Production & Operations Management — Recitation 5

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Overview

Example: Sunco Oil

Problem Statement: Sunco oil has three different processes that can be used to manufacture various type of gasoline. Each process involves blending oils in the company's catalytic cracker.

- *Running process 1* for an hour costs \$5 and requires two barrels of crude oil 1 and three barrels of crude oil 2. The output is two barrels of gas 1 and one barrel of gas 2;
- *Running process 2* for an hour costs \$4 and requires one barrels of crude oil 1 and three barrels of crude oil 2. The output is three barrels of gas 2;
- *Running process 3* for an hour costs \$1 and requires two barrels of crude oil 2 and three barrels of gas 2. The output is two barrels of gas 3.

Example: Sunco Oil (c.t.d)

Problem Statement (c.t.d): Each week 200 barrels of crude oil 1, at $2 per barrel, and 300 barrels of crude oil 2, at $3 per barrel can be purchased. All gas produced can be sold at the following per barrel prices: gas 1, $ 9; gas 2 $ 10; gas 3, $ 24. Assume that only 100 hours of time on the catalytic cracker are available each week.

- **Question:** Create the process model table, define decision variables, and develop the *objective function* (O.F.), *constraints* (C.S.), *non-negative constraints* (N.C.S.)?

Example: Sunco Oil - Identifying Problem Type

This is a process model because:

- It has multiple processes;
- Products are consumed by other products;
- Consumed products are are also sold separately (as a standalone product)

Example: Sunco Oil - Process Model Table

Step 1: create process model table:

		Activities				
		Process 1 x1	**Process 2 x2**	**Process 3 x3**	**Available Resources**	**Cost / Price**
Resources	**Crude 1**	2	1	0	200	$2/barrel
	Crude 2	3	3	2	300	$3/barrel
	Cracker Hours	1	1	1	100	
Direct Cost		$5.00	$4.00	$1.00		
Products	**Gas 1**	2				$9/ barrel
	Gas 2	1	3	-3		$10/ barrel
	Gas 3			2		$24/ barrel

Example: Sunco Oil - Decision Variables

Step 2: identify decision variables:

- x_1 =the number of times to run process 1 this week;
- x_2 =the number of times to run process 2 this week;
- x_3 =the number of times to run process 3 this week.

Example: Sunco Oil - Objective Function

Step 3: Establish objective function. Think in the following way, we are trying to maximize the profit, what is profit ?

$$\text{Profit} = \text{Total Revenue} - \text{Total Cost}$$

The revenue is made whenever Gas 1, Gas 2, Gas 3 are sold. Imagine, for example, we run x_1 times process 1, then we will get $2x_1$ gas 1 each time, if the selling price of Gas 1 is \$9, then by working on process 1, we get $\$9 \times 2x_1$. Similarly,

$$\begin{aligned}\text{Revenue} &= \$9 \times 2x_1 + \$10[1x_1 + 3x_2 - 3x_3] + \$24 \times 2_x 3 \\ &= 28x_1 + 30x_2 + 18x_3\end{aligned}$$

Remark

Notice that Gas 2 sales include the production from processes 1 and 2 minus the consumption in process 3.

Example: Sunco Oil - Objective Function (c.t.d)

Step 3 (c.t.d): We have done the part for the revenue, now cost. There are two sources of cost one is direct cost incurred by working on each process, the other is the cost of the crude oil. We firstly compute the direct cost:

$$\text{Direct Cost} = 5x_1 + 4x_2 + 1x_3 \tag{1}$$

Then the crude oil cost. Consider the cost of Crude 1, whenever we work on process 1, we need crude 1 two barrels, and for process 2, we need crude oil just one barrel, process does not require crude oil 1, thus the cost of crude 1 is $2x_1 + 1_x2$. Similarly, For the direct cost:

$$\text{Crude oil Cost} = \$2(2x_1 + 1x_2) + \$3(3x_1 + 3x_2 + 2x_3] \tag{2}$$

$$= 28x_1 + 30x_2 + 18x_3 \tag{3}$$

Thus, by combining (1) and (2), the total cost is $18x_1 + 15x_2 + 7x_3$.

Example: Sunco Oil - Objective Function (c.t.d)

Step 3 (c.t.d): Now we can calculate the profit and establish the objective function.

$$\begin{aligned}\text{Profit} &= \text{Total Revenue} - \text{Total Cost} \\ &= 28x_1 + 30x_2 + 18x_3 - (18x_1 + 15x_2 + 7x_3) \\ &= 10x_1 + 15x_2 + 11x_3\end{aligned}$$

Objective Function (O.F):

$$\max.\ 10x_1 + 15x_2 + 11x_3$$

Example: Sunco Oil - Constraints

Step 4: develop the constraints. There are two kinds of constraints:

- Resource Constraints:

$$\begin{aligned} &\text{Crude Oil 1} && 2x_1 + 1x_2 + 0x_3 \leq 200 \text{ barrels} \\ &\text{Crude Oil 2} && 3x_1 + 3x_2 + 2x_3 \leq 300 \text{ barrels} \\ &\text{Cracker Hours} && 1x_1 + 1x_2 + 1x_3 \leq 100 \text{ hours} \end{aligned}$$

- Minimum Production Constraints

$$\text{Gas 2} \quad 1x_1 + 3x_2 - 3x_3 \geq 0$$

Remark

The minimum production constraints is implicit. When we work on process 3, we actually consume Gas 2, since we can not borrow Gas 2, thus we can at most consume as much as produced by process 1 and process 2.

Example: Sunco Oil - Non-negative Constraints

Step 5: Complete the model by setting up the non-negative constraints

$$x_1 \geq 0$$
$$x_2 \geq 0$$
$$x_3 \geq 0$$

or

$$x_1, x_2, x_3 \geq 0$$

Example: Sunco Oil - Complete LP Model

Linear Programming Model for Process Model - Sunco Oil

$$X = \begin{cases} x_1: & \text{the number of times to run process 1 this week;} \\ x_2: & \text{the number of times to run process 2 this week;} \\ x_3: & \text{the number of times to run process 3 this week;} \end{cases}$$

$$\begin{aligned} \max. \quad & 10x_1 + 15x_2 + 11x_3 \\ \text{subject to:} \quad & 2x_1 + 1x_2 + 0x_3 \leq 200 \\ & 3x_1 + 3x_2 + 2x_3 \leq 300 \\ & 1x_1 + 1x_2 + 1x_3 \leq 100 \\ & 1x_1 + 3x_2 - 3x_3 \geq 0 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Example: Sunco Oil - Excel Solution 1

Excel Formulas:

Decision Variables						
Variable Name	Process 1	Process 2	Process 3			
Variables	x1	x2	x3			
Variable Values	0	100	7.105427357601E-15			
O. F.						
Parameters	10	15	11	Value of O.F. =	=SUMPRODUCT(B6:D6,B9:D9)	
Constraints				LHS	OP	RHS
Crude Oil 1	2	1	0	=SUMPRODUCT(B12:D12, B$6:D$6)	<=	200
Crude Oil 2	3	3	2	=SUMPRODUCT(B13:D13, B$6:D$6)	<=	300
Time	1	1	1	=SUMPRODUCT(B14:D14, B$6:D$6)	<=	100
Min Gas 2	1	3	-3	=SUMPRODUCT(B15:D15, B$6:D$6)	>=	0

Example: Sunco Oil - Excel Solution 2

Excel Solutions:

Decision Variables						
Variable Name	Process 1	Process 2	Process 3			
Variables	x1	x2	x3			
Variable Values	0	100	0			
O. F.						
Parameters	10	15	11	Value of O.F. =	1500	
Constraints				LHS	OP	RHS
Crude Oil 1	2	1	0	100	<=	200
Crude Oil 2	3	3	2	300	<=	300
Time	1	1	1	100	<=	100
Min Gas 2	1	3	-3	300	>=	0

The Process Model Table

Activities /Processes /Actions

Resources / Products	Process 1	....	Process L	Available Resources	Cost / Price
Resource 1					Cost of Res. 1
...					
Resource n					Cost of Res. n
Cost per action	Cost per running Process 1 one time		Cost per running Process L one time		
Product 1					Price of Prod. 1
...					
Product m					Price of Prod. m

Amount of Product 1 consumed or produced each time we run Process 1

Amount of Resources 1 consumed each time we run Process 1

Solution Steps

The following is the steps to formulate process model:

- Create the process model table as shown in the previous slide;
- Define the decision variables as the number of time to run each process during this production cycle;
- Develop the objective function. Note profit usually is not given explicitly, one need to calculate the cost and revenue. For cost and revenue, also not usually given, in most of the case it involves some simple calculations;
- Develop the constraints, pay attention to the **implicit constraint**;
- Non-negativity constraints.

Exercise: Chemco

Problem Statement: Chemco produces three different products 1,2 and 3. Each pound of raw material costs $25. The raw material undergoes processing and produces 3 ounces of product 1 and 1 ounce product 2. It costs $1 and takes two hours of labor to process each pound of raw materials. that can be used to manufacture various type of gasoline. Each process involves blending oils in the company's catalytic cracker.

- Each ounce of product 1 can be used in three ways. Firstly, it can be sold for $10 per ounce. Second, it can be processed into one ounce of product 2. this requires 2 hours of labor and costs $1. Third, it can processed into one ounce of product 3. This requires three hours of labor and it costs $ 2;
- Each ounce of product 2 can be used in one of two ways. first, it can be sold for $20 per ounce. Second, it can be processed into one ounce of product 3. This requires one hour of labor and costs $6. Product 3 is sold for $30 per ounce.

Exercise: Chemco (c.t.d)

Question: The maximum number of ounces of products 1, 2, and 3 that can be sold are, respectively, 5000, 5000, and 3000. A maximum of 25,000 hour of labor are available. Determine how Chemco can maximize its profit.

Exercise: Chemco - Process Model Table

Step 1: create process model table:

	Process raw materials into products 1 and 2 X1	Process Product 1 into product 2 X2	Process Product 1 into product 3 X3	Process Product 2 into product 3 X4	Available / maximum to sell	Cost/ Price
Raw material	1 lb					$25
Labor	2 hrs.	2 hrs.	3 hrs.	1 hr.	25,000	
Direct Cost	$1	$1	$2	$6		
Product 1	+3 Oz	-1 Oz	-1 Oz	0	5,000 Oz	$10 /Oz
Product 2	+1 Oz	+1 Oz	0 Oz	-1 Oz	5,000 Oz	$20 /Oz
Product 3	0	0	+1	+1 Oz	3,000 Oz	$30 /Oz

Exercise: Chemco - Decision Variables

Step 2: identify decision variables:

- x_1 = how many times to process raw materials into products 1 and 2;
- x_2 = how many times to process product 1 into product 2;
- x_3 = how many times to process product 1 into product 3;
- x_4 = how many times to process product 2 into product 2.

Exercise: Chemco - Objective Function

Step 3: Establish objective function.

$$\text{Profit} = \text{Total Revenue} - \text{Total Cost}$$

$$\begin{aligned}
\text{Total Revenue} &= \text{sum (sales of each product} \times \text{ product price)} \\
&= (3x_1 - 1x_2 - 1x_3 + 0x_4) \times \$10 \quad \text{row for product 1} \\
&+ (1x_1 + 1x_2 + 0x_3 - 1x_4) \times \$20 \quad \text{row for product 2} \\
&+ (0x_1 + 0x_2 + 1x_3 + 1x_4) \times \$30 \quad \text{row for product 3} \\
&= \$50x_1 + \$10x_2 + \$20x_3 + \$10x_4
\end{aligned}$$

$$\text{Total Cost} = \$25 \times 1x_1 + (\$1x_1 + \$1x_2 + \$2x_3 + \$6x_4)$$

$$\text{Objective Function} \quad \max \$24x_1 + \$9x_2 + \$18x_3 + \$4x_4$$

Exercise: Chemco - Constraints

Step 4: develop the constraints.

- Resource Constraints: **None !**
- Labor (we take constraint from the labor raw)

$$\text{Labor} \quad 2x_1 + 2x_2 + 3x_3 + 1x_4 \leq 25,000 \text{ hours}$$

- Limits on sales (from the product rows)

$$\begin{aligned}
&\text{Product 1 Sales Limit} \quad 3x_1 - 1x_2 - 1x_3 + 0x_4 \leq 5,000 \text{ Oz} \\
&\text{Product 2 Sales Limit} \quad x_1 + 1x_2 + 0x_3 - 1x_4 \leq 5,000 \text{ Oz} \\
&\text{Product 3 Sales Limit} \quad 0x_1 + 0x_2 + 1x_3 + 1x_4 \leq 3,000 \text{ Oz}
\end{aligned}$$

Exercise: Chemco - Constraints (c.t.d) and Non-negative Constraints

Implicit Constraints: we need to make sure that production is more than consumption for products 1 and 2. These are the product rows that have negative numbers in them:

$$\begin{aligned} &\text{Product 1} \quad 3x_1 - 1x_2 - 1x_3 + 0x_4 \geq 0 \\ &\text{Product 2} \quad 1x_1 + 1x_2 + 0x_3 - 1x_4 \geq 0 \end{aligned}$$

Non-negative Constraints:

$$x_1, x_2, x_3, x_4 \geq 0$$

Exercise: Chemco - Complete LP Model

Linear Programming Model for Process Model - Chemco

$$X = \begin{cases} x_1: & \text{how many times to process raw materials into products 1 and} \\ x_2: & \text{how many times to process product 1 into product 2;} \\ x_3: & \text{how many times to process product 1 into product 3;} \\ x_4: & \text{how many times to process product 2 into product 3;} \end{cases}$$

$$\begin{aligned} \max. \; & 24x_1 + 9x_2 + 18x_3 + 4x_4 \\ \text{subject to: } & 2x_1 + 2x_2 + 3x_3 + 1x_4 \leq 25,00 \\ & 3x_1 - 1x_2 - 1x_3 + 0x_4 \leq 5,000 \\ & 1x_1 + 1x_2 + 0x_3 - 1x_4 \leq 5,000 \\ & 0x_1 + 0x_2 + 1x_3 + 1x_4 \leq 3,000 \\ & 3x_1 - 1x_2 - 1x_3 + 0x_4 \geq 0 \\ & 1x_1 + 1x_2 + 0x_3 - 1x_4 \geq 0 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Exercise: Chemco - Excel Solution 1

Excel Formulas:

Decision Variables								
Var Name	Raw Materials into Products 1 and 2	Product 1 into Product 2	Product 1 into Product 3	Product 2 into Product 3				
Math Symbol	x1	x2	x3	x4				
Var Value	3250	1750	3000	0				
O.F.								
	24	9	18	4	max profit	=SUMPRODUCT(B9:E9,B6:E6)		
Constraints					**LHS**	**OP**	**RHS**	**Units**
Labor	2	2	3	1	=SUMPRODUCT(B12:E12,B$6:E$6)	<=	25000	Hours
Product 1 Sales Limit	3	-1	-1	0	=SUMPRODUCT(B13:E13,B$6:E$6)	<=	5000	Oz
Product 2 Sales Limit	1	1	0	-1	=SUMPRODUCT(B14:E14,B$6:E$6)	<=	5000	Oz
Product 3 Sales Limit	0	0	1	1	=SUMPRODUCT(B15:E15,B$6:E$6)	<=	3000	Oz
Product 1 not negative	3	-1	-1	0	=SUMPRODUCT(B16:E16,B$6:E$6)	>=	0	Oz
Product 2 not negative	1	1	0	-1	=SUMPRODUCT(B17:E17,B$6:E$6)	>=	0	Oz

Exercise: Chemco - Excel Solution 2

Excel Solutions:

Chemco Problem

Decision Variables								
Var Name	Raw Materials into Products 1 and 2		Product 1 into Product 2		Product 1 into Product 3		Product 2 into Product 3	
Math Symbol	x1		x2		x3		x4	
Var Value		3250		1750		3000		0
O.F.								
	$	24.00	$	9.00	$	18.00	$	4.00

max profit: 147750

Constraints					**LHS**	**OP**	**RHS**	**Units**
Labor	2	2	3	1	19000	<=	25000	Hours
Product 1 Sales Limit	3	-1	-1	0	5000	<=	5000	Oz
Product 2 Sales Limit	1	1	0	-1	5000	<=	5000	Oz
Product 3 Sales Limit	0	0	1	1	3000	<=	3000	Oz
Product 1 not negative	3	-1	-1	0	5000	>=	0	Oz
Product 2 not negative	1	1	0	-1	5000	>=	0	Oz

References

Ahmed, Mahmoud (2014)

Lecture Slides of Operations Management

Thank You !!!