

public static void fName (int a, char c)

{ SOP("Hello");

SOP(a);
SOP(c);

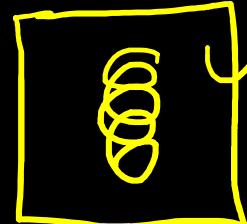
return;

}

main
fName();

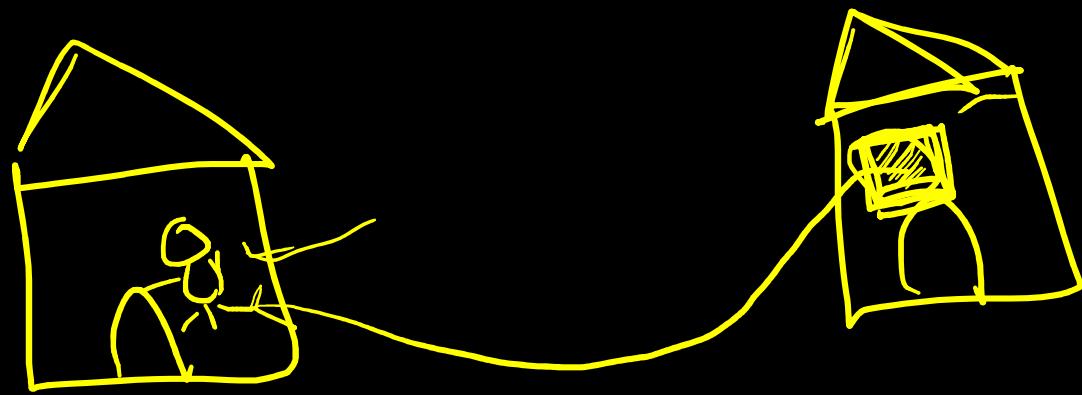
fName
a, c
return a;

IP

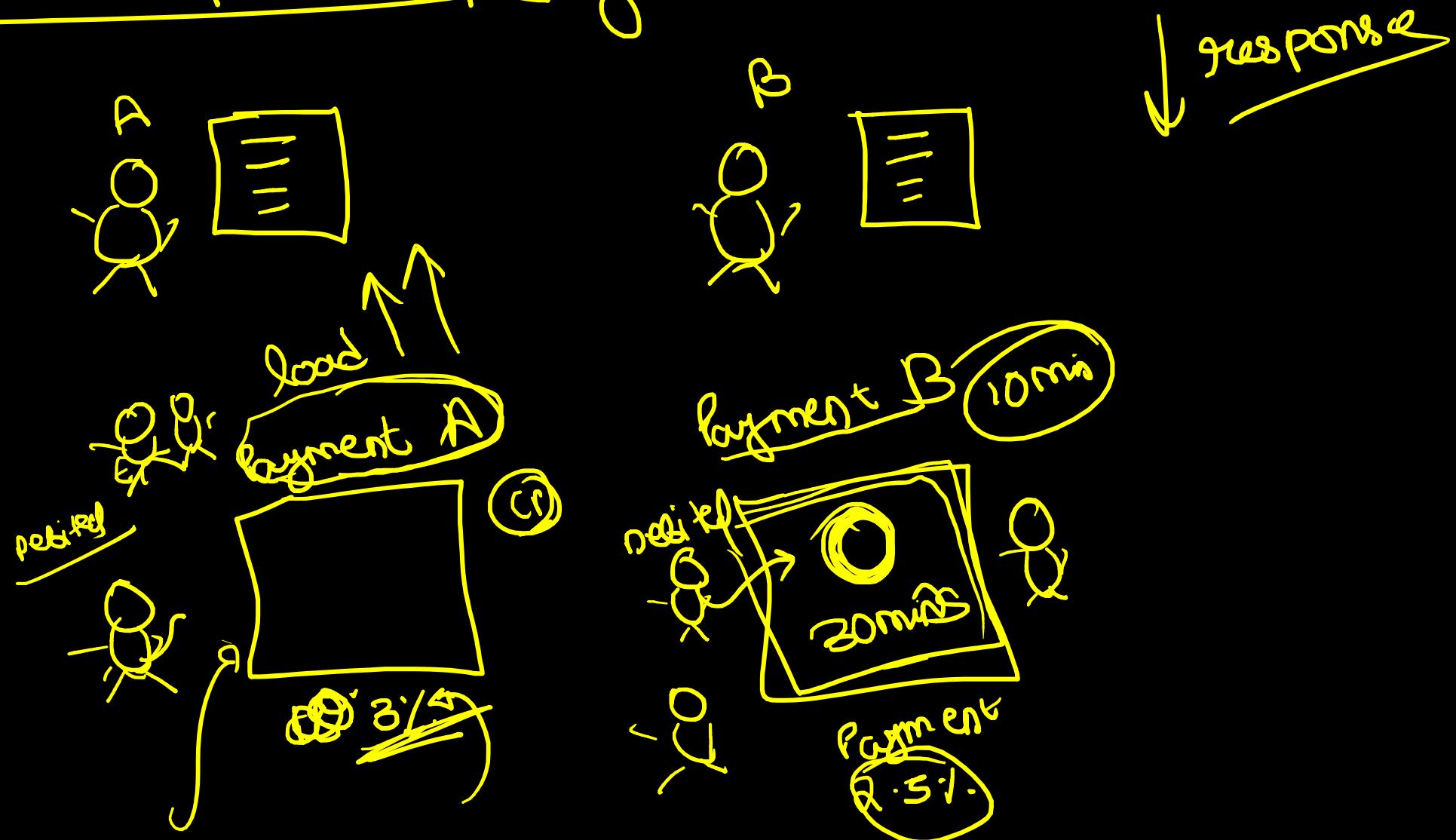


Hello
S
'D'

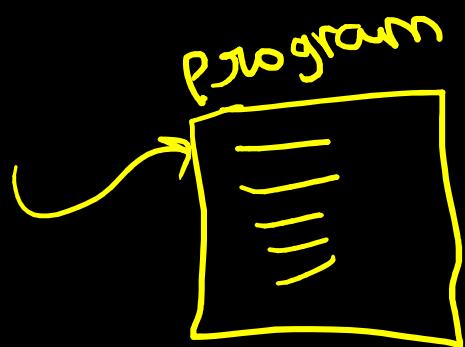
main
fName(b, 'c');
SOP(a);



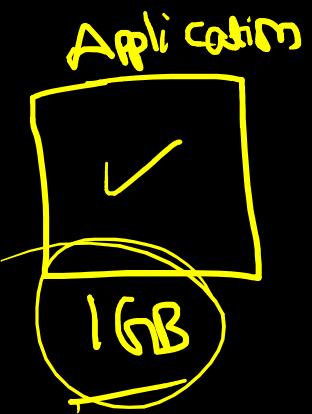
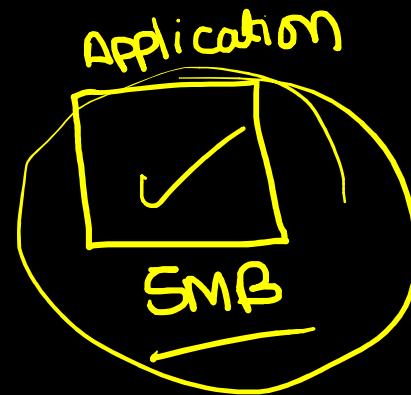
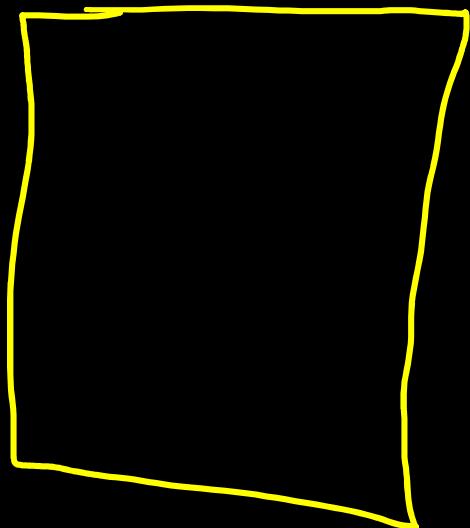
Time and Space Complexity

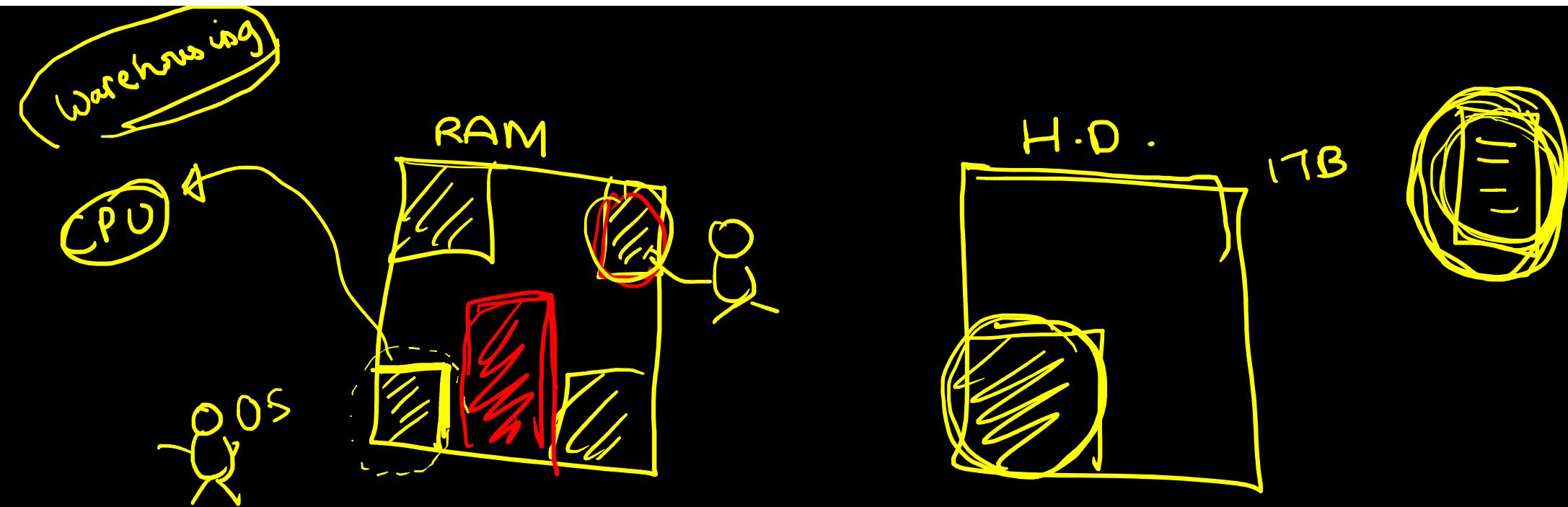


Space



RAM

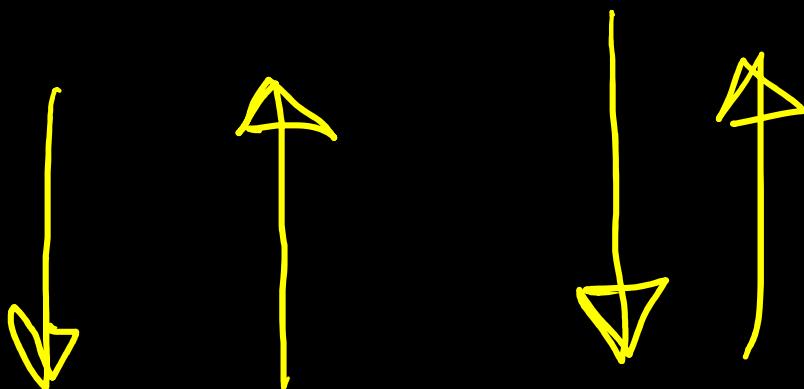


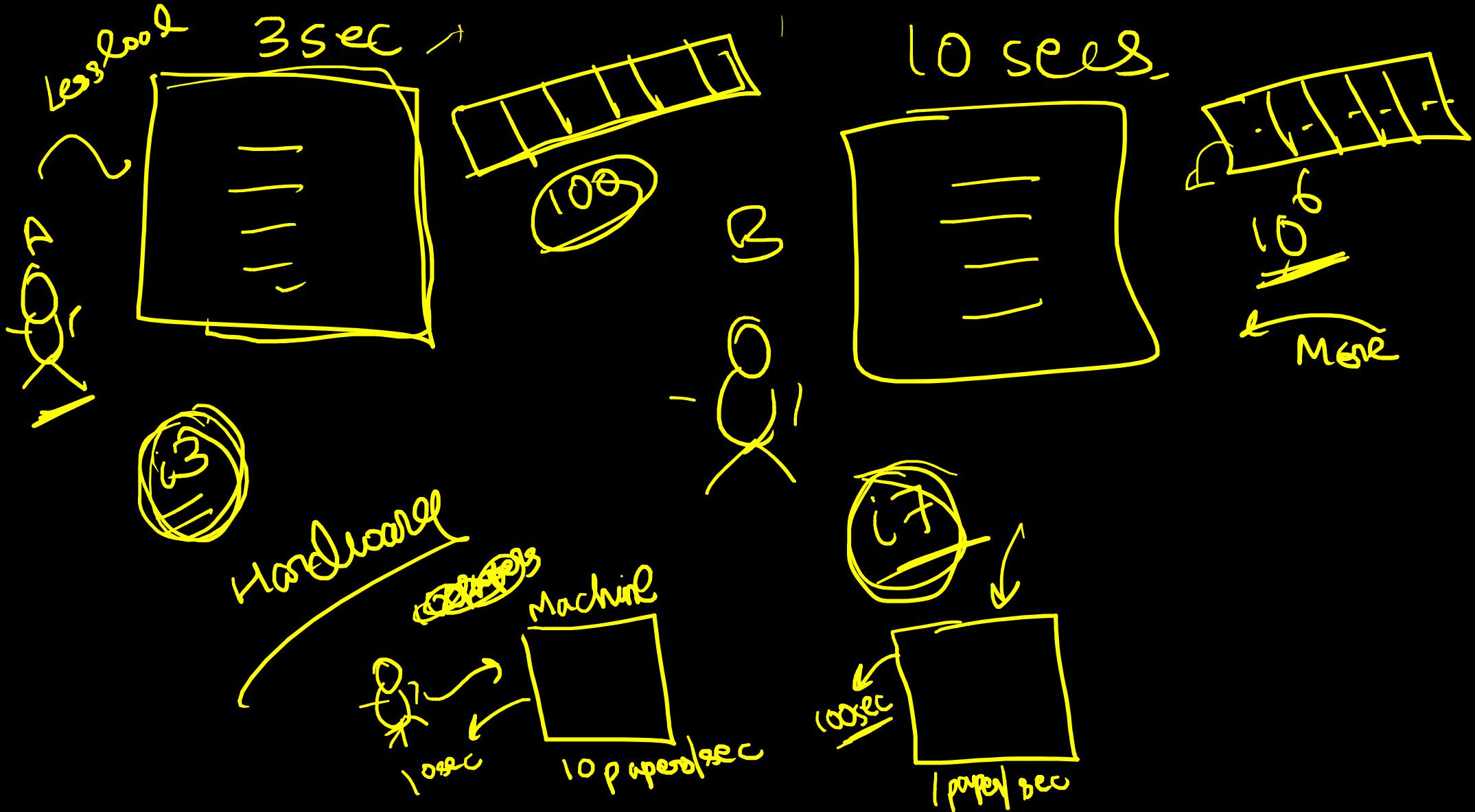


P.P.T
25m

Time & Space

Time / Space





Time Complexity

→ Time taken by the P program

as a fn of i/p size.

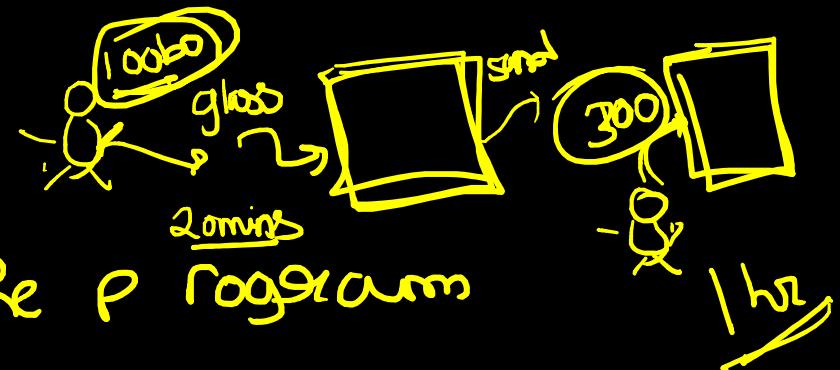
$$T = f(n)$$

B

bottles
100 → 500 min

B

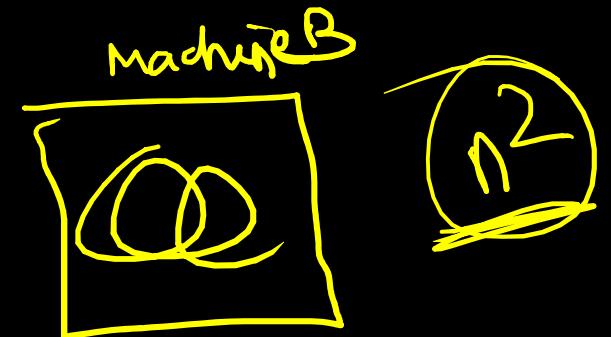
1000 → 5000 min





100 → 500 sec
1000 → 5000 sec

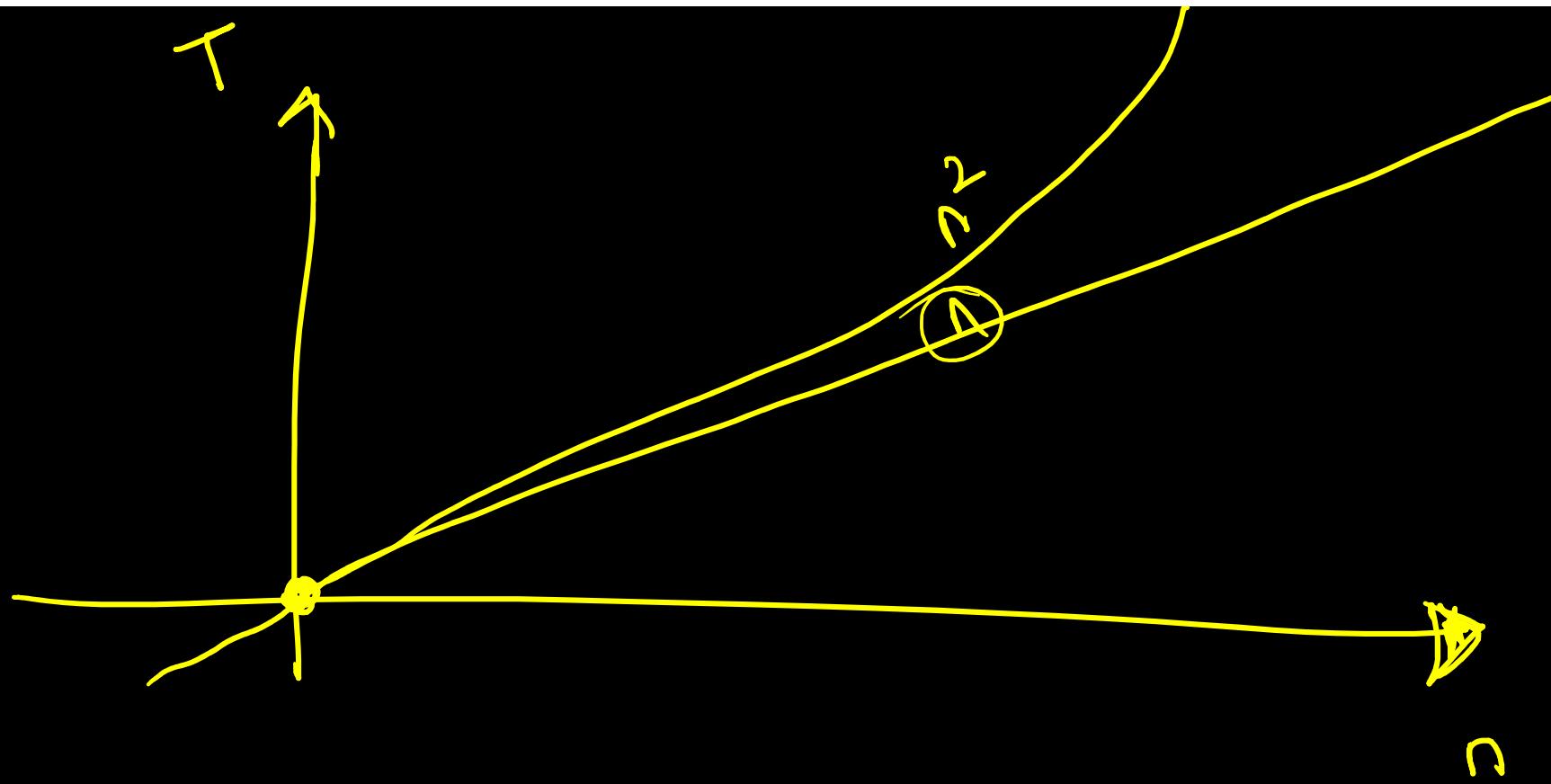
$T \propto n^1$



5 → 25 sec
10 → 100 sec²

15 → 225

$T \propto n^2$



int n; ↪

$$n = \cancel{10^5} 10^5$$

int a = 0; ↪

```
for (int i = 0; i < n; i++)  
    a = a + i;
```

100 times

$$10^5$$



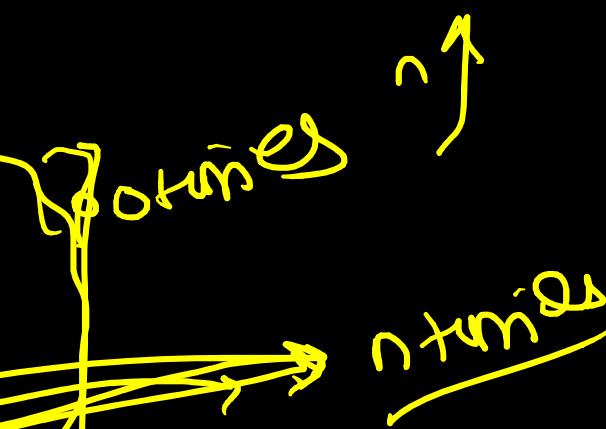
$T \propto n$

```
int n = sc.nextInt();
```

```
int a=0;
```

```
for( int i=0; i<n; i++ )
```

```
    a=a+i;
```



n times

T α n

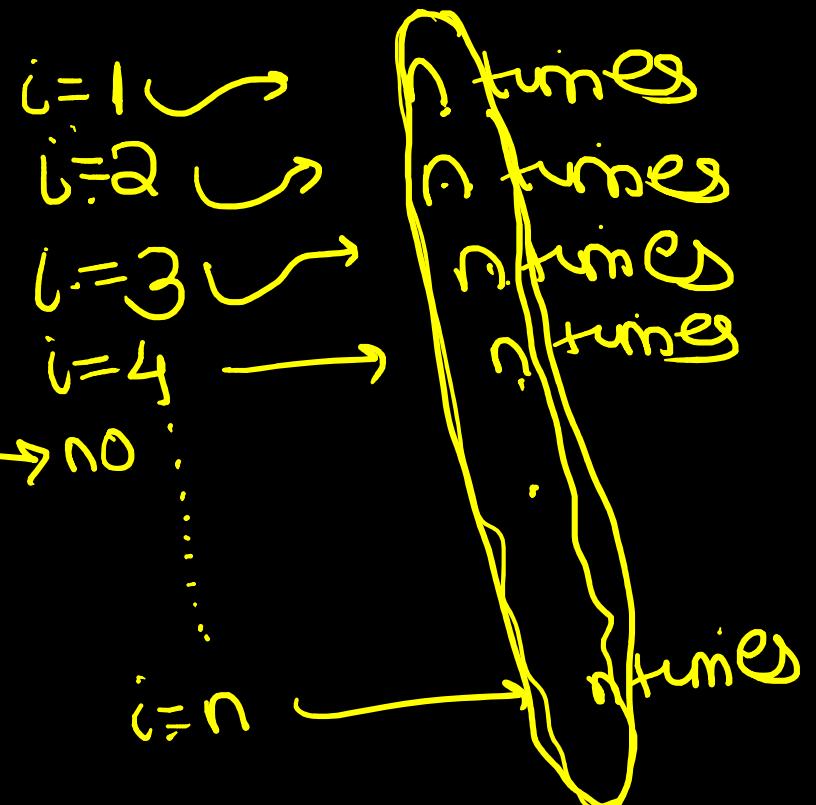
int n; ✓
int a = 0 ✓

for (int i=1; i<=n; i++)
{ for (int j=1; j<=i; j++)
{ cout << "Hello"; } }

n=3 → 9
n=4 → 16

$T \propto n^2$

$$n + n + n + n + \dots = \underline{\underline{n}}$$



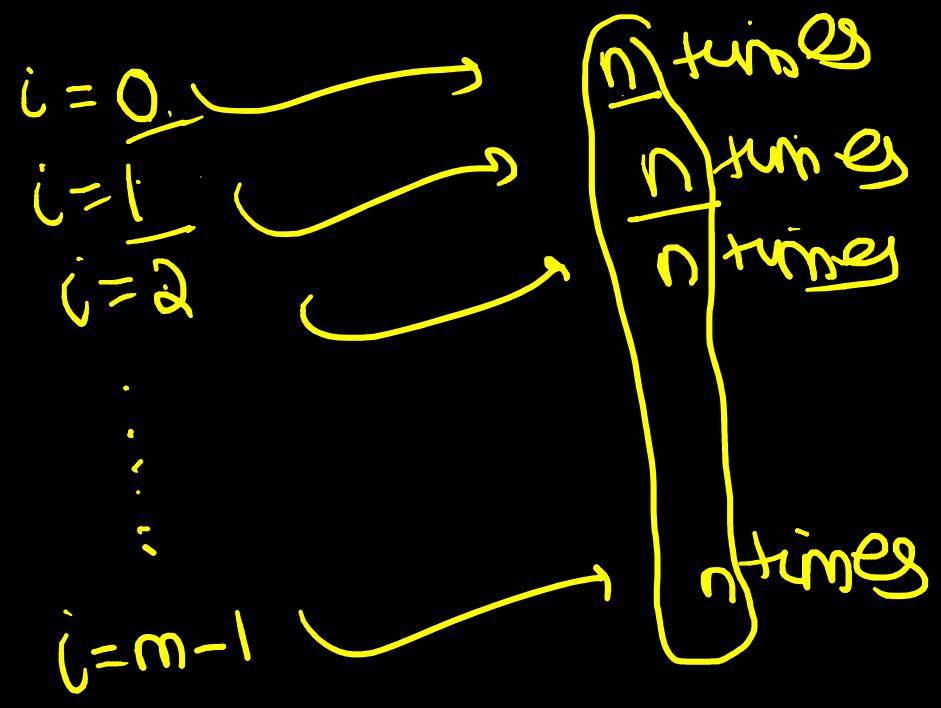
```

for( i=0; i<m; i++)
{
    for( j=0; j<n; j++)
    {
        cool("Hi")
    }
}

```

~~loop~~

~~if else~~



$n + n + n + n +$

n times $m * n$

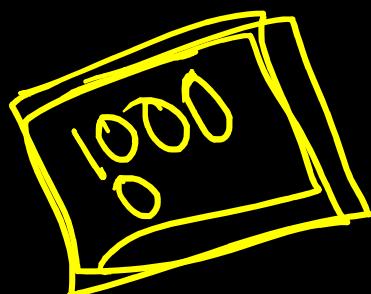
Space $\sim f(n)$

12 Bytes



$n=100$

Space's constant



so

int i

int a = 0

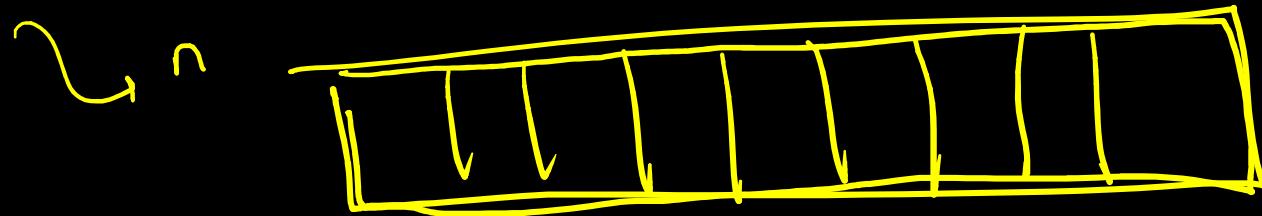
for (int i = 0; i < n; i++)

{ a = a + i; }

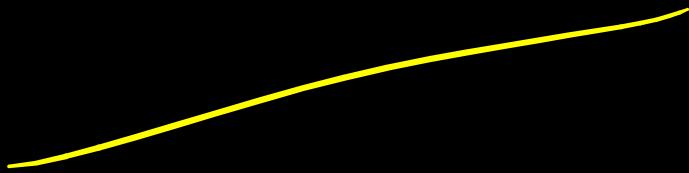
$2^{31} - 1$

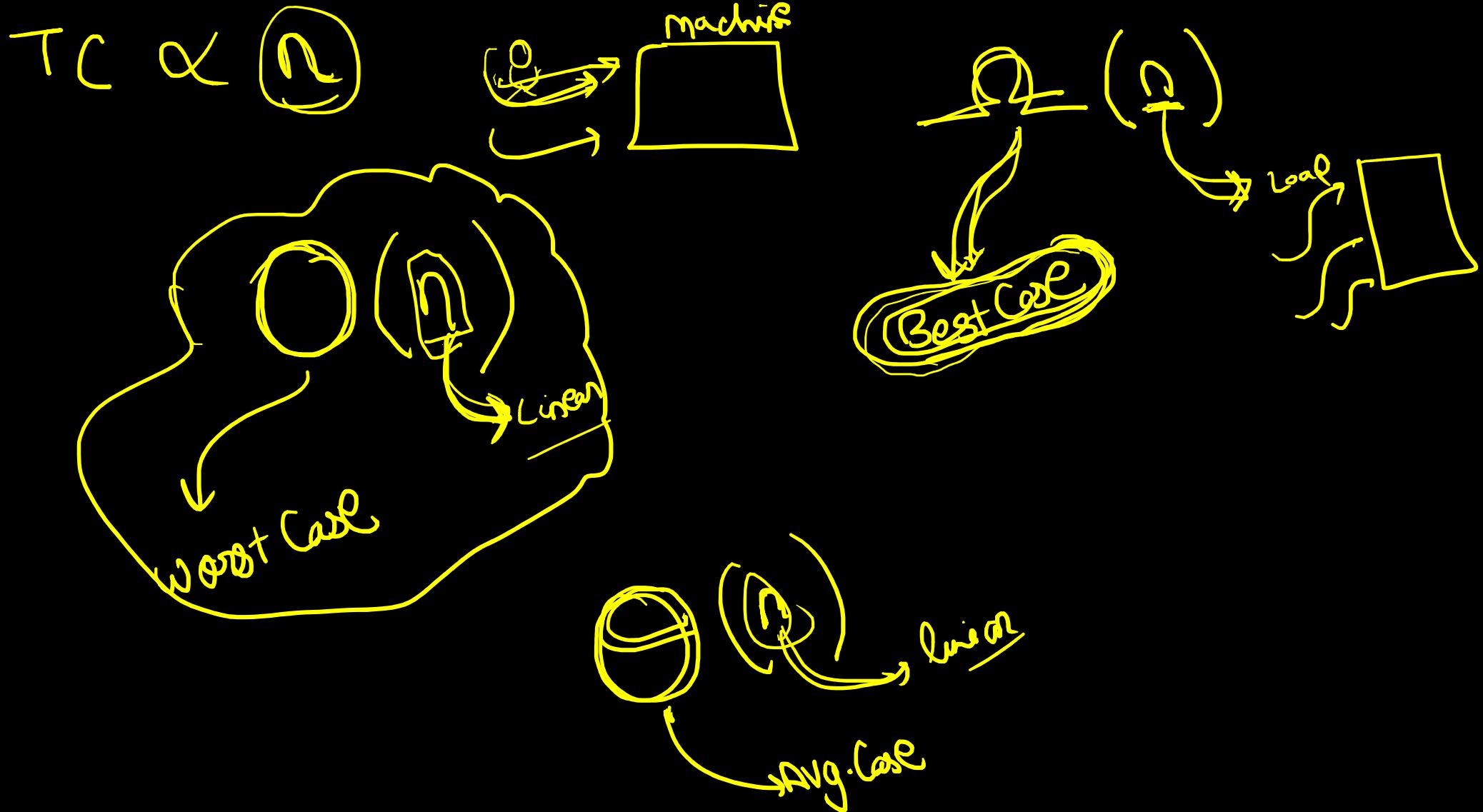


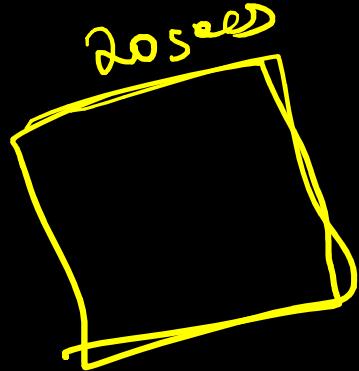
$n = 1000000$



S X O

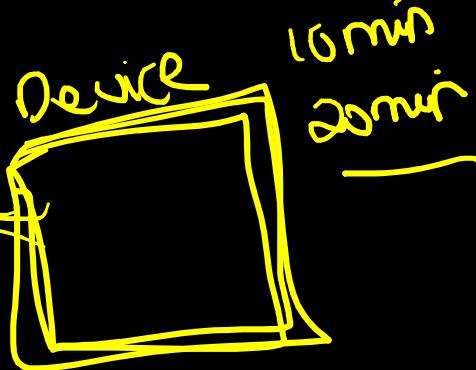
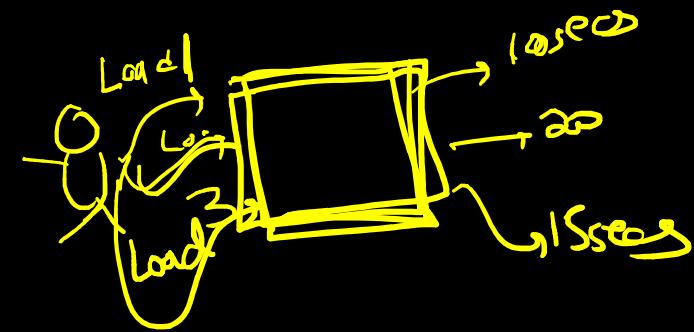


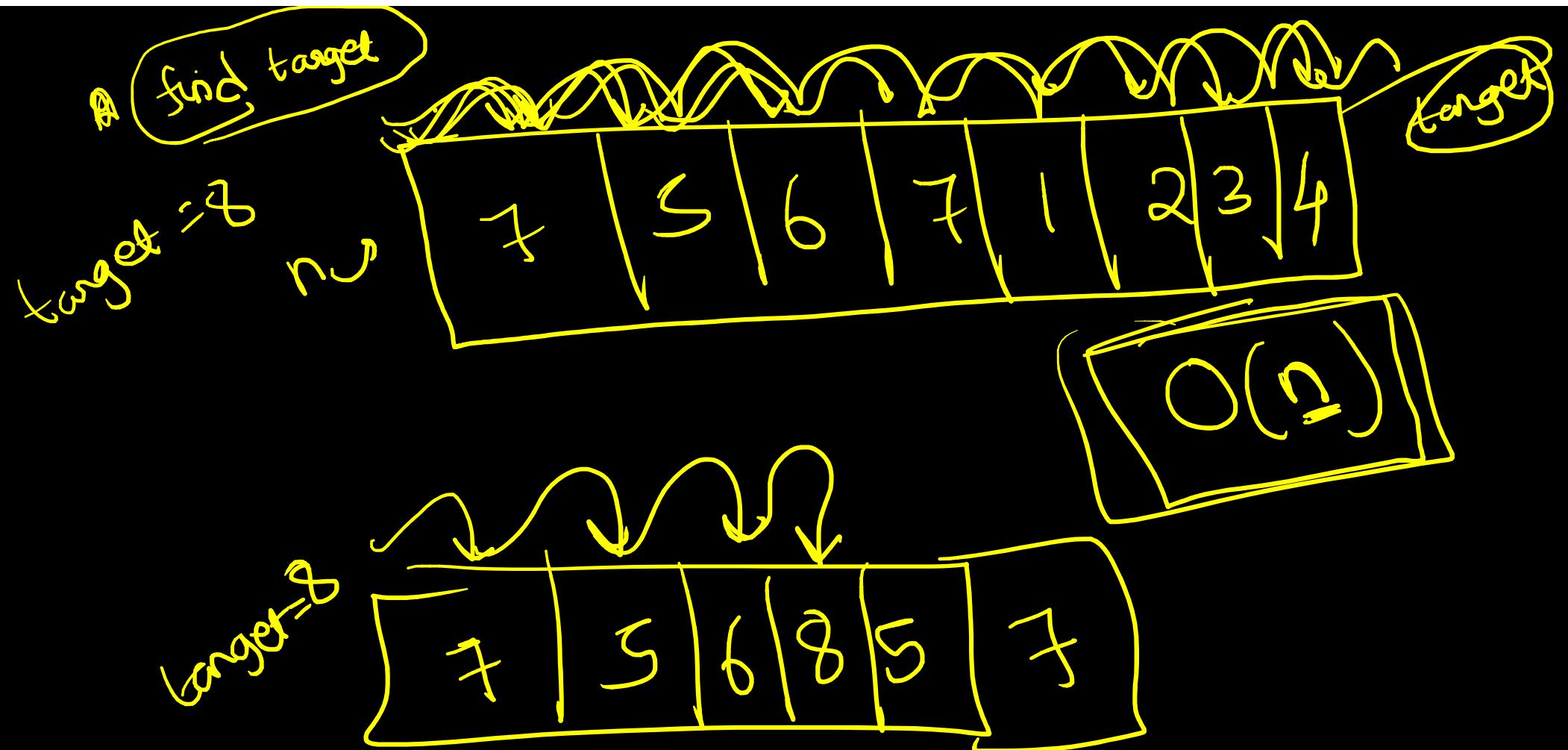


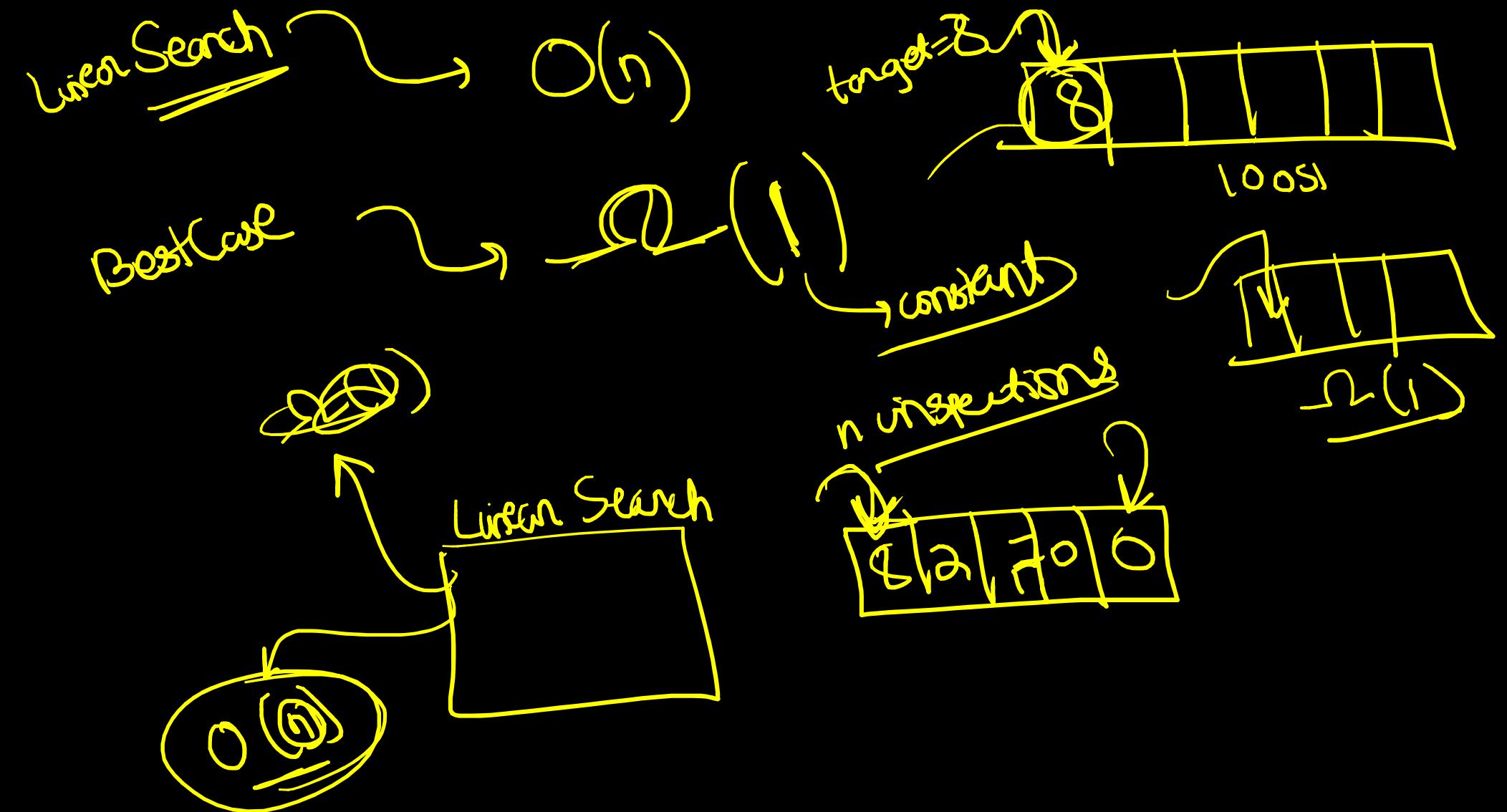


```
for (int i=0; i<n; i++)  
{ sop("needle"); }
```

$O(n)$
 $\Omega(n)$
 $\Theta(n)$

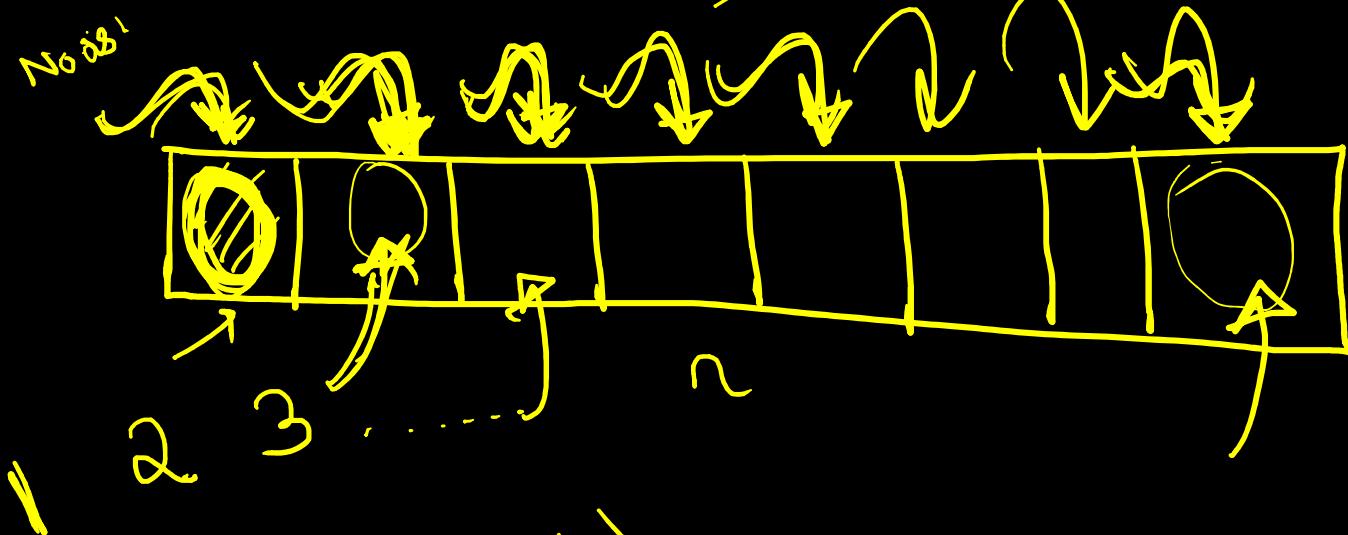






Best $O(1)$

ℓ_2 Linear search
 ℓ_1 $O(n)$ \rightarrow w.c.
 ℓ_0 $O(1)$ \rightarrow b.c.



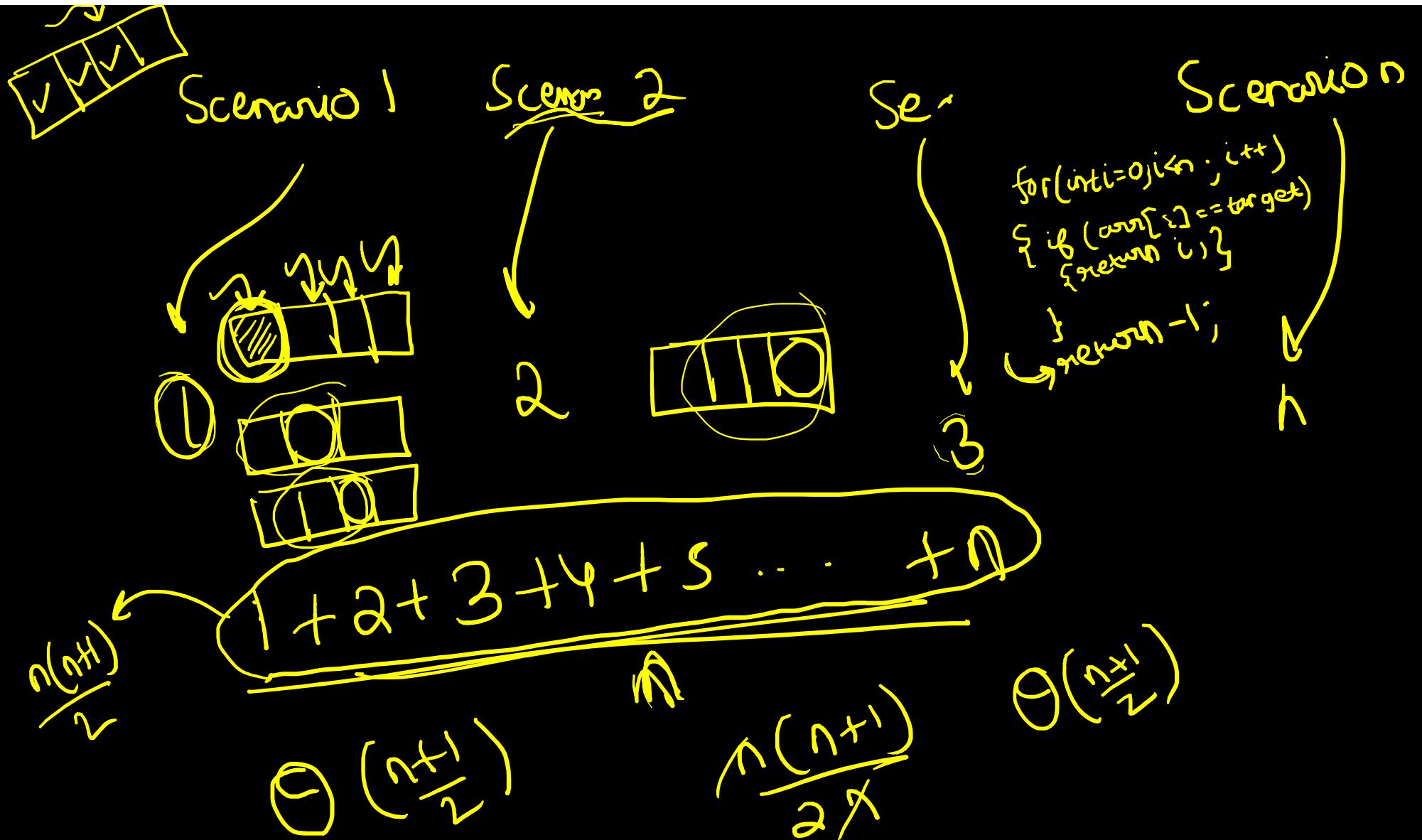
$O(\frac{n+1}{2})$

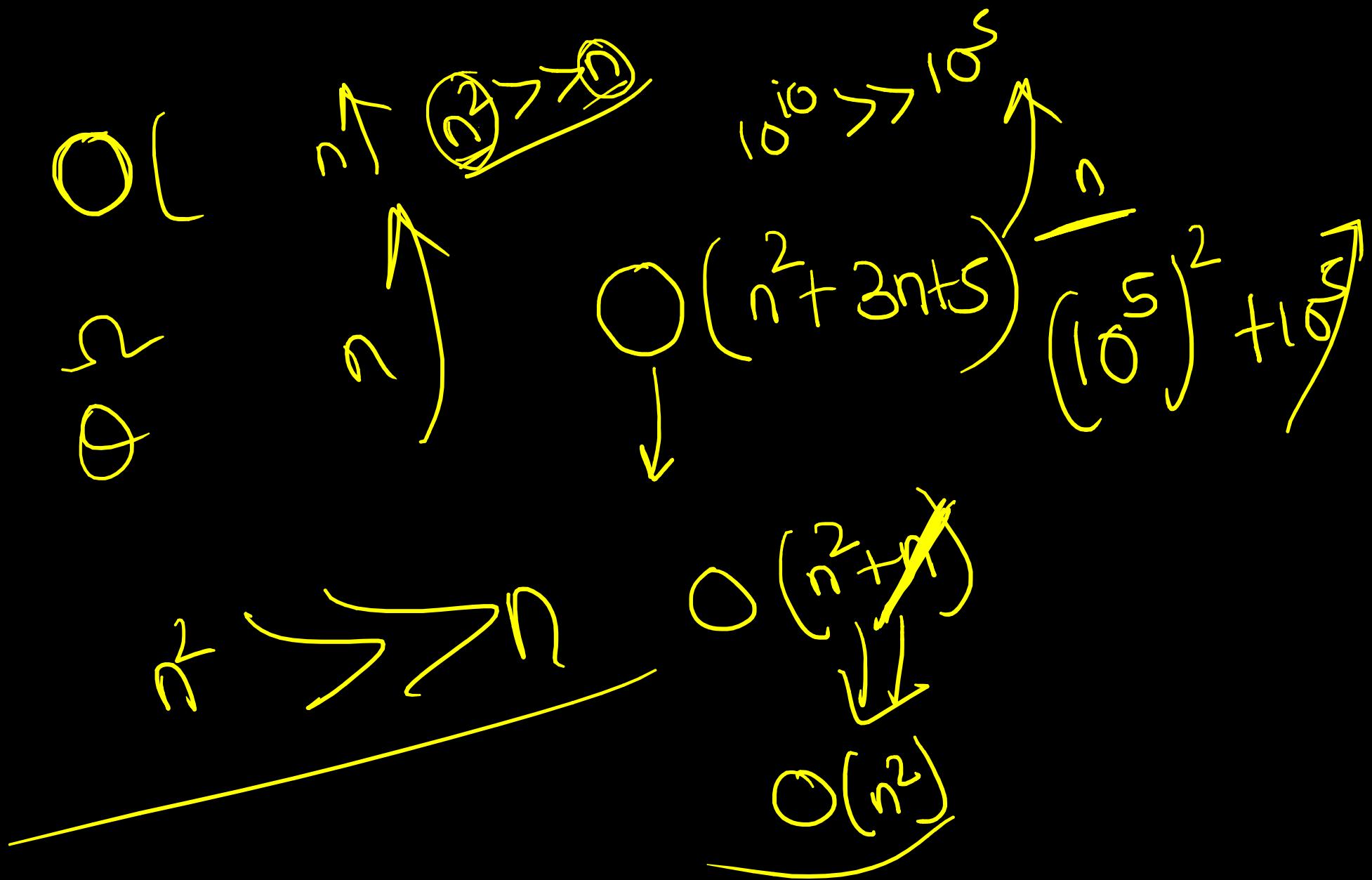
$(\frac{n+1}{2})$

$O(\cdot)$

$$\begin{aligned} & 1 + 2 + 3 \\ & + 4 + 5 \dots \\ & + n \end{aligned}$$

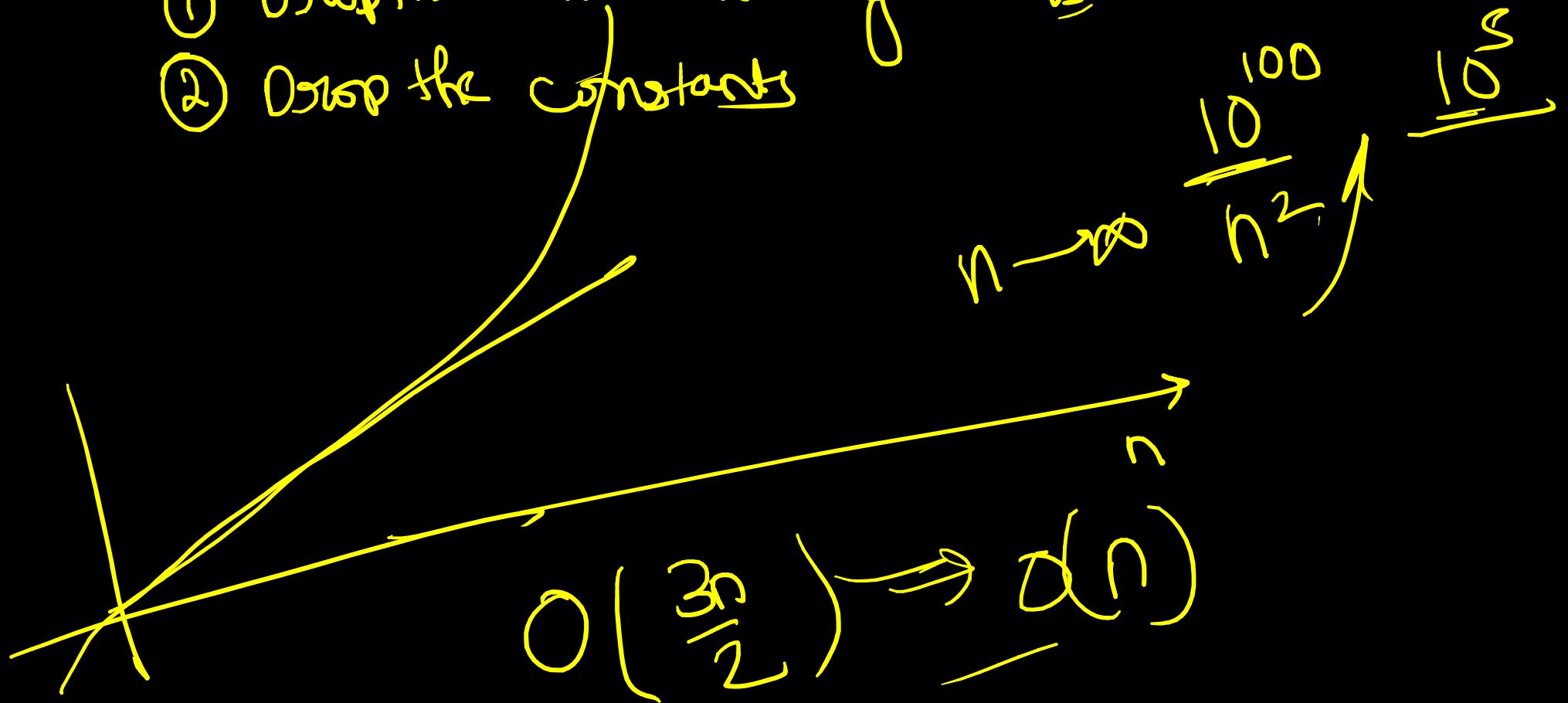
$$= \frac{n(n+1)}{2}$$





① Drop the non dominating terms

② Drop the constants



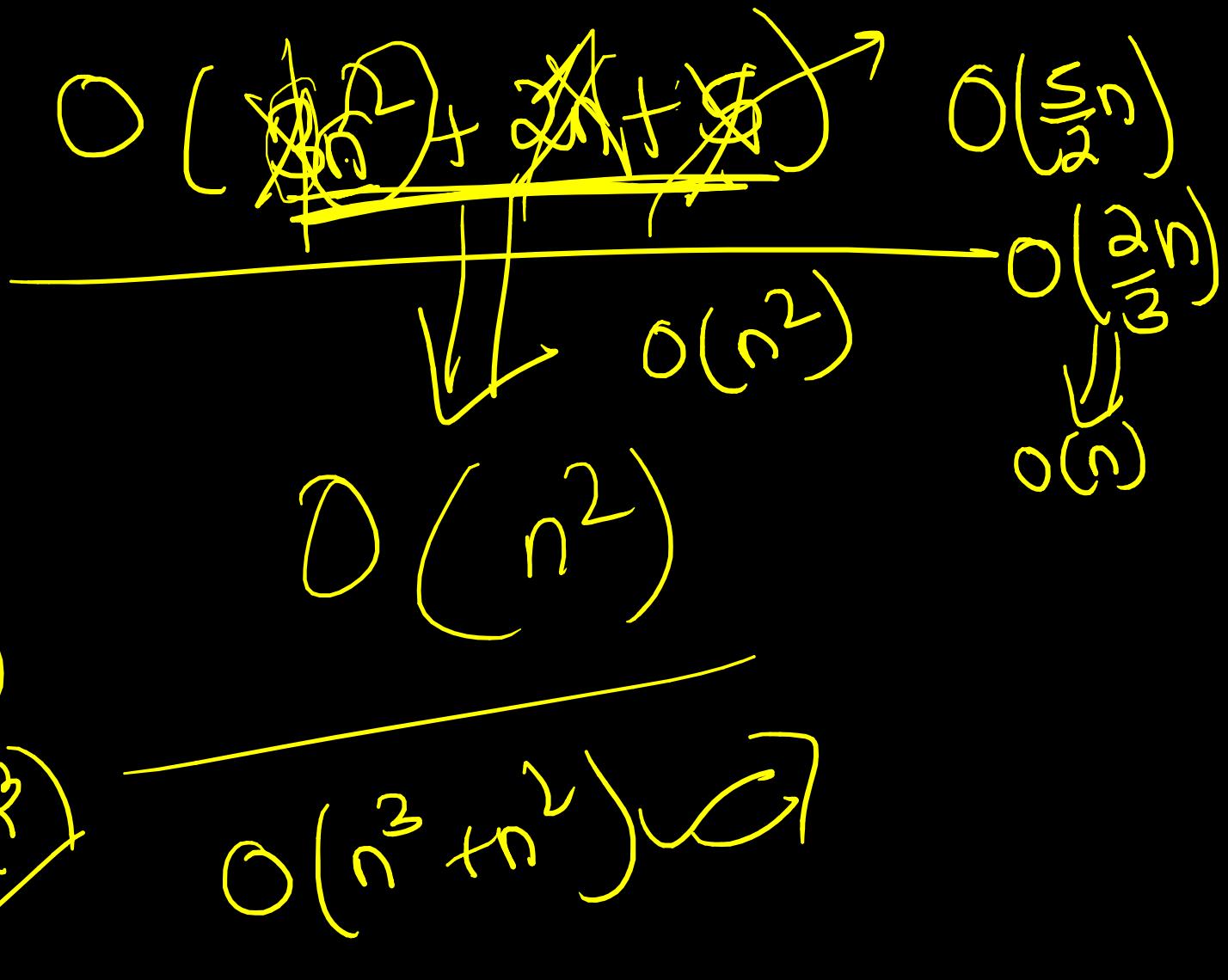
$\binom{100000}{0005}$

100

10^3

$O\left(n^{\frac{3}{2}} + n^2\right)$

$O(n^3)$



$\text{for } (i = 0 \rightarrow m)$ $\xrightarrow{\text{m times}}$
 $\{ \underline{\text{sop}(r @ i)} \} \checkmark$ $\xrightarrow{\mathcal{O}(m)}$

$\text{for } (i = 0 \rightarrow n)$ $\xrightarrow{\text{n times}}$ $\xrightarrow{\mathcal{O}(m+n)}$
 $\{ \underline{\text{sop}(i)} \} \checkmark$

```
for (i=0 → n)
{
    for(j=0 → n)
    {
        }
}
```

```
{for( j=0 → n)
{
    }
```

$$O(n^2 + n)$$



$$O(n^2)$$

$$O(1)$$

$$O(n^2)$$

```
for ( i=0; i<n; i++ )
```

```
{ for ( j=0; j<i; j++ ) } i times
```



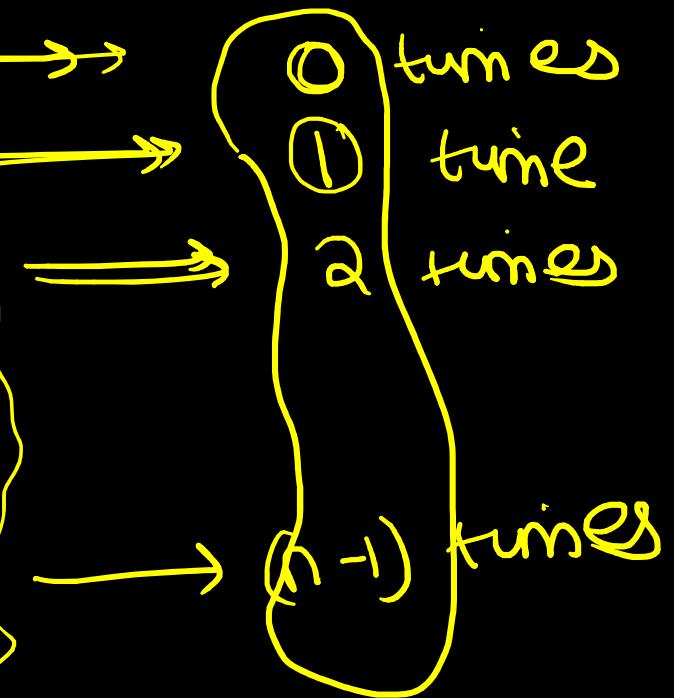
```
}
```

$$\frac{n(n+1)}{2}$$

$$(n-1)(n-1+1) \quad 2$$

$$\frac{n(n-1)}{2}$$

$j=0$ i dist
0 1 2 ... $i-1$



{ $1 + 2 + 3 + \dots + (n-1)$ }

$$\frac{n(n-1)}{2}$$

$$O(n^2)$$

$$1+2+3+4+\dots+(n-1)$$

$$\frac{k(k+1)}{2}$$

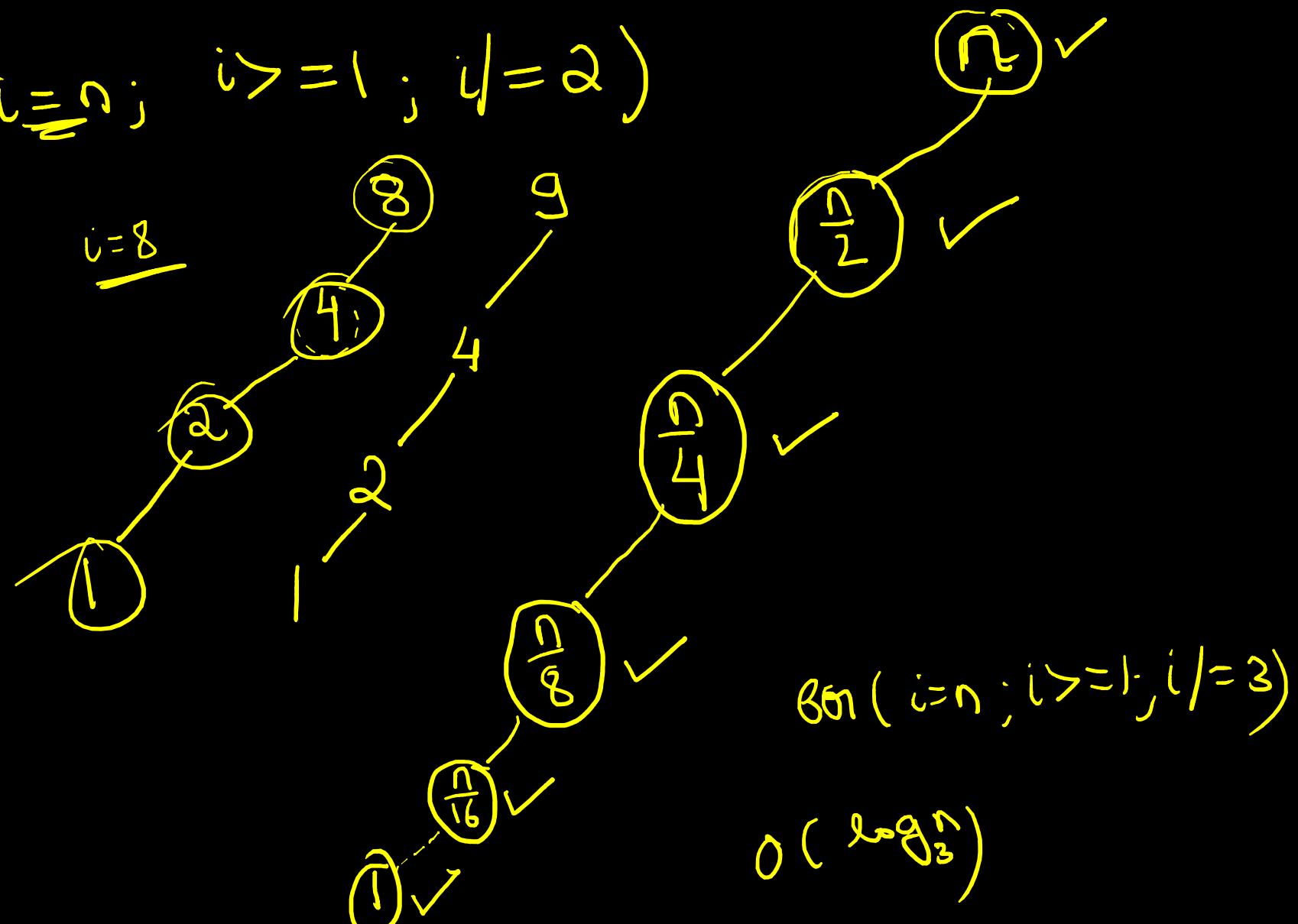
$$\frac{(n-1)(n-1+1)}{2}$$

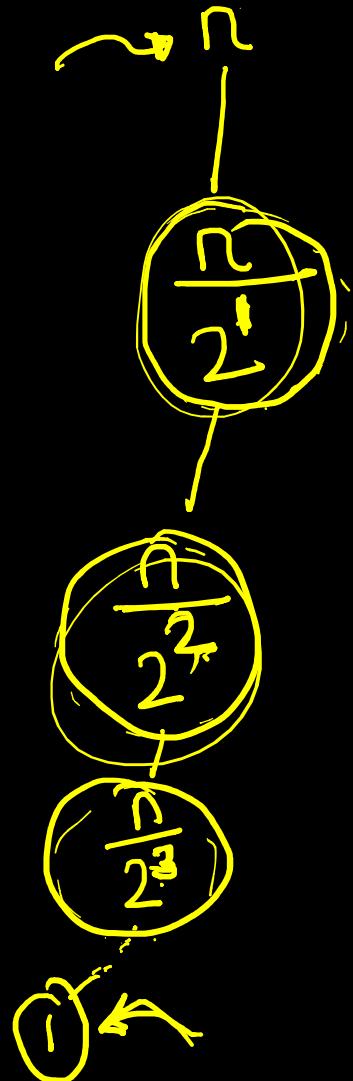
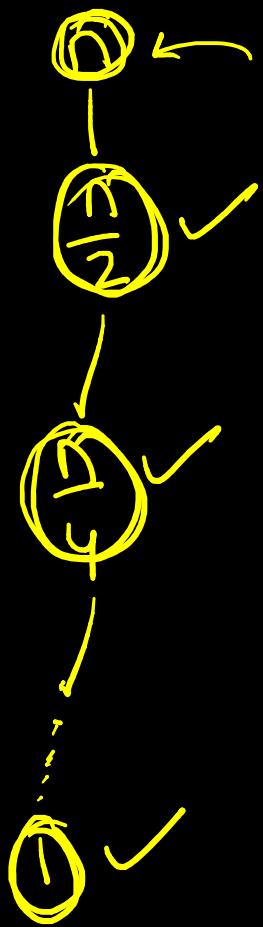
$$\mathcal{O}(n^2)$$

$$\mathcal{O}\left(\frac{n(n-1)}{2}\right)$$

for(int i=n; i>=1 ; i/=2)

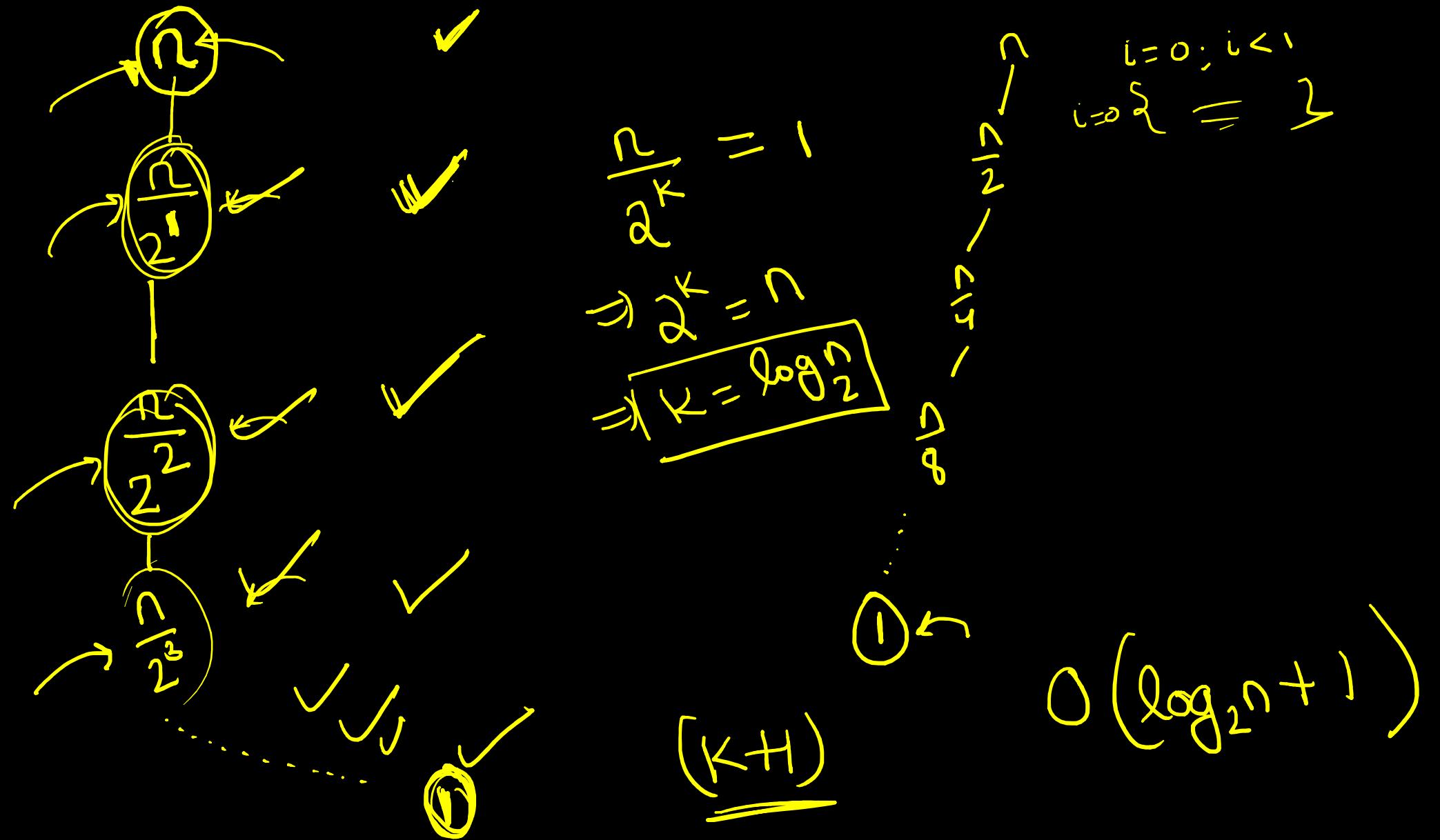
{ }





$$\frac{n}{2^k} = 1$$
$$\Rightarrow 2^k = n$$
$$\Rightarrow k = \log_2 n$$

$$O(k+1)$$
$$= O(\log n + 1)$$
$$= O(\log n)$$



for (i=n; i>=1; i--)

$O(\log n)$

~~$i*=2$~~
 \log

```
i=n  
while(i>=1)  
{ i=i/2;  
  a=a+i;  
}  
y
```

$O(\log_2 n)$

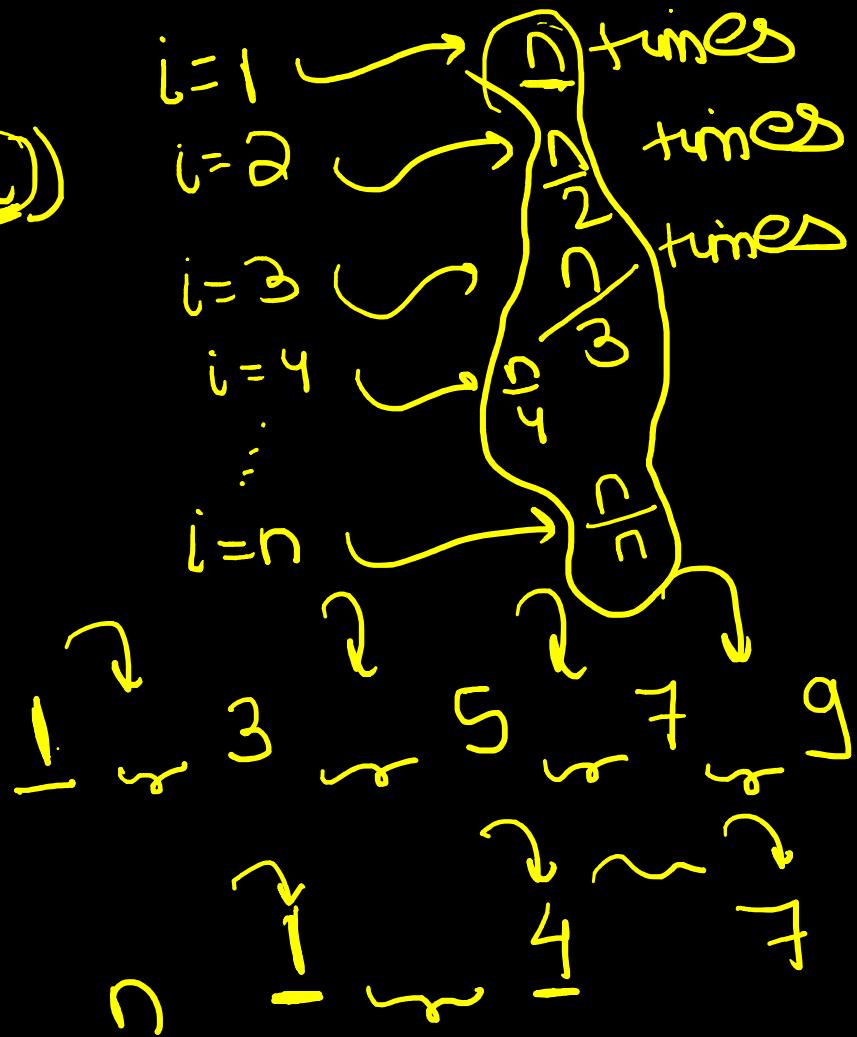
```

for(int i=1; i<=n; i++)
{
    for( int j=i; j<=n; j+=i)
    {
        j=1
        j=2
        j=3
        j=4
        ...
    }
}

```

$j=1, 2, 3, 4, \dots, n$

1 2 3 4 ... n



$$O\left(n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \frac{n}{5} + \dots\right)$$

$$n \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{c} \right)$$

$$O(n \log n)$$

$$O(n \alpha^n)$$

$$\int_1^n \frac{1}{x} dx = \log x \Big|_1^n = \log n - \underline{\log 1} \quad \begin{matrix} \log n \\ \text{ln } n \\ e \end{matrix}$$

$$\frac{1}{1} + \frac{1}{1.0000} + \frac{1}{1.00} + \frac{1}{1.1} + \frac{1}{1.5} + \frac{1}{2} + \frac{1}{2.00} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots < \log_2^n$$

$O(\cdot)$

$$n \left(1 + \frac{1}{2} + \frac{1}{3} + \dots \right)$$

$O(n \log n)$

$O(n)$ < $O(n^2)$ < $O(n^3)$

$\log n$ < n

$$\sqrt{n} > \log n$$

$$\log \sqrt{n}$$

$$\log(\log n)$$

$$\Rightarrow \frac{1}{2} \log n$$

$$\underbrace{\log(\log n)}_{\text{underbrace}}$$

$$g(n) = \begin{cases} n & n < 1000 \\ n^2 & n \geq 1000 \end{cases}$$

$$\cancel{\Theta(n^2)}$$

$$g_1(n) = \begin{cases} n & n \leq 100 \\ n^4 & n > 100 \end{cases}$$

$$\cancel{g_1 + g_2} \quad \Theta(n^4 + \cancel{n^2})$$

$$g_2(n) = \begin{cases} n^3 & n \leq 100 \\ n^2 & n > 100 \end{cases}$$

$$\Theta(n^4)$$

$O(1)$ \rightarrow T.C

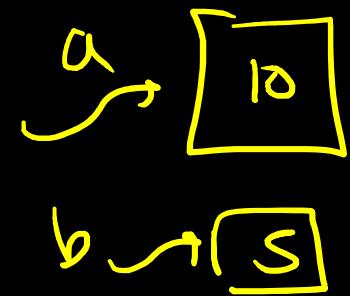
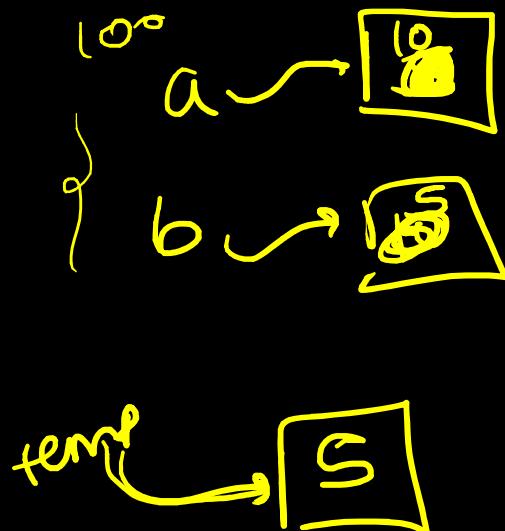
max a,b

if ($a > b$)
{ max=a; }
else
{ max=b; }

$O(1)$

Swap 2 variables

Step
 $\rightarrow \text{int } a = 5;$
 $\rightarrow \text{int } b = 10;$
 $\rightarrow \text{int temp} = @$
 $a = b; \rightarrow$
 $b = \text{temp};$



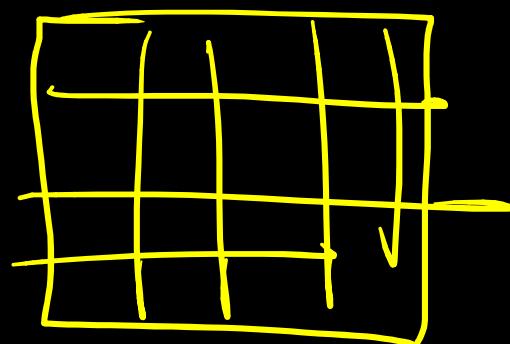
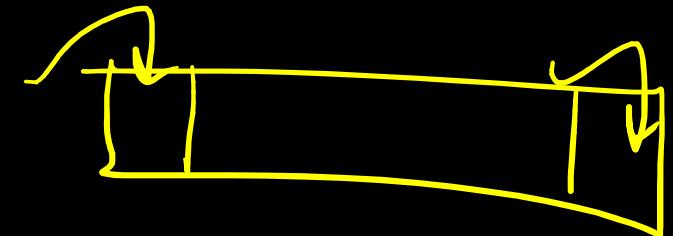
$O(1)$ $O(1)$
 \equiv

$a = 5;$
 $b = 5;$
 $a = b;$

sum of first and last element

$O(1)$

$$\text{Sum} = arr[0] + arr[n-1]$$



$m, \text{ rows}$
 $n, \text{ cols}$

$O(m * n)$

```
for ( i=1; i<=n ; i*=2 )
```

```
{ }
```

$O(\log n)$



$$\begin{aligned} 2^K &= n \\ \Rightarrow K &= \underline{\log n} \end{aligned}$$

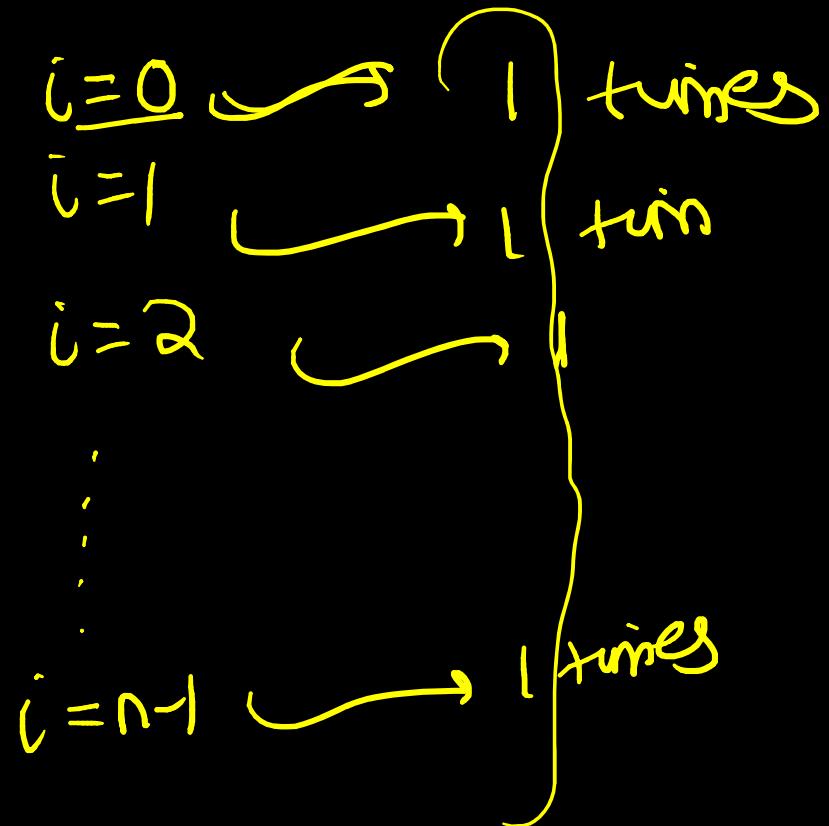
```
for ( i=1 ; i<=n ; i++ )  
{   for( j=1 ; j<=n ; j*=2 )  
    {  
    }  
}
```

$O(n \log n)$

```

for( i=0; i<=n; i++) n
{
    for(j=0; j<=n; j++)
        {
            cout << "Hello";
            break;
        }
}

```



$$\begin{array}{c}
| + | + | + | \\
\curvearrowright n \times n
\end{array}
\qquad O(n)$$

```

for ( i = 0 ; i < n ; i++)
{
    for ( j = 0 ; j < n ; j++)
    {
        if ( i < n - 1)
            break;
    }
}

```

$i = 0 \rightarrow 1$

$i = 1 \rightarrow 1$

⋮

$i = n - 2 \rightarrow 1$

$i = n - 1 \rightarrow n \text{ times}$

$\Rightarrow O(n)$

$| + | + \dots + |$
 $\underbrace{|}_{(n-1)}$

$(n-1) + n + \dots + n$
 $O(2n-1)$

$O(n)$

$O(n^2)$

$O(n^3)$

$O(\log n)$

$O(n \log n)$

$O(1)$

\checkmark $O(1) < O(\log n) < O(\sqrt{n}) < O(2)$
 $< O(n \log n) < O(n^2) < O(n^3)$
 $< O(2^n) < O(3^n)$

~~100~~

$n \log n > n$.

$O(n^2)$

$O(n)$

```
int a = 0;
```

```
int i = N;
```

```
while (i > 0)
```

```
{  
    a += i;  
    i /= 2;  
}
```

T.C $\rightarrow O(\log_2 N)$

$O(a)$

$$a = N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + 1$$

```
int [] arr = new int[a];
```

$$a = N + \frac{N}{2} + \frac{N}{4} + \dots + 1$$

$$\begin{aligned}
 & 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \\
 \Rightarrow & n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \frac{n}{16} + \dots \downarrow \\
 & \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{n} \right) \xrightarrow{\text{G.P}} 1 \\
 & a = 2n \\
 & a = 1 \\
 & q = \frac{1}{2} \\
 & \frac{1}{1 - \frac{1}{2}} = 2 \\
 & O(2n) \xrightarrow{\text{Simplification}} O(n)
 \end{aligned}$$

~~if this is a perfect square~~

(n)

```
for( i=1; i<2; ++)  
{ if( i*i == n )  
    { retun true; }  
}  
return false;
```

1, 4, 9, 16, 25, 36, 49
 $4 \times 4 = 16$

~~i = n~~

Perfect squares $\Theta(\sqrt{n})$

Non-perfect $\Theta(n)$

$O(n)$

$$i = \sqrt{n}$$

(A) ...

$$i = \sqrt{n} * \sqrt{n} = n$$

n perfect

$$i = \sqrt{n}$$

(*)

$$i = \sqrt{n}$$

$$\downarrow 2, 3, 4, \dots 7, 8 \rightarrow 50$$
$$(\sqrt{n} + 1)(\sqrt{n} + 1) > n$$

n
non-perf

math sqrt

```
for ( i=1; i<= n; i++ )
```

```
{ if ( i*i == n )  
{ return true; }
```

```
}  
return false;
```



$O(\sqrt{n})$

$i = 1 \dots$

$$\sqrt{n} * \sqrt{n} = c$$

$$(\sqrt{n} + 1)(\sqrt{n} + 1) > n$$

```
for (i=1; i<n; i++)  
{ if ( i*i == n)  
    { return true; }  
    if ( i*i > n)  
    { return false; }  
}
```

$$\textcircled{\sqrt{n}} \quad \frac{\sqrt{n} + 1}{\sqrt{n}}$$
$$O(\sqrt{n})$$