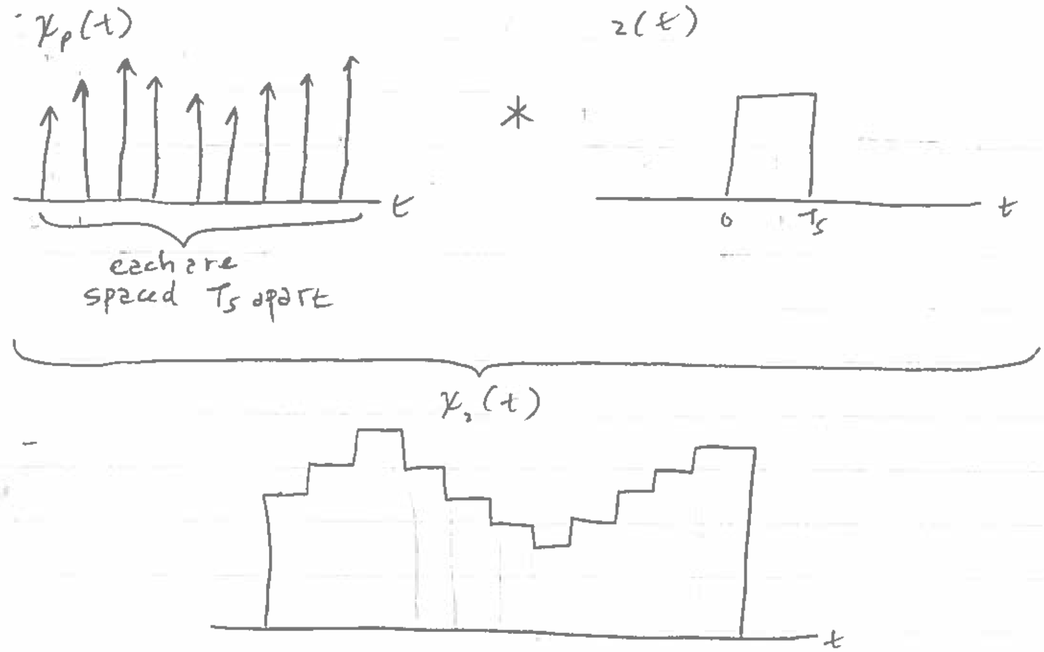
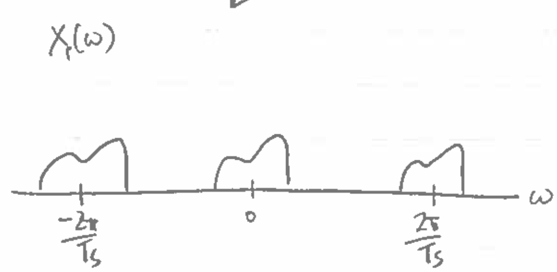
Problem Set #8 Jay Woo

1 g)



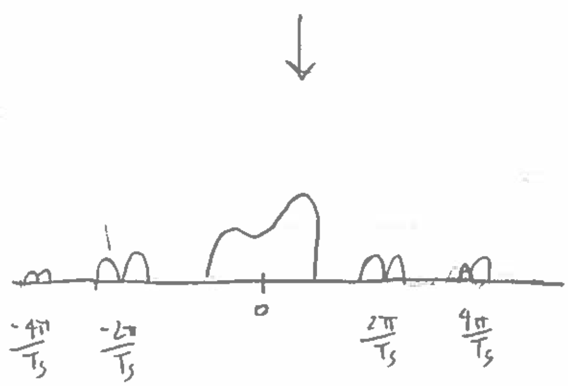
Since the impulses in $x_p(t)$ are spaced apart by T_s, the area in the convolution window will always be the size of one of the impulses as it sweeps through.

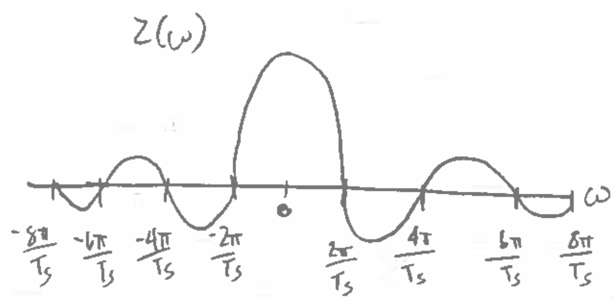
h) $X_z(\omega) = X_p(\omega)\, Z(\omega)$



According to Siddhartan's notes:

$$Z(\omega) = \underbrace{\frac{2\sin(\omega T_s/2)}{\omega}}_{\text{only part that contributes to amplitude}} \cdot e^{jT_s/2}$$



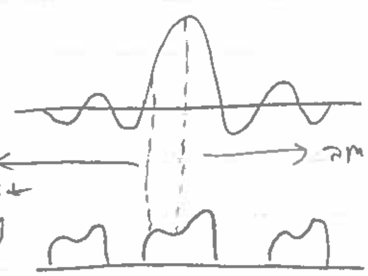


i) $\bar{X}(\omega) = X_z(\omega)\,H(\omega)$ $\qquad\qquad$ $\hat{X}(\omega) = X_p(\omega)\,H(\omega)$

Only the first period is preserved. All higher frequencies are cut.

j) $\hat{X}(\omega)$ is exactly the same as one period of $X_p(\omega)$, whereas $\bar{X}(\omega)$ is only an approximation, since $X_p(\omega)$ has been multiplied by $Z(\omega)$. As ω increases, $\bar{X}(\omega)$ becomes less and less of a good approximation.

some of the amplitude is lost as ω goes away from 0.



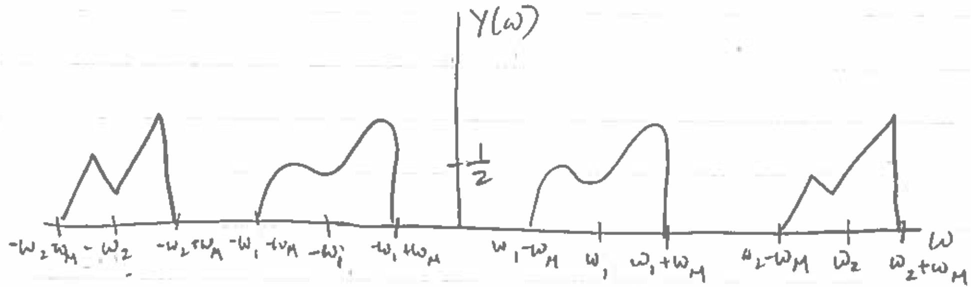
amplitude is completely preserved

k) $$\frac{\bar{X}(\omega)}{\hat{X}(\omega)} = \frac{X_z(\omega)\cancel{H(\omega)}}{X_p(\omega)\cancel{H(\omega)}} = \frac{\cancel{X_p(\omega)}\,Z(\omega)}{\cancel{X_p(\omega)}} = \frac{2\sin(\omega T_s/2)}{\omega}\cdot e^{jT_s/2}$$

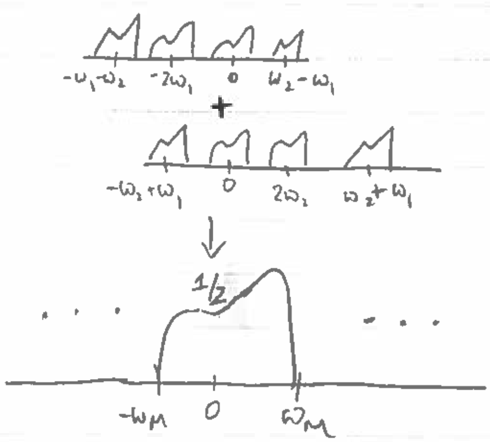
$$\frac{\bar{X}(\omega_M)}{\hat{X}(\omega_M)} = \frac{2\sin\left(\frac{\pi}{T_s}\frac{T_s}{2}\right)}{\left(\frac{\pi}{T_s}\right)} e^{jT_s/2} = \frac{2T_s\sin(\pi/2)}{\pi} e^{jT_s/2} = \boxed{\frac{2T_s}{\pi} e^{jT_s/2}}$$

2 a) $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

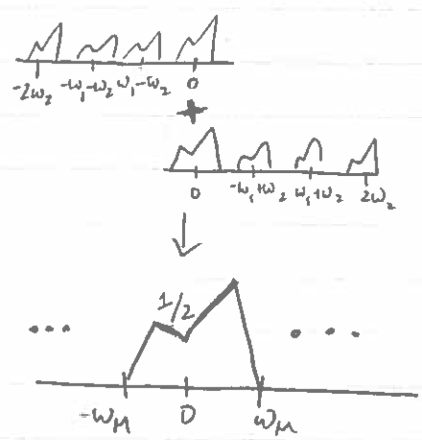
$Y(\omega) = X_1(\omega)\left[\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)\right] + X_2(\omega)\left[\pi\delta(\omega-\omega_2) + \pi\delta(\omega+\omega_2)\right]$



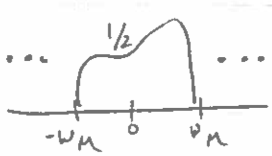
b) $y(t)\cos(\omega_1 t)$



$y(t)\cos(\omega_2 t)$



c) In order to get back the original signals $x_1(t)$ and $x_2(t)$ from $y(t)$, we can multiply $y(t)$ with a cosine wave at an appropriate frequency and apply a lowpass filter to remove all higher frequencies. From part (b), we got:



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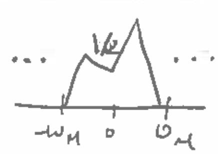


By filtering out all frequencies higher than ω_M and by dividing all the amplitudes by 2, we can get the original Fourier transforms: $X_1(\omega)$ and $X_2(\omega)$

3 a) $v_{in}(t) = v_R(t) + v_{out}(t) + v_L(t)$

$$v_{in}(t) = RC\frac{d}{dt}v_{out}(t) + v_{out}(t) + LC\frac{d^2}{dt^2}v_{out}(t)$$

b) $V_{in}(\omega) = j\omega RC\, V_{out}(\omega) + V_{out}(\omega) - \omega^2 LC\, V_{out}(\omega)$

$$= V_{out}(\omega)\left[j\omega RC + (1-\omega^2 LC)\right]$$

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{j\omega RC + (1-\omega^2 LC)}$$

c) $|H(\omega)| = \frac{1}{\sqrt{(\omega RC)^2 + (1-\omega^2 LC)^2}}$

$$= \frac{1}{\sqrt{\omega^2R^2C^2 + 1 - 2\omega^2LC + \omega^4L^2C^2}}$$

$$= \left(\omega^4L^2C^2 + \omega^2(R^2C^2 - 2LC) + 1\right)^{-1/2}$$

d) The denominator must be minimized, so the derivative of the denominator should be zero.

can ignore the square root

$$\frac{d}{d\omega}\left(\omega^4L^2C^2 + \omega^2(R^2C^2 - 2LC) + 1\right) = 0$$

$$4\omega^3L^2C^2 + 2\omega(R^2C^2 - 2LC) = 0$$

$$\omega(2\omega^2L^2C + (R^2C - 2L)) = 0$$

$$2\omega^2L^2C = 2L - R^2C$$

$$\omega^2 = \frac{2L-R^2C}{2L^2C}$$

$$\omega = 0,\ \pm\sqrt{\frac{2L-R^2C}{2L^2C}}$$

The second derivative should also be negative:

$$\frac{d}{d\omega}\left(2\omega^3L^2C + \omega(R^2C - 2L)\right) = 6\omega^2L^2C + R^2C - 2L$$

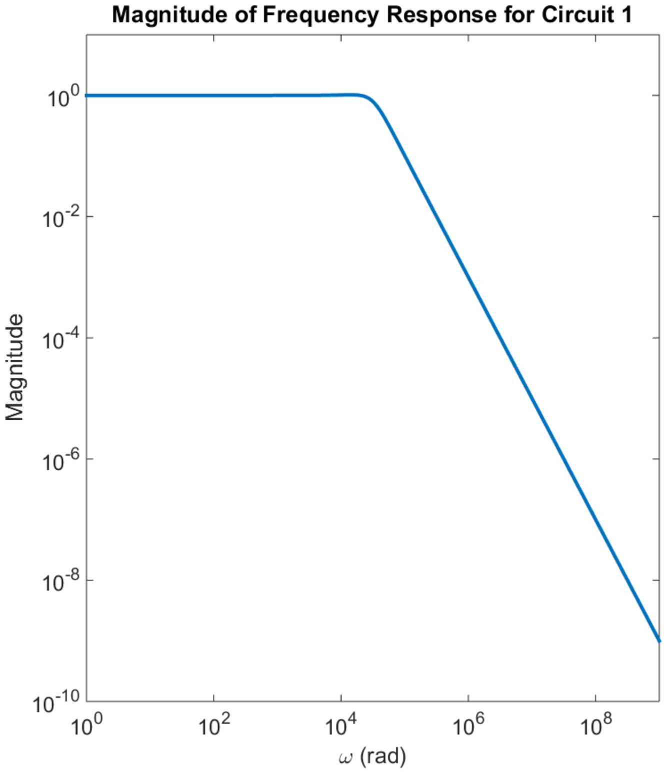
$$3\cancel{6}\left(\frac{2L-R^2C}{\cancel{2}L^2C}\right)\cancel{L^2C} + R^2C - 2L = 6L - 3R^2C + R^2C - 2L$$

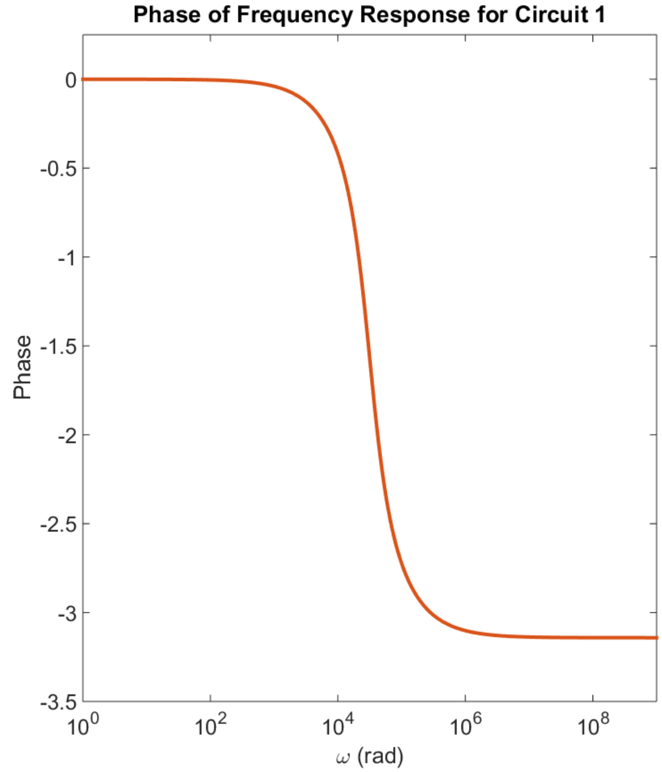
$$= 4L - 2R^2C$$

[since R is usually much larger than L and C, this whole expression will probably be negative.

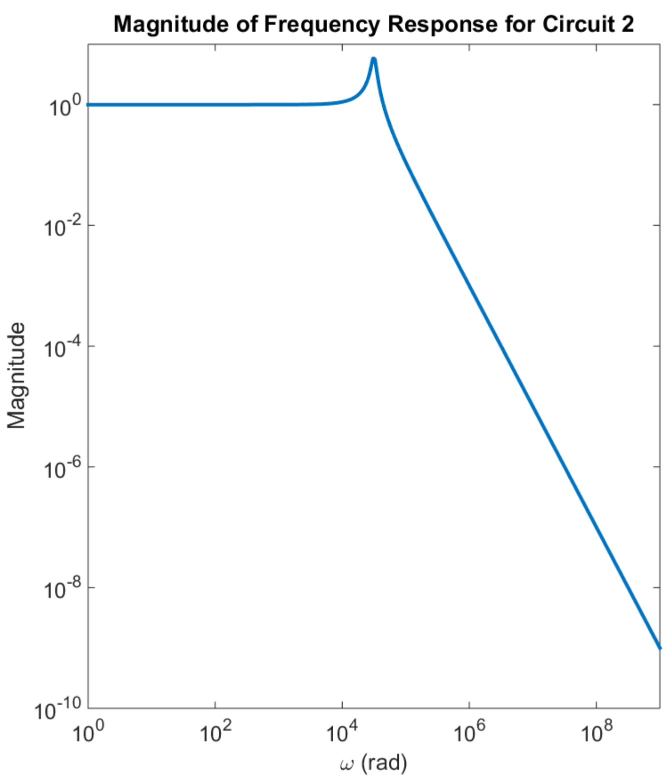
$$\boxed{\omega = \pm\sqrt{\frac{2L-R^2C}{2L^2C}}}$$

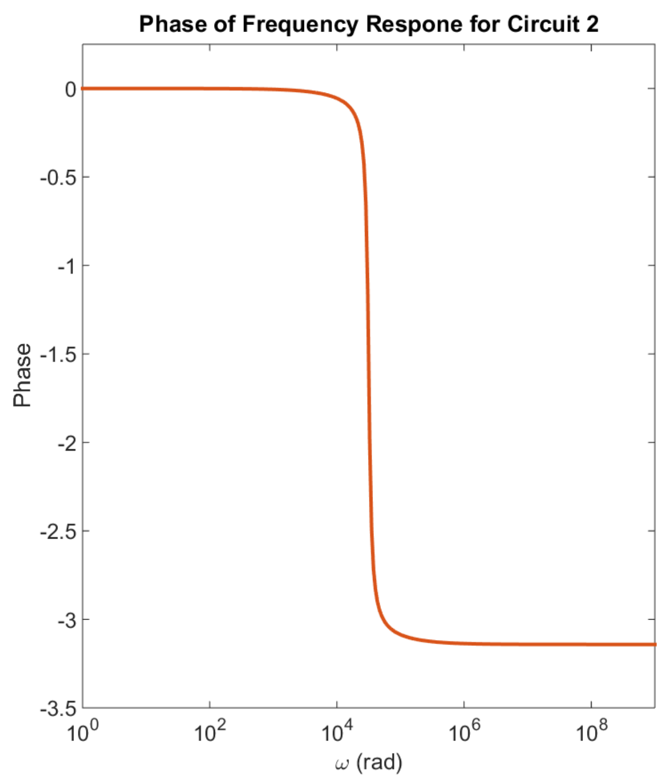
RLC Circuit 1 – C = 10^{-7} F, L = 10^{-2} H, R = 400 Ω





RLC Circuit 2 – C = 10^{-7} F, L = 10^{-2} H, R = 50 Ω





```
w = logspace(0, 9, 300);
C = 10^-7;
L = 10^-2;
R = 400; % 50 for circuit #2

H = 1 ./ (1 + 1i*w*R*C - w.^2*L*C);

figure;
subplot(1,2,1);
loglog(w, abs(H));

subplot(1,2,2);
semilogx(w, angle(H));
```