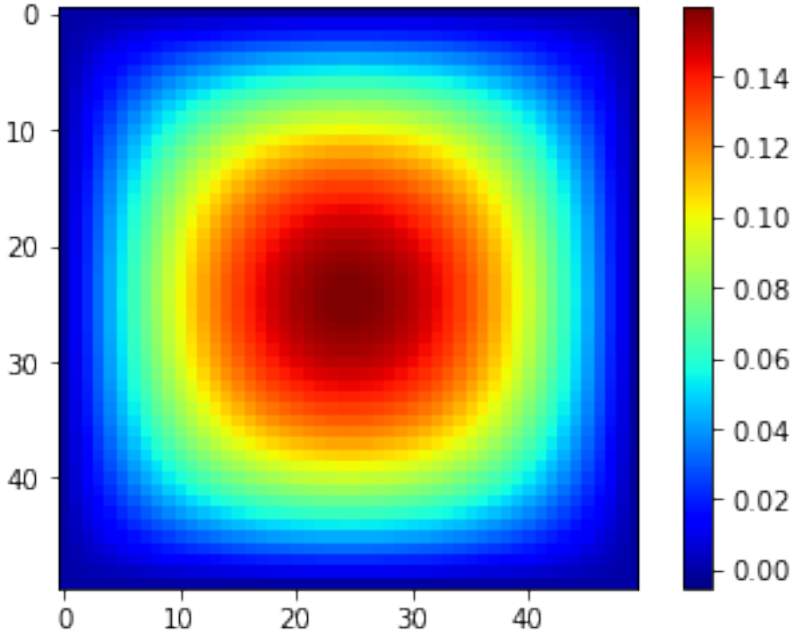
[11]:
```
cs=plt.contour(T[:,25,:],cmap='jet')
plt.clabel(cs,inline=1,fontsize=10)
plt.xlabel('z axis')
plt.ylabel('x axis')
plt.title('Temp distribution in z-x plane')
plt.colorbar()
plt.show()
```



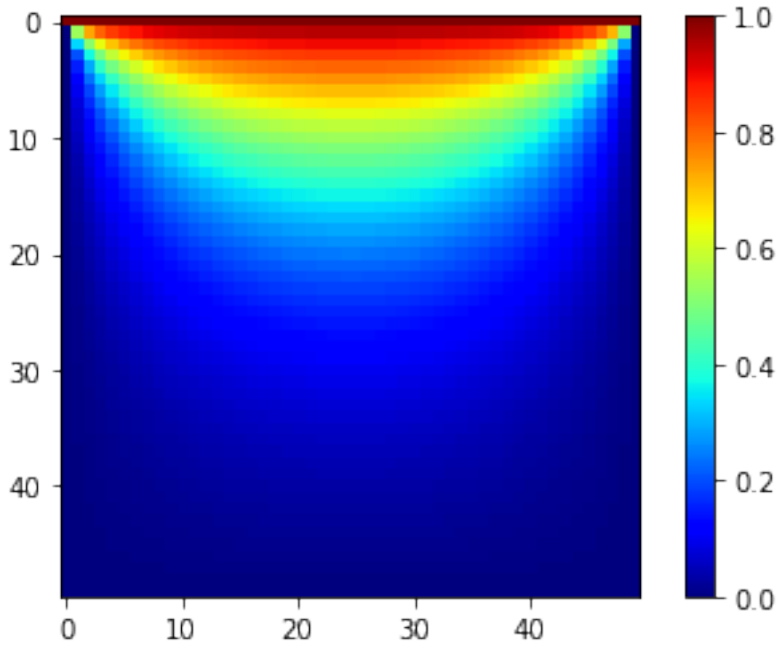
[12]:
```
cs=plt.contour(T[:,:,25],cmap='jet')
plt.clabel(cs,inline=1,fontsize=10)
plt.xlabel('z axis')
plt.ylabel('y axis')
plt.title('Temp distribution in z-y plane')
plt.colorbar()
plt.show()
```



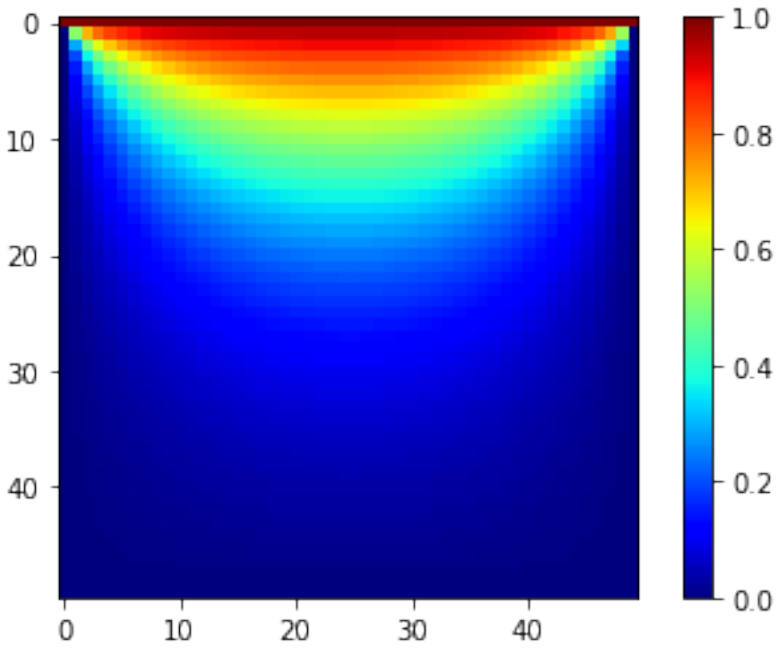
[19]:
```
plt.imshow(T[25,:,:],cmap='jet')
plt.colorbar()
plt.show()
```



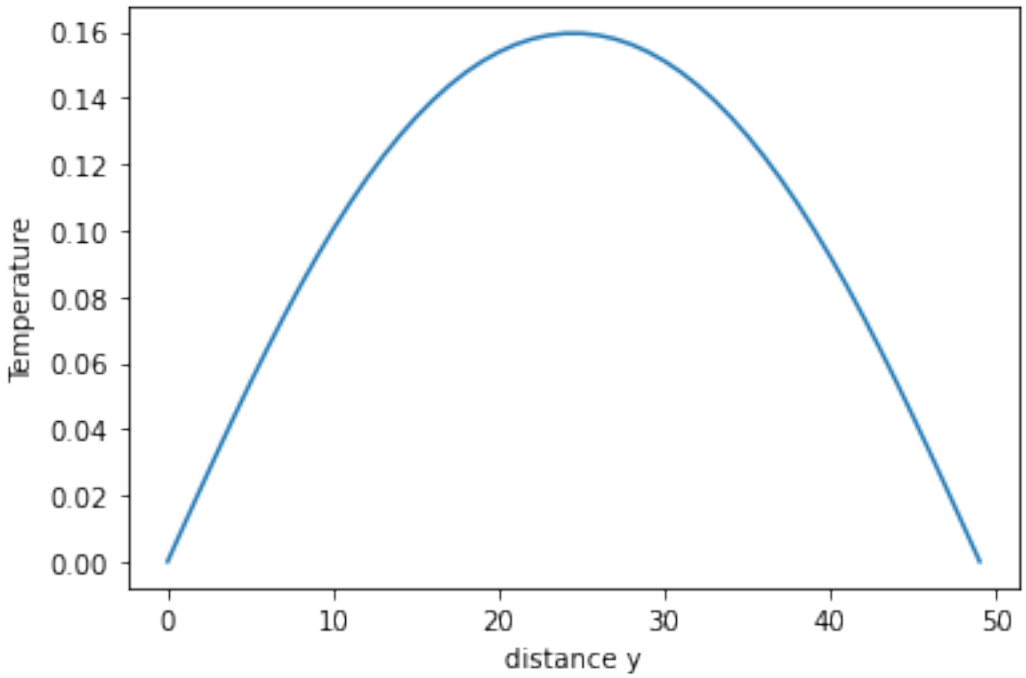
[16]:
```
plt.imshow(T[:,:,25],cmap='jet')
plt.colorbar()
plt.show()
```



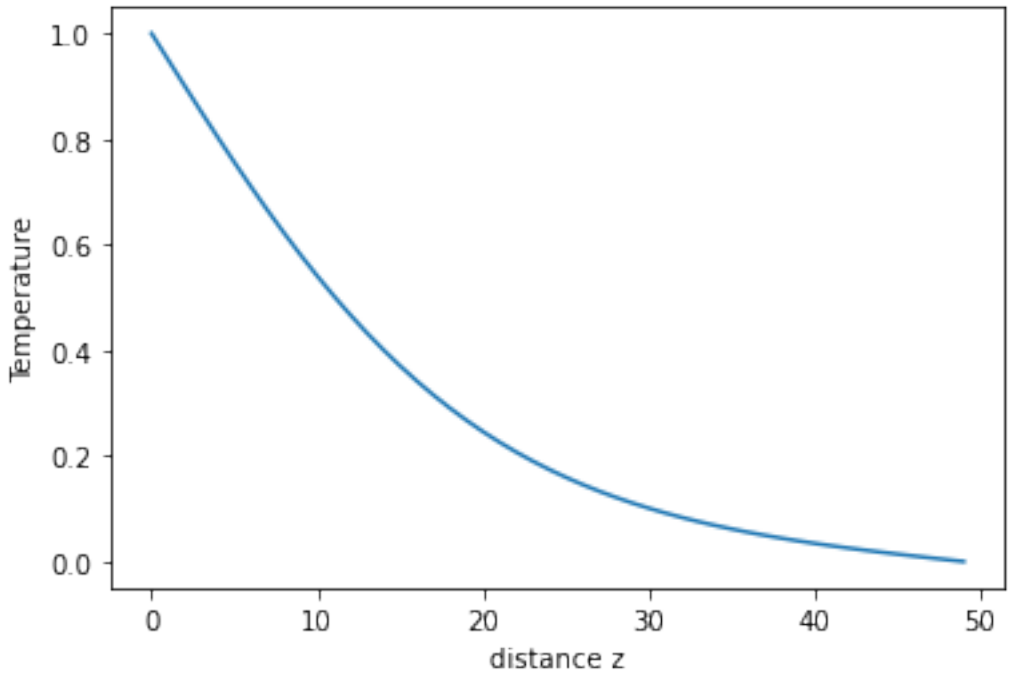
[17]:
```
plt.imshow(T[:,25,:],cmap='jet')
plt.colorbar()
plt.show()
```



[23]:
```
plt.plot(T[25,:,25])
plt.ylabel('Temperature')
plt.xlabel('distance y')
plt.show()
```



[24]:
```
plt.plot(T[:,25,25])
plt.xlabel('distance z')
plt.ylabel('Temperature')
plt.show()
```



[ ]: