# FACTORIAL ANALYSIS OF VARIANCE DESIGN

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# FACTORIAL ANOVA DESIGN

- IN FACTORIAL ANOVA DESIGN, WE ARE INTERESTED IN THE MAIN AND INTERACTION EFFECTS ON THE RESPONSE VARIABLE.

- ILLUSTRATION OF FACTORIAL ANOVA DESIGN

  ASSUMING THAT FACTOR A HAS 3 LEVELS AND FACTOR B HAS 4 LEVELS

  THE FACTORIAL ANOVA DESIGN MAY BE WRITTEN 3 X 4

  THE EFFECTS UNDER INVESTIGATION ARE:

  MAIN EFFECTS: A, B

  INTERACTION EFFECT: A X B

# TWO –WAY FACTORIAL ANOVA

- WE CAN ILLUSTRATE THE TWO FACTORS INTO COLUMNS AND ROWS AS FOLLOWS

| | | FACTOR-B | | | ai | ai^2 |
|---|---|---|---|---|---|---|
| | b1 | b2 | b3 | b4 | **Row Total** | |
| | a1 | | | | | |
| **FACTOR-A** | a2 | | | | | |
| | a3 | | | | | |
| **Column Total** | **bj** | | | | | |
| | bj^2 | | | | | |

# TERMS IN THE FACTORIAL DESIGN

- THE NUMBER OF REPLICATE PER CELL IS DEFINED AS “r” WHERE $r \geq 2$
- THE NUMBER OF LEVELS IN FACTOR A IS “a” WHERE $a \geq 2$
- THE NUMBER OF LEVELS IN FACTOR B IS “b” WHERE $b \geq 2$
- THE TOTAL OF REPLICATE RESPONSE PER CELL IN ROW i by COLUMN j IS DENOTED

$$\left[ a_i b_j \right]$$

# SUM OF SQUARES IN TWO-WAY FACTORIAL ANOVA

- SUM SQUARE OF FACTOR A (SSA) $SSA = \frac{1}{b.r}\sum[a_i]^2 - \frac{T^2}{N}$
- SUM SQUARE OF FACTOR B (SSB) $SSB = \frac{1}{a.r}\sum[b_j]^2 - \frac{T^2}{N}$
- SUM SQUARE OF INTERACTION (AXB), SSAB= SSA.B – SSA - SSB
- WHERE SSA.B IS DEFINED AS

$$SSA.B = \frac{1}{r}\sum[a_i b_j]^2 - \frac{T^2}{N}$$

# SUM SQUARES CONTD.

- SUM SQUARE OF ERROR (SSE) = SSTOT – SSA.B
- SUM SQUARE TOTAL (SSTOT) OR TOTAL SUM OF SQUARE

$$SSTOT = \sum_{i,j} y_{ij}^2 - \frac{T^2}{N}$$

# TWO-WAY FACTORIAL TABLE

| SOURCE | DF | SS | MS | F |
|---|---|---|---|---|
| A | a-1 | SSA | SSA/a-1=MSA | MSA/MSE |
| B | b-1 | SSB | SSB/b-1 =MSB | MSB/MSE |
| AB | (a-1)(b-1) | SSAB | SSAB/(a-1)(b-1)=MSAB | MSAB/MSE |
| RESIDUAL | ab(r-1) | SSE | SSE/ab(r-1)=MSE=$S^2$ | |
| TOTAL | N-1 | SSTOT | | |

# EXAMPLE

**An engineer suspects that the surface finish of a metal part is influenced by the feed rate and the depth of cut. He selects three feed rates and four depths of cut. He then conducts a factorial experiment and obtains the following data:**

- **What is the purpose of the experiment? and what type?**
- **Formulate two research questions assuming you are the researcher .**
- **Conduct appropriate analysis of variance to answer the hypotheses formulated**
  **Use alpha =0.5. Use both manual calculation and SPSS to conduct the analysis**
  **Prepare a brief report on the conclusion of your findings**
  **Prepare the appropriate residual plots and comment on the model's adequacy**

# EXAMPLE

| | | | Depth of Cut (in) | | |
|---|---|---|---|---|---|
| | | 0.15 | 0.18 | 0.2 | 0.25 |
| | | 74 | 79 | 82 | 99 |
| | 0.2 | 64 | 68 | 88 | 104 |
| | | 60 | 73 | 92 | 96 |
| | | | | | |
| | | 92 | 98 | 99 | 104 |
| Feed Rate | 0.25 | 86 | 104 | 108 | 110 |
| (in/min) | | 88 | 88 | 95 | 99 |
| | | | | | |
| | | 99 | 104 | 108 | 114 |
| | 0.3 | 98 | 99 | 110 | 111 |
| | | 102 | 95 | 99 | 107 |

# SOLUTION

- PURPOSE OF EXPERIMENT: TO INVESTIGATE THE FEECTS OF DEPTH OF CUT, FEED RATE AND THE INTERACTION
- TYPE OF ANOVA:  FACTORIAL
- RESEARCH QUESTIONS
- 1.  IS THERE ANY RELATIONSHIP BETWEEN DEPTH CUT AND THE RESPONSE VARIABLE?
- 2. IS THERE ANY RELATIONSHIP BETWEEN FEED RATE  AND THE RESPONSE VARIABLE?
- 3. IS THERE ANY EFFECT OF INTERACTION AND THE RESPONSE VARIABLE?

# SOLUTION

- **HYPOTHESES**
- **FACTOR A (Feed Rate)**

$$H_0 : \alpha_1 = \alpha_2 = \alpha_3$$

$$H_1 : \alpha_1 \neq \alpha_2 \neq \alpha_3$$

- **FACTOR B (Depth of cut)**

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4$$

$$H_1 : \beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_4$$

- **INTERACTION (AB)**

$$H_0 : (\alpha\beta)_{ij} = 0$$

$$H_1 : (\alpha\beta)_{ij} \neq 0$$

# CALCULATION OF TOTALS

| | 0.15 | 0.18 | 0.2 | 0.25 | | ai | ai^2 |
|---|---|---|---|---|---|---|---|
| | 74 | 79 | 82 | 99 | | 979 | 958441 |
| | 64 | 68 | 88 | 104 | | | |
| 0.2 | 60 | 73 | 92 | 96 | | | |
| | 92 | 98 | 99 | 104 | | 1171 | 1371241 |
| | 86 | 104 | 108 | 110 | | | |
| 0.25 | 88 | 88 | 95 | 99 | | | |
| | 99 | 104 | 108 | 114 | | 1246 | 1552516 |
| | 98 | 99 | 110 | 111 | | | |
| 0.3 | 102 | 95 | 99 | 107 | | | |
| | | | | | | | |
| bj | 763 | 808 | 881 | 944 | | | |
| bj^2 | 582169 | 652864 | 776161 | 891136 | | | |

# [aibj]

| | | | |
|---|---|---|---|
| 198 | 220 | 262 | 299 |
| 266 | 290 | 302 | 313 |
| 299 | 298 | 317 | 332 |

# [aibj]^2

| | | | |
|---|---|---|---|
| 39204 | 48400 | 68644 | 89401 |
| 70756 | 84100 | 91204 | 97969 |
| 89401 | 88804 | 100489 | 110224 |

# SUM SQUARES AND TOTALS

**T = 3396**

**N =36**

**CORRECTION FACTOR = T^2/N = (3396)^2/36 =320356**

**SSTOTAL = 326888 -320356 = 6532**

**b = 4**

**a = 3**

**r = 3**

**SSA = [958441 +1371241 +1552516 ]/12 - 320356 = 3160.5**

**SSB = [582169 + 652864 + 776161 + 891136 ]/9 - 320356 =2125.11**

**SSA.B = 978596/3 - 320356 = 5842.67**

**SSAB = SSA.B –SSA – SSB = 5842.67 – 3160.5 – 2125.11 = 557.06**

**SSE = SSTOT – SSA.B = 6532 – 5842.67 = 689.33**

# MEAN SQUARE CALCULATIONS

- MSA, FACTOR A = 3160.5/2 =1580.25
- MSB, FACTOR B = 2125.11/3 = 708.37
- MSAB, INTERACTION = 557.06/6 = 92.84
- MSE, ERROR=RESIDUAL = 689.33/24 = 28.72

# F - CALCULATIONS

- **FACTOR A F = MSA/MSE = 1580.25/28.72 = 55.0226**
- **FACTOR B F = MSB/MSE = 708.37/28.72 = 24.664**
- **INTERACTION F =MSAB/MSE = 92.84/28.72 = 3.2325**

# HYPOTHESES TESTING

**LEVEL OF SIGNIFICANCE IS 5%**

**F-TABLE FOR FACTOR A,** $F(2, 24, 0.05)$ **= 3.402826**

**F-TABLE FOR FACTOR B,** $F(3, 24, 0.05)$ **= 3.008787**

**F-TABLE FOR INTERACTION (AB),** $F(6, 24, 0.05)$ **=2.508189**

**DECISIONS: REJECT THE NULL HYPOTHESIS FOR ALL , F-CAL > F-TABLE**

# CONCLUSION

- At the 5% level of significance, there is an evidence that there are effects of the depth of cut , feed rate and interaction of both factors on the surface finish of metal part. Further post hoc or multiple comparison of the means for each factor level to identify the least and greatest effect on the surface finish.

- Alternatively we are 95% confident that the depth of cut, feed rate and interaction of both factors have effects on the surface finish of metal part tested in the study.

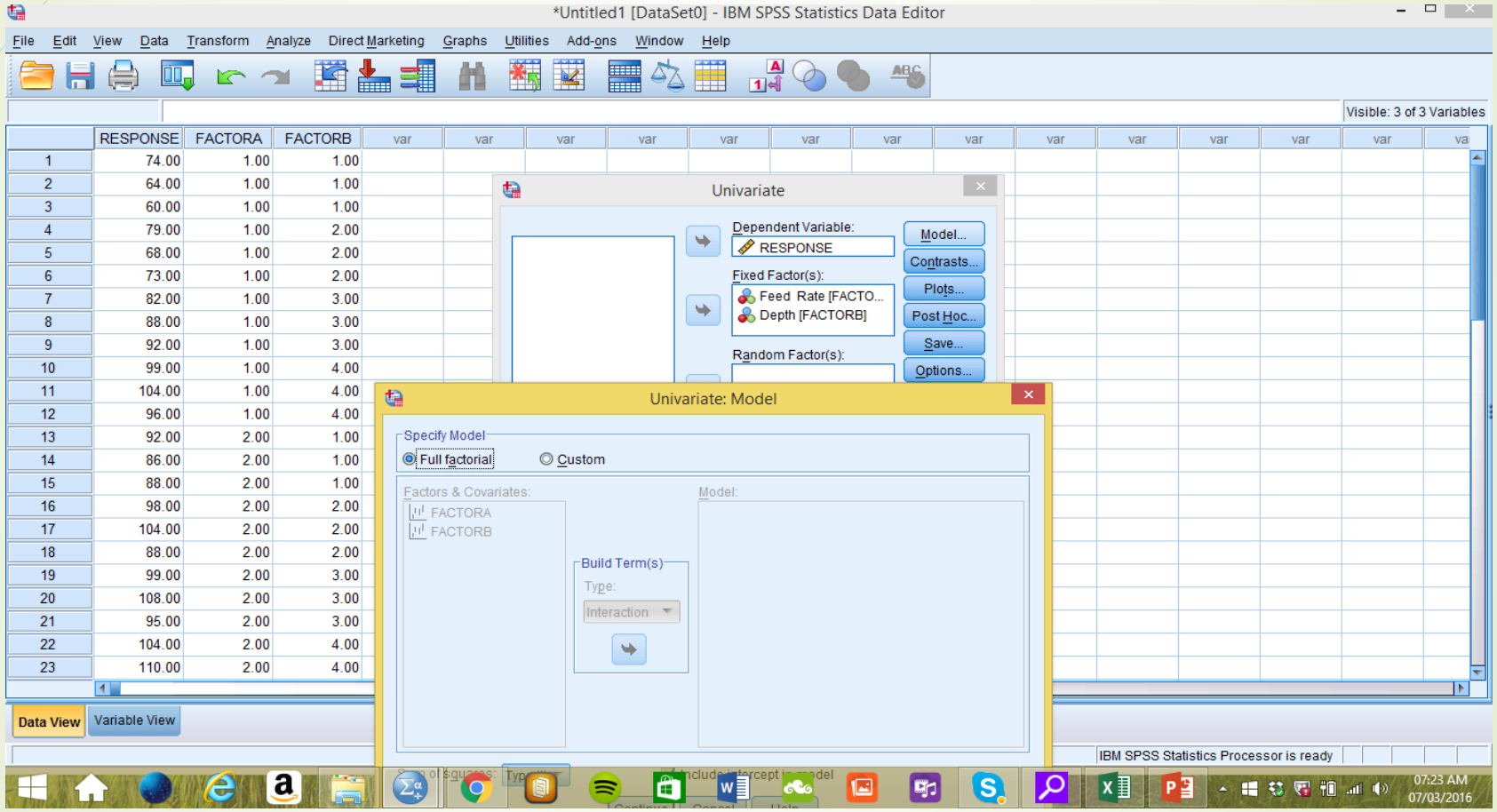
# SPSS

| RESPONSE | FACTOR A | FACTOR B |
|---|---|---|
| 74 | 1 | 1 |
| 64 | 1 | 1 |
| 60 | 1 | 1 |
| 79 | 1 | 2 |
| 68 | 1 | 2 |
| 73 | 1 | 2 |
| 82 | 1 | 3 |
| 88 | 1 | 3 |
| 92 | 1 | 3 |
| 99 | 1 | 4 |
| 104 | 1 | 4 |
| 96 | 1 | 4 |

# SPSS

| RESPONSE | FACTOR A | FACTOR B |
|---|---|---|
| 92 | 2 | 1 |
| 86 | 2 | 1 |
| 88 | 2 | 1 |
| 98 | 2 | 2 |
| 104 | 2 | 2 |
| 88 | 2 | 2 |
| 99 | 2 | 3 |
| 108 | 2 | 3 |
| 95 | 2 | 3 |
| 104 | 2 | 4 |
| 110 | 2 | 4 |
| 99 | 2 | 4 |
| 99 | 3 | 1 |
| 98 | 3 | 1 |
| 102 | 3 | 1 |
| 104 | 3 | 2 |
| 99 | 3 | 2 |
| 95 | 3 | 2 |
| 108 | 3 | 3 |
| 110 | 3 | 3 |
| 99 | 3 | 3 |
| 114 | 3 | 4 |
| 111 | 3 | 4 |
| 107 | 3 | 4 |

# SPSS - GENERAL LINEAR MODEL, UNIVARIATE



# SPSS OUTPUT

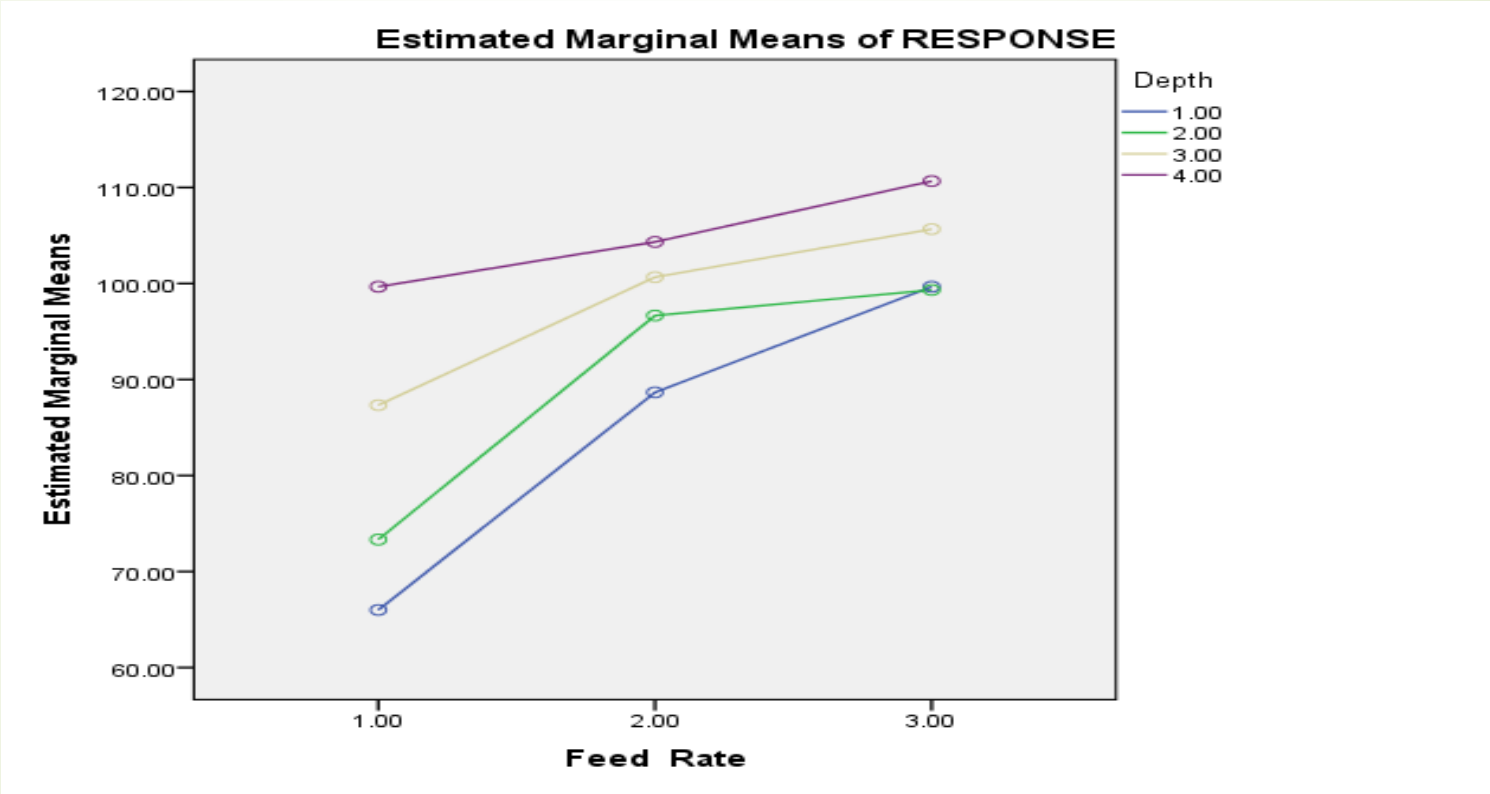
**Tests of Between-Subjects Effects**

Dependent Variable: RESPONSE

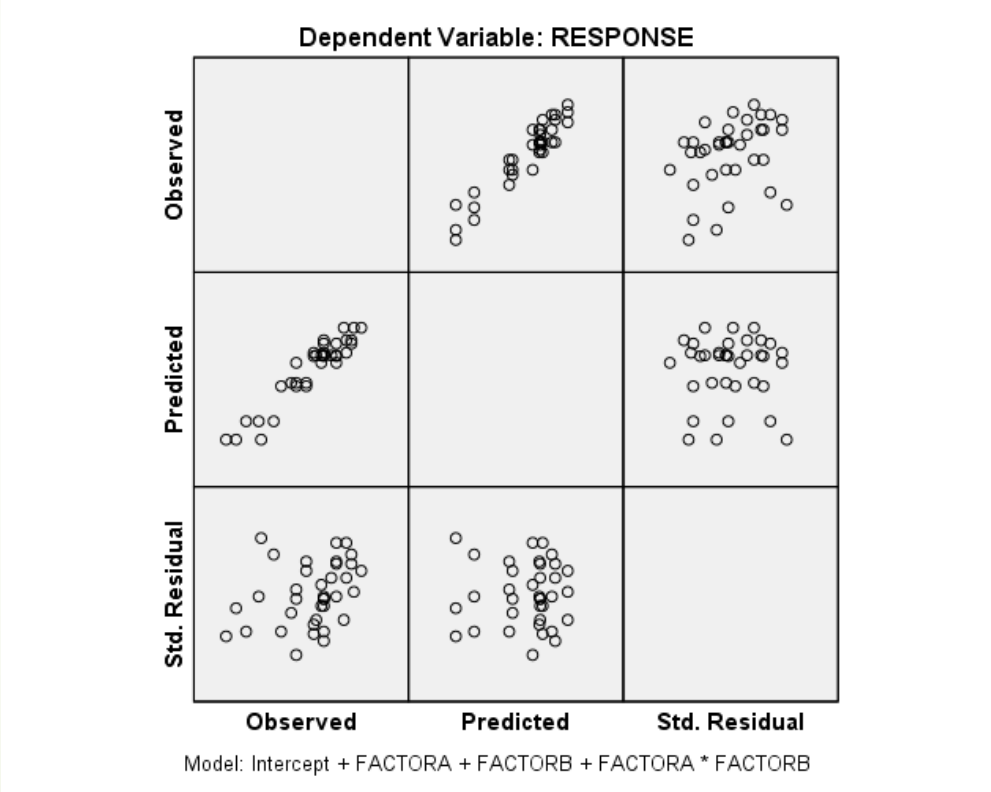
| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Corrected Model | 5842.667[a] | 11 | 531.152 | 18.493 | .000 |
| Intercept | 320356.000 | 1 | 320356.000 | 11153.594 | .000 |
| **FACTORA** | **3160.500** | 2 | 1580.250 | 55.018 | .000 |
| **FACTORB** | **2125.111** | 3 | 708.370 | 24.663 | .000 |
| **FACTORA * FACTORB** | **557.056** | 6 | 92.843 | 3.232 | .018 |
| Error | **689.333** | 24 | 28.722 | | |
| Total | 326888.000 | 36 | | | |
| Corrected Total | **6532.000** | 35 | | | |

a. R Squared = .894 (Adjusted R Squared = .846)

# INTERACTION BETWEEN DEPTH 2 &4



# RESIDUAL PLOT



# DESCRIPTIVE STATISTICS

- **2. Feed Rate**
- Dependent Variable: RESPONSE

| Feed Rate | Mean | Std. Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| 1.00 | 81.583 | 1.547 | 78.390 | 84.776 |
| 2.00 | 97.583 | 1.547 | 94.390 | 100.776 |
| 3.00 | 103.833 | 1.547 | 100.640 | 107.026 |

# DESCRIPTIVE STATISTICS

- **3. Depth**
- Dependent Variable: RESPONSE

| Depth | Mean | Std. Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| 1.00 | 84.778 | 1.786 | 81.091 | 88.465 |
| 2.00 | 89.778 | 1.786 | 86.091 | 93.465 |
| 3.00 | 97.889 | 1.786 | 94.202 | 101.576 |
| 4.00 | 104.889 | 1.786 | 101.202 | 108.576 |

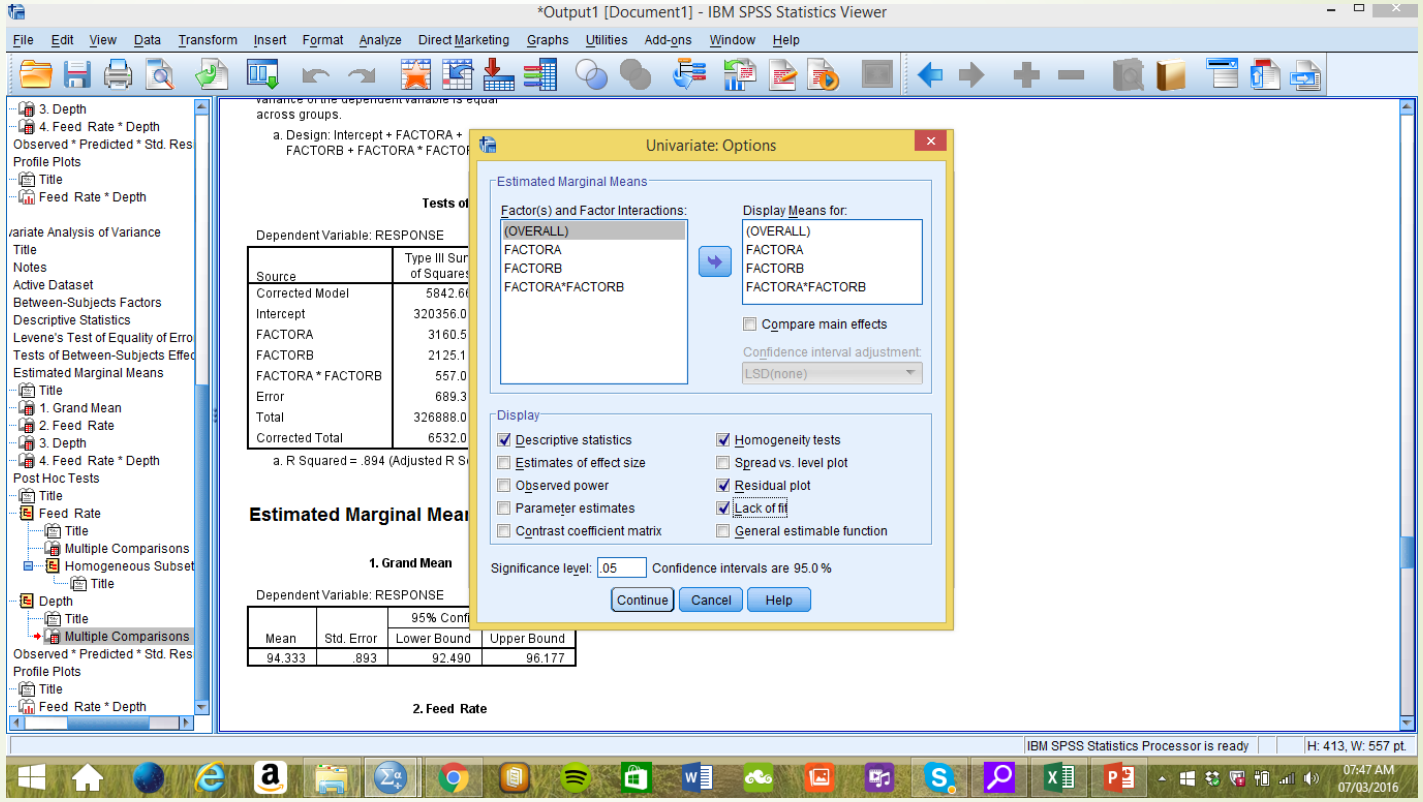
# MULTIPLE COMPARISONS-FEED RATE

| Multiple Comparisons | | | | | | |
|---|---|---|---|---|---|---|
| Dependent Variable: RESPONSE<br>LSD | | | | | | |
| (I) Feed Rate | | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval | |
| | | | | | Lower Bound | Upper Bound |
| 1.00 | 2.00 | -16.0000* | 2.18793 | .000 | -20.5157 | -11.4843 |
| | 3.00 | -22.2500* | 2.18793 | .000 | -26.7657 | -17.7343 |
| 2.00 | 1.00 | 16.0000* | 2.18793 | .000 | 11.4843 | 20.5157 |
| | 3.00 | -6.2500* | 2.18793 | .009 | -10.7657 | -1.7343 |
| 3.00 | 1.00 | 22.2500* | 2.18793 | .000 | 17.7343 | 26.7657 |
| | 2.00 | 6.2500* | 2.18793 | .009 | 1.7343 | 10.7657 |

# MULTIPLE COMPARISON –DEPTH OF CUT

| Multiple Comparisons | | | | | | |
|---|---|---|---|---|---|---|
| Dependent Variable: RESPONSE<br>LSD | | | | | | |
| (I) Depth | | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval | |
| | | | | | Lower Bound | Upper Bound |
| 1.00 | 2.00 | -5.0000 | 2.52640 | .059 | -10.2142 | .2142 |
| | 3.00 | -13.1111* | 2.52640 | .000 | -18.3254 | -7.8969 |
| | 4.00 | -20.1111* | 2.52640 | .000 | -25.3254 | -14.8969 |
| 2.00 | 1.00 | 5.0000 | 2.52640 | .059 | -.2142 | 10.2142 |
| | 3.00 | -8.1111* | 2.52640 | .004 | -13.3254 | -2.8969 |
| | 4.00 | -15.1111* | 2.52640 | .000 | -20.3254 | -9.8969 |
| 3.00 | 1.00 | 13.1111* | 2.52640 | .000 | 7.8969 | 18.3254 |
| | 2.00 | 8.1111* | 2.52640 | .004 | 2.8969 | 13.3254 |
| | 4.00 | -7.0000* | 2.52640 | .011 | -12.2142 | -1.7858 |
| 4.00 | 1.00 | 20.1111* | 2.52640 | .000 | 14.8969 | 25.3254 |
| | 2.00 | 15.1111* | 2.52640 | .000 | 9.8969 | 20.3254 |
| | 3.00 | 7.0000* | 2.52640 | .011 | 1.7858 | 12.2142 |
| Based on observed means.<br>The error term is Mean Square(Error) = 28.722. | | | | | | |
| *. The mean difference is significant at the .05 level. | | | | | | |

# DISPLAY OF UNIVARIATE OPTIONS



# ASSUMPTION OF EQUALITY OF VARIANCE

| Levene's Test of Equality of Error Variances a | | | | |
|---|---|---|---|---|
| Dependent Variable: RESPONSE | | | | |
| F | df1 | df2 | Sig. | |
| 0.772 | 11 | 24 | 0.663 | |

**Tests the null hypothesis that the error variance of the dependent variable is equal across groups.**

**a Design: Intercept + FACTORA + FACTORB + FACTORA * FACTORB**