Production & Operations Management — Recitation 2

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Overview

What does it mean by solving the LP problem

Two major steps need to be made:

- Step 1: Extract information from the problem statement and formulate the model
- Step 2: Analyze and solve the LP formulated by using:
  (i) Graphical Solution: Only applicable in 2-dimension;
  (ii) Computer Solution: make use of Excel Solver (or others);
  (iii) Simplex Method: usually be taught in mathematics department and will not be touched in this course

Linear Function / Linear Combination

The linear programming differs from other optimization models in the format of constraints functions and objective functions. In linear programming, only linear function is 'good' for O.F. and C.S..

Remark

Linear function is of the format $a_0 + a_1 \times x_1 + a_2 \times x_2 + \cdots a_n \times x_n$, where a_i's are constant and x_i's are decision variables for $i = 1, 2, ..., n$. (Note a_i can be negative, which implicitly include the possibility of subtraction !!!)

Formulate the model

We will focus on formulate the model:

- **List and define the decision variables (D.V.)**: these typically represent quantities/activities specified in the question of the problem.
- **State the objective function (O.F.)**: Starting with max (for profit) or min (for cost), it is a linear function of decision variables (but not necessary all decision variables).
- **Develop the constraints (C.S.)** Constraints are system of linear equalities or inequalities, i.e., the left hand side (LHS) is a linear function of some but not necessary all decision variables and the right hand side (RHS) is a numerical value, they are connected by operators $\leq, \geq$ or $=$.
- **Write Non-negativity constraints (N.C.)** : Most of times, decision variables take only zero or positive number. $(x_i \geq 0)$

Example 1: Equipment Bags

Problem statement: Our firm makes equipment bags for professional camera equipment. We have two models: regular (selling price: $10) and deluxe (selling price: $9). We have a prior pricing agreement with our distributor, who is willing to buy as much of each kind of bag as we could conceivably make.

Four steps in making a bag:

- cutting and dyeing the material
- sewing
- finishing
- inspection and packing

Example 1: Equipment Bags (c.t.d)

Each regular bag requires:

- $\frac{7}{10}$ hour in cutting and dyeing
- $\frac{1}{2}$ hour in sewing
- 1 hour in finishing
- $\frac{1}{10}$ hour in inspection and packing

Each deluxe bag requires:

- 1 hour in cutting and dyeing
- $\frac{5}{6}$ hour in sewing
- $\frac{2}{3}$ hour in finishing
- $\frac{1}{4}$ hour in inspection and packing

Example 1: Equipment Bags (c.t.d)

Director of manufacturing estimates that following work hours will be available for manufacturing the bags during next production cycle:

- 630 hours for cutting and dyeing
- 600 hours for sewing
- 708 hours for finishing
- 135 hours for inspection and packing

Question: **How many** bags of each type should the firm produce to **maximize** its profit ?

Example 1: Equipment Bags (c.t.d) - Table

Step 0: extract and organize the information in the problem in a table (and/or a network diagram)

We introduce the Product-Mix-Table

Resources/ Sales Commitments	Products				Resource Availability, Commitment
	Product 1	Product 2	...	Product n	
Resource 1					
...					
Resource n					
Sales commitment					
Cost					
Revenue					
Profit					

Example 1: Equipment Bags (c.t.d) - Table

In our case the product-mix-table looks like the following:

Operation / Resources	Regular bag	Deluxe bag	Available time in Hours
Cut & dye time	7/10	1	630
Sewing time	1/2	5/6	600
Finishing time	1	2/3	708
Inspect and packing time	1/10	1/4	135
Profit	$10	$9	

Example 1: Equipment Bags (c.t.d) - Decision variable

Step 1: identify decision variable (D.V.)

Recall the question is: '**How many** bags of each type should the firm produce ...'. This tells us that we should define the two variables x_1 and x_2 as follows:

- $x_1 =$ the number of regular bags made in this production cycle
- $x_2 =$ the number of deluxe bags made in this production cycle

Remark

In your answer, you must write a complete definition like above. It includes an amount, a unit, and a time frame.

Example 1: Equipment Bags (c.t.d) - Decision variable

Modification of the table with adding decision variables:

Operation / Resources	Regular bag x_1	Deluxe bag x_2	Available time in Hours
Cut & dye time	7/10	1	630
Sewing time	1/2	5/6	600
Finishing time	1	2/3	708
Inspect and packing time	1/10	1/4	135
Profit	$10	$9	

Example 1: Equipment Bags (c.t.d) - Objective Function

Step 2: define the objective function (O.F.)

Again, from the question '... to **maximize** its profit ', we see immediately the objective function starts from maximization. Then, we also observe from the table that we have profit for producing each product. Therefore,

$$\begin{aligned} \text{The total profit} &= \$10 \times \text{number of regular bags}(x_1) \\ &+ \$9 \times \text{number of deluxe bags}(x_2) \end{aligned}$$

Mathematically, the O.F. is expressed as:

$$\max 10x_1 + 9x_2$$

Example 1: Equipment Bags (c.t.d) - the constraints

Step 3: establish the constraints (C.S.)

From the product-mix-table, we shall notice that each resource line in the table represents a constraint where:

- The LHS is a linear combination of decision variables in the following sense: decision variables are multiplied by the required resource that can be read off from the table to form one term and take summation over those terms;
- The RHS is the quantity in the available column;
- The operator is $\leq$ for operations and resource and $\geq$ for sales commitment.

Example 1: Equipment Bags (c.t.d) - the constraints

There are four steps for each steps we have a limited number and hours to distribute. For cutting and dyeing the operation can not exceed 630 hours. And

- **The LHS**: the number of hours required to cut and dye regular and deluxe bags: $(\frac{7}{10})x_1 + 1x_2$;
- **The operator**: the consumption of the time has a limit so it is $\leq$;
- **The RHS**: The limited hours is 630 hours.

Operation / Resources	Regular bag x_1	Deluxe bag x_2	Available time in Hours
Cut & dye time	7/10	1	630
Sewing time	1/2	5/6	600
Finishing time	1	2/3	708
Inspect and packing time	1/10	1/4	135
Profit	\$10	\$9	

Example 1: Equipment Bags (c.t.d) - the constraints

Consequently, the mathematical statement of the first constraint reads:

$$\text{(Cutting and Dyeing)} \quad \frac{7}{10}x_1 + 1x_2 \leq 630 \text{ hours}$$

Similarly, the other constraints reads

$$\text{(Sewing)} \quad \frac{1}{2}x_1 + \frac{5}{6}x_2 \leq 600 \text{ hours}$$

$$\text{(Finishing)} \quad 1x_1 + \frac{2}{3}x_2 \leq 708 \text{ hours}$$

$$\text{(Inspection \& Packing)} \quad \frac{1}{10}x_1 + \frac{1}{4}x_2 \leq 135 \text{ hours}$$

Example 1: Equipment Bags (c.t.d) - non-negative constraints

Step 4: complete the program by adding non-negative constraints (N.C.)

This is the easiest one to formulate. Since the product is bag, our decision can only be making some or not making, thus a non-negative constraint is required:

$$x_1 \geq 0$$
$$x_2 \geq 0$$

Or, simply

$$x_1, x_2 \geq 0$$

Example 1 - the complete LP program

$$x_1 = \text{ the number of regular bags made in this production cycle}$$
$$x_2 = \text{ the number of deluxe bags made in this production cycle}$$

$$\text{Max } 10x_1 + 9x_2$$

$$\begin{aligned}
\text{Subject to: } \frac{7}{10}x_1 + 1x_2 &\le 630 \\
\frac{1}{2}x_1 + \frac{5}{6}x_2 &\le 600 \\
1x_1 + \frac{2}{3}x_2 &\le 708 \\
\frac{1}{10}x_1 + \frac{1}{4}x_2 &\le 135 \\
x_1, x_2 &\ge 0
\end{aligned}$$

Example 2: Gourmet Gulfco - Description

Problem statement:

Gulfco is a company that produces chemicals. In its production process, Gulfco mixes 3 raw materials to produce two products: Fuel additive and Solvent base. To produce one ton of Fuel additive, the company mixes 2/5 ton of material 1 and 3/5 ton of material 3. To produce Solvent base, the company mixes 1/2 ton of material 1, 1/5 ton of material 2, and 3/10 ton of material 3. The profit of each ton of Fuel additive produced is \$40 and the profit of each ton of Solvent base produced is \$30.

Example 2: Gourmet Gulfco - Description (c.t.d)

Gulfco has limited quantities of the raw material as specified following:

- The amount of Material 1 available for next production cycle is 40 tons
- The amount of Material 2 available for next production cycle is 10 tons
- The amount of Material 3 available for next production cycle is 42 tons

Question: formulate a linear programming model for the next production cycle (to maximize the profit).

Example 2: Gourmet Gulfco (c.t.d) - Table

Step 0: set up the product-mix-table

Resources	Fuel Additive	Solvent Base	Available Materials in tons
Material 1			
Material 2			
Material 3			
Profit			

Step 0: set up the product-mix-table

Resources	Fuel Additive	Solvent Base	Available Materials in tons
Material 1	2/5	1/2	40
Material 2	0	1/5	10
Material 3	3/5	3/10	42
Profit	$40	$30	

Remark

The production-mix-table is required for assignments

Example 2: Gourmet Gulfco (c.t.d) - Decision variable

Step 1: identify decision variable (D.V.)

Note: Different from the bag problem, this problem doesn't ask directly 'how many tons to produce from each product', it is clear that we need to decide on that to find the maximum profit.

Example 2: Gourmet Gulfco (c.t.d) - Decision variable

Step 1: identify decision variable (D.V.)

For this reason, we define the two variables x_1 and x_2 as follows:

- $x_1 =$ the amount of Fuel Additive in tons made in the next production cycle
- $x_2 =$ the amount of Solvent Base in tons made in the next production cycle

Remark

Again, in your answer, you must write a complete definition like above. It includes an amount, a unit, and a time frame !!!

Example 2: Gourmet Gulfco (c.t.d) - Decision variable

Modification of the table with adding decision variables:

Resources	Fuel Additive x_1	Solvent Base x_2	Available Materials in tons
Material 1	2/5	1/2	40
Material 2	0	1/5	10
Material 3	3/5	3/10	42
Profit	$40	$30	

Example 2: Gourmet Gulfco (c.t.d) - Objective Function

Step 2: define the objective function (O.F.) The profit obtained for each ton of Fuel Additive is \$40, while that for each ton of Solvent Base is \$30. Thus,

$$\begin{aligned}\text{The total profit} &= \$40 \times \text{amount in tons of Fuel Additive}(x_1) \\ &+ \$30 \times \text{amount in tons of Solvent Base}(x_2)\end{aligned}$$

Mathematically, the O.F. is expressed as:

$$\max 40x_1 + 30x_2$$

Example 2: Gourmet Gulfco (c.t.d) - the constraints

Step 3: establish the constraints (C.S.) From the product-mix-table,

Resources	Fuel Additive x_1	Solvent Base x_2	Available Materials in tons
Material 1	2/5	1/2	40
Material 2	0	1/5	10
Material 3	3/5	3/10	42
Profit	$40	$30	

We can develop the following constraints:

Step 3: establish the constraints (C.S.)

From the product-mix-table,

Resources	Fuel Additive x_1	Solvent Base x_2	Available Materials in tons
Material 1	2/5	1/2	40
Material 2	0	1/5	10
Material 3	3/5	3/10	42
Profit	\$40	\$30	

We can develop the following constraints:

$$\text{(Material 1)} \quad \frac{2}{5}x_1 + \frac{1}{2}x_2 \leq 40$$

$$\text{(Material 2)} \quad 0x_1 + \frac{1}{5}x_2 \leq 10$$

$$\text{(Material 3)} \quad \frac{3}{5}x_1 + \frac{3}{10}x_2 \leq 42$$

Example 2: Gourmet Gulfco (c.t.d) - non-negative constraints

Step 4: complete the program by adding non-negative constraints (N.C.) This is the easiest one to formulate.

$$x_1 \geq 0$$
$$x_2 \geq 0$$

Or, simply

$$x_1, x_2 \geq 0$$

Example 2 - the complete LP program

$$\begin{aligned} x_1 &= \text{ the amount of Fuel Additive in tons made in the next production cycle} \\ x_2 &= \text{ the amount of Solvent Base in tons made in the next production cycle} \end{aligned}$$

$$\text{Max } 40x_1 + 3x_2$$

$$\begin{aligned} \text{Subject to: } & \frac{2}{5}x_1 + \frac{1}{2}x_2 \leq 40 \\ & 0x_1 + \frac{1}{5}x_2 \leq 10 \\ & \frac{3}{5}x_1 + \frac{3}{10}x_2 \leq 42 \\ & x_1, x_2 \geq 0 \end{aligned}$$

References

Ahmed, Mahmoud (2014)
Lecture Slides of Operations Management

Thank You !!!