# Production & Operations Management — Recitation 3

Jianing Yao

Rutgers University, the State University of New Jersey

*yaojn_1@hotmail.com*

September 16, 2014

# Overview

# Linear Programing Assumptions

It is **not** that every problem can modeled as a linear programming problem (although you can simplify the problem to LP, still not all the time).There are lives beyond linear programming, such as, nonlinear programming, stochastic programming, e.t.c.. A natural question one may ask is that what's the assumptions of linear programming ?

# Linear Programing Assumptions - Linearity

The linear programming differs from other optimization models in the format of constraints functions and objective functions. In linear programming, only linear function is 'good' for O.F. and C.S..

## Remark

Linear function is of the format $a_0 + a_1 \times x_1 + a_2 \times x_2 + \cdots a_n \times x_n$, where $a_i$'s are constant and $x_i$'s are decision variables for $i = 1, 2, ..., n$. (Note $a_i$ can be negative, which implicitly include the possibility of subtraction !!!)

# Linear Programing Assumptions - Divisibility& Non-negativity

When considering **non-negative constraints**, the only requirement is non-negative but not restricted to integer ! Later on, we will learn a little bit about integer programming, in which we need to have not only non-negativity but also integrality constraints. For example, in the bags problem, we can only allowed to make integer-valued number of bags, or famous '0-1' bags problem (I will explain).

# Linear Programing Assumptions - Certainty

In linear programming, all the parameters in objective functions and constraints are deterministic (known). There is **no uncertainty** on the parameter, for example, if the constraint is

$$3x_1 + Yx_2 \leq 2$$

where $Y$ is random variable (unknown) is not allowed. Actually this formulation is wrong !

**Remark**

We shall know the distribution of $Y$ (if you learn probability), and that should be formulated as a stochastic programming problem (graduate level course).

# Gourmet Gulfco - Description

**Problem statement:**

Gulfco is a company that produces chemicals. In its production process, Gulfco mixes 3 raw materials to produce two products: Fuel additive and Solvent base. To produce one ton of Fuel additive, the company mixes 2/5 ton of material 1 and 3/5 ton of material 3. To produce Solvent base, the company mixes 1/2 ton of material 1, 1/5 ton of material 2, and 3/10 ton of material 3. The profit of each ton of Fuel additive produced is \$40 and the profit of each ton of Solvent base produced is \$30.

**Gulfco has limited quantities of the raw material as specified following:**

- The amount of Material 1 available for next production cycle is 40 tons
- The amount of Material 2 available for next production cycle is 10 tons
- The amount of Material 3 available for next production cycle is 42 tons

Question: formulate a linear programming model for the next production cycle (to maximize the profit).

# Recall The Product-Mix-Table

How does the *Product-Mix-Table* looks like in the Gourmet Gulfco case ?

- In this gourmet problem, three materials to produce two products
- Given profit directly, no need to compute from the cost and revenue
- No other sales and production constraints

| Resources/ Sales Commitments | Products | | | | Resource Availability, Commitment |
|---|---|---|---|---|---|
| | Product 1 | Product 2 | ... | Product n | |
| Resource 1 | | | | | |
| ... | | | | | |
| Resource n | | | | | |
| Sales commitment | | | | | |
| Cost | | | | | |
| Revenue | | | | | |
| Profit | | | | | |

**Step 0: set up the product-mix-table**

| Resources | Fuel Additive | Solvent Base | Available Materials in tons |
|---|---|---|---|
| Material 1 | | | |
| Material 2 | | | |
| Material 3 | | | |
| **Profit** | | | |

# Gourmet Gulfco (c.t.d) - Table

**Step 0: set up the product-mix-table**

| Resources | Fuel Additive | Solvent Base | Available Materials in tons |
|---|---|---|---|
| Material 1 | 2/5 | 1/2 | 40 |
| Material 2 | 0 | 1/5 | 10 |
| Material 3 | 3/5 | 3/10 | 42 |
| **Profit** | $40 | $30 | |

**Remark**

The production-mix-table is required for assignments

**Step 1: identify decision variable (D.V.)**

**Note:** Different from the bag problem, this problem doesn't ask directly 'how many tons to produce from each product', it is clear that we need to decide on that to find the maximum profit.

**Step 1: identify decision variable (D.V.)**

For this reason, we define the two variables $x_1$ and $x_2$ as follows:

- $x_1 =$ **the amount of Fuel Additive** in **tons** made in the **next production cycle**
- $x_2 =$ **the amount of Solvent Base** in **tons** made in the **next production cycle**

**Remark**

**Very important !!!**. Include an amount, a unit, and a time frame !!!

Modification of the table with adding decision variables:

| Resources | Fuel Additive $x_1$ | Solvent Base $x_2$ | Available Materials in tons |
|---|---|---|---|
| Material 1 | 2/5 | 1/2 | 40 |
| Material 2 | 0 | 1/5 | 10 |
| Material 3 | 3/5 | 3/10 | 42 |
| **Profit** | \$40 | \$30 | |

# Gourmet Gulfco (c.t.d) - Objective Function

**Step 2: define the objective function (O.F.)** The profit obtained for each ton of Fuel Additive is \$40, while that for each ton of Solvent Base is \$30. Thus,

$$\begin{aligned}\text{The total profit} &= \$40 \times \text{amount in tons of Fuel Additive}(x_1) \\ &+ \$30 \times \text{amount in tons of Solvent Base}(x_2)\end{aligned}$$

Mathematically, the O.F. is expressed as:

$$\max 40x_1 + 30x_2$$

# Gourmet Gulfco (c.t.d) - the constraints

**Step 3: establish the constraints (C.S.)** From the product-mix-table,

| Resources | Fuel Additive $x_1$ | Solvent Base $x_2$ | Available Materials in tons |
|---|---|---|---|
| Material 1 | 2/5 | 1/2 | 40 |
| Material 2 | 0 | 1/5 | 10 |
| Material 3 | 3/5 | 3/10 | 42 |
| **Profit** | $40 | $30 | |

We can develop the following constraints:

**Step 3: establish the constraints (C.S.)**

From the product-mix-table,

| Resources | Fuel Additive $x_1$ | Solvent Base $x_2$ | Available Materials in tons |
|---|---|---|---|
| Material 1 | 2/5 | 1/2 | 40 |
| Material 2 | 0 | 1/5 | 10 |
| Material 3 | 3/5 | 3/10 | 42 |
| Profit | \$40 | \$30 | |

We can develop the following constraints:

$$\text{(Material 1)} \quad \frac{2}{5}x_1 + \frac{1}{2}x_2 \leq 40$$

$$\text{(Material 2)} \quad 0x_1 + \frac{1}{5}x_2 \leq 10$$

$$\text{(Material 3)} \quad \frac{3}{5}x_1 + \frac{3}{10}x_2 \leq 42$$

**Step 4: complete the program by adding non-negative constraints (N.C.)** This is the easiest one to formulate.

$$x_1 \geq 0$$
$$x_2 \geq 0$$

Or, simply

$$x_1, x_2 \geq 0$$

# Gourmet Gulfco (c.t.d) - the complete LP program

$$x_1 = \text{ the amount of Fuel Additive in tons made in the next production cycle}$$
$$x_2 = \text{ the amount of Solvent Base in tons made in the next production cycle}$$

$$\text{Max } 40x_1 + 3x_2$$

$$\begin{aligned}
\text{Subject to: } & \frac{2}{5}x_1 + \frac{1}{2}x_2 \leq 40 \\
& 0x_1 + \frac{1}{5}x_2 \leq 10 \\
& \frac{3}{5}x_1 + \frac{3}{10}x_2 \leq 42 \\
& x_1, x_2 \geq 0
\end{aligned}$$

Yellow Region Decision value will surface when you solve the problem (placeholder) Blue/Light Blue Region Input all known parameters

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | Gulfco Problem | | |
| 2 | | | | | | | |
| 3 | Decision Variables | | | | | | |
| 4 | Variable names | Fuel additive | Solvent base | | | | |
| 5 | Math Symbol | x1 | x2 | | | | |
| 6 | Variable Value | | | | | | |
| 7 | | | | | | | |
| 8 | O. F. | | | | | | |
| 9 | Profit | 40 | 30 | Max Profit | 0 | | |
| 10 | | | | | | | |
| 11 | Constraints | | | LHS | OP | RHS | Units |
| 12 | Material 1 | 0.4 | 0.5 | 0 | <= | 40 | Tons |
| 13 | Material 2 | 0 | 0.2 | 0 | <= | 10 | Tons |
| 14 | Material 3 | 0.6 | 0.3 | 0 | <= | 42 | Tons |
| 15 | | | | | | | |

*Small Modification*

*Last Row in product mix table*

*Availability*

*Product 1, Product 2*

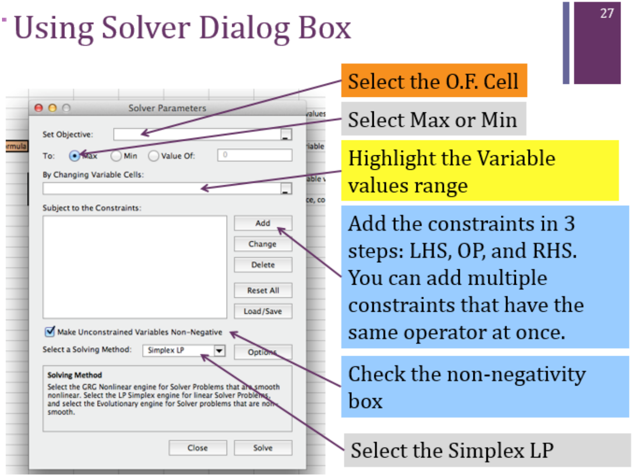
**In this step, you are essentially establish the product-mix-table on the EXCEL !!**

Use function SUMPRODUCT with absolute reference, the $ sign

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | Gulfco Problem | | |
| 2 | | | | | | | |
| 3 | Decision Varia | | | | | | |
| 4 | Variable names | Fuel additive | olvent bas | | | | |
| 5 | Math Symbol | x1 | x2 | | | | |
| 6 | Variable Value | | | | | | |
| 7 | | | | | | | |
| 8 | O. F. | | | | | | |
| 9 | Profit | 40 | 30 | Max Profit | =SUMPRODUCT(B9:C9,B6:C6) | | |
| 10 | | | | | | | |
| 11 | Constraints | | | LHS | OP | RHS | Units |
| 12 | Material 1 | =2/5 | =1/2 | =SUMPRODUCT(B12:C12,B$6:C$6) | <= | 40 | Tons |
| 13 | Material 2 | | =1/5 | =SUMPRODUCT(B13:C13,B$6:C$6) | <= | 10 | Tons |
| 14 | Material 3 | =3/5 | =3/10 | =SUMPRODUCT(B14:C14,B$6:C$6) | <= | 42 | Tons |

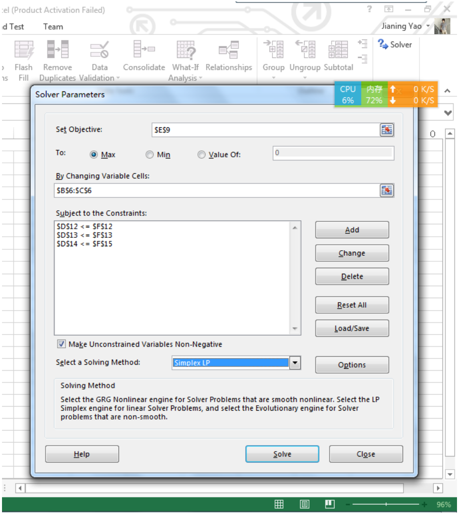
**In yellow region values are delayed, but one should use them as there are values. At the end, the solution will tell what they really are.**

Introduction of LP solver !



# Computer Solution - Solution 2

Using Solvers in Excel to solve Gourmet Gulfco Problem



Here comes the solution !

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | Gulfco Problem | | |
| 2 | | | | | | | |
| 3 | **Decision Variables** | | | | | | |
| 4 | Variable names | Fuel additive | Solvent base | | | | |
| 5 | Math Symbol | x1 | x2 | | | | |
| 6 | Variable Value | 50 | 40 | (Decision Variable) | | | |
| 7 | | | | | | | |
| 8 | **O.F.** | | | | | | |
| 9 | Profit | 40 | 30 | Max Profit | 3200 | (Optimal Value) | |
| 10 | | | | | | | |
| 11 | **Constraints** | | | **LHS** | **OP** | **RHS** | **Units** |
| 12 | Material 1 | 0.4 | 0.5 | 40 | <= | 40 | Tons |
| 13 | Material 2 | 0 | 0.2 | 8 | <= | 10 | Tons |
| 14 | Material 3 | 0.6 | 0.3 | 42 | <= | 42 | Tons |
| 15 | | | | (LHS result) | | | |
| 16 | | | | | | | |

# Binding Constraints

The LHS value represents the actual resources consumed to achieve the optimal solution.

**Definition**

A constraint that has LHS $=$ RHS is called **Binding**.

# Binding Constraints

When resources consumed are less than available or when production is more than sales commitment, we have either **slack** or **surplus**

**Definition**

In a "Less than or Equals" constraint,

$$\text{Slack} = \text{RHS} - \text{LHS}$$

While in a "Greater than or Equals" constraint,

$$\text{Slack} = \text{LHS} - \text{RHS}$$

Binding if and only if no Slack or Surplus.

# Binding, Surplus and Slack (c.t.d)

In Gourmet Gulfco problem,

- Which constraints are binding ?
- Which constraint has surplus or slack and how much ?

# Binding, Surplus and Slack (c.t.d)

**Solution**

- First and third constraints are binding since RHS $-$ LHS $= 0$ for each of them;
- For material 2, there is a slack of $10 - 8 = 2$ tons

**Remark**

The constraint has a slack not surplus since we have a "less than or equals" situation.

# References

Ahmed, Mahmoud (2014)
Lecture Slides of Operations Management

Thank You !!!