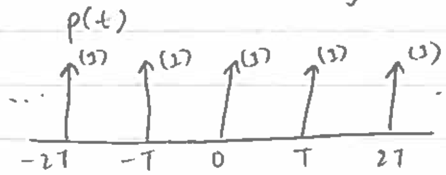
Problem Set #7 Jay Woo

1 a)



Series of impulses from $t = -\infty \rightarrow \infty$

b) $C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt = \frac{1}{T}\cdot 1(e^{-j\frac{2\pi}{T}k(0)}) = \frac{1}{T}$

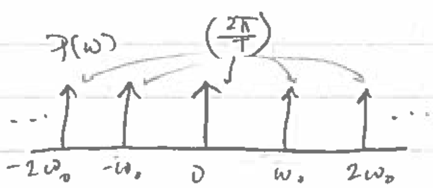
$$\boxed{x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j\frac{2\pi}{T}kt}}$$

c) $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt} = \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 kt}$

$$\boxed{X(\omega) = 2\pi C_k\, \delta(\omega - \omega_0 k)}$$

d) $\left.\begin{matrix} C_k = \frac{1}{T} \\ \omega_0 = \frac{2\pi}{T} \end{matrix}\right\}$ $\boxed{P(\omega) = \frac{2\pi}{T}\,\delta\left(\omega - \frac{2\pi}{T}k\right)}$

e)



p(t) - Increasing/decreasing T increases/decreases the spacing between the impulses

P(ω) - Increasing T decreases the spacing between the impulses and decreases the magnitude of the impulses

These make sense, since if the period decreases, the frequency increases, so higher frequency components are required to characterize the signal. The opposite is also true when the period increases.

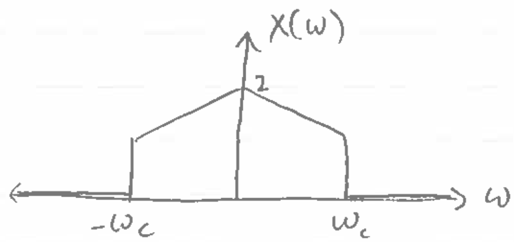
2 a) $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}\,d\omega$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi j t}\left[e^{j\omega t}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j t}\left(e^{j\omega_c t} - e^{-j\omega_c t}\right) = \frac{\sin(\omega_c t)}{\pi t} = \frac{\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c t}{\pi}\right)$$

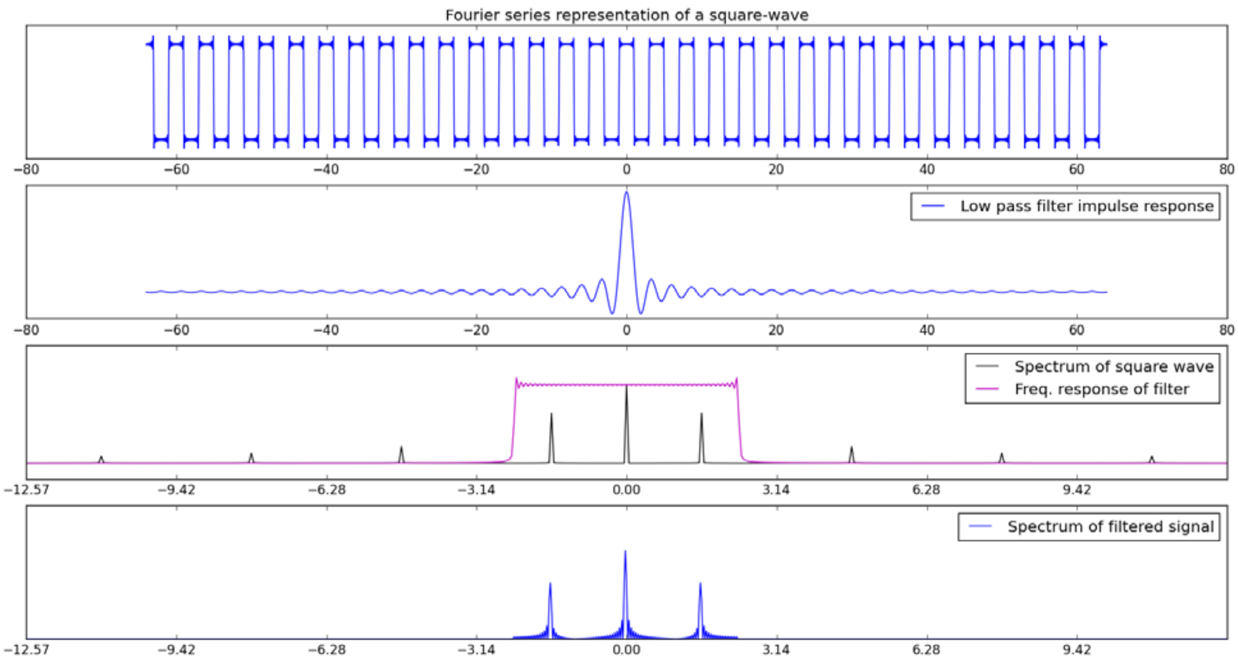
b) $Y(\omega) = H(\omega)\,X(\omega)$



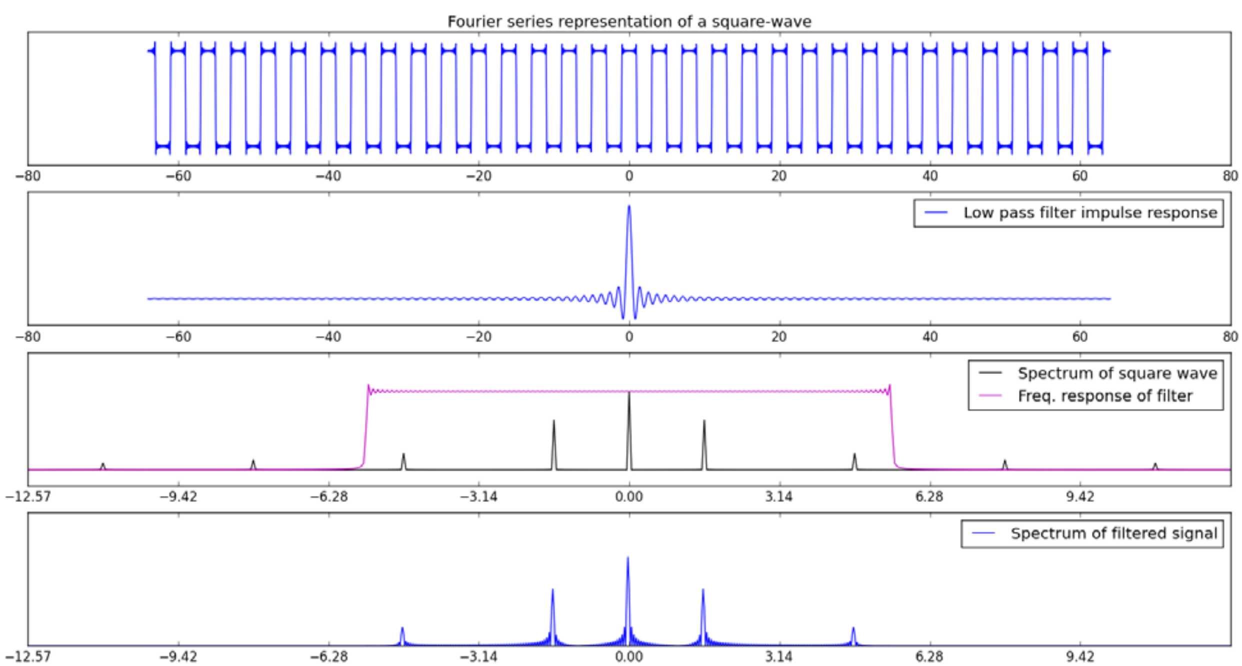
c) This LTI system acts as a lowpass filter because it preserves the amplitudes at frequencies below the threshold value ω_c and cuts out all higher frequencies.

d) CODE + PLOTS ON NEXT PAGE

Cutoff Frequency = 0.75 π



Cutoff Frequency = 1.75 π



Code:

```
####################################################################################
######   This is the cutoff frequency
######   Originally set to 0.75pi, you should change this accordingly.
####################################################################################
w_c = np.pi*0.75


####################################################################################
######  You should change the following line of code so that h is the impulse
######  response of the low-pass filter
######  As an example, to make an impulse response which equals cosine(w_c t),
######  you would type h = np.cos(w_c*ts)
######  Note that numpy's sinc(t) function implements sin(Pi t)/(Pi t).
####################################################################################
h = w_c / np.pi * np.sinc(w_c*ts/np.pi)
```

$h(t) = \cos(\omega_c t) \rightarrow \pi\,\delta(\omega - \omega_c) + \pi\,\delta(\omega + \omega_c)$

TWO IMPULSES

3 $y(t) = x(t)\,h(t) \rightarrow Y(\omega) = \frac{1}{2\pi} X * H(\omega)$

$Y(\omega) =$

$=$