March-April: 2017.

Q:1 Answer the following questions:

1. Define equivalent sets with illustration.

→. If the elements of one set can be put into one-to-one correspondence with the element another set then the two sets are called equivalent sets.

→. It is denoted as $A \equiv B$ or $A \cong B$

→. e.g :- If $A = \{1, 2, 3, 4, 5\}$
$B = \{1, 4, 9, 16, 25\}$

Then $A \equiv B$ (B is related to A as square)

2. Explain symmetric difference of two non-empty sets with illustration.

→. let U be the universal set and P(U) be the power set of U.

→. let $A \in P(U)$ and $B \in P(U)$

→. Then the set of element which belongs to A or to B but not to both A and B is called the symmetric difference set of A and B and is it is denoted by $A \Delta B$

→. e.g $A \Delta B = (A \cup B) - (A \cap B)$

3. Define complement of set with illustration.

→. The complement of a set A is a

set of all elements which do not belong to set A but belong to universal set U.

→ i.e. $A' = \{x \mid x \notin A \text{ but } x \in U\}$

or $A' = U - A$

4. Define onto function with illustration.

→ If the functions $f : A \to B$ is such that each element in B is f image of at least one element in A then f is a function of A onto B.

e.g $A = \{2 \times 1, 3 \times 2, 4 \times 8\}$

$B = \{2, 6, 32\}$ Here A is onto B.

5. Find $f\left(\frac{2}{3}\right) - f\left(\frac{3}{2}\right)$ for $f(x) = x^2 + x + 1$

$f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) + 1$

$= \frac{4}{9} + \frac{2}{3} + 1$

$= \frac{4 + 6 - 9}{9}$

$= \frac{1}{9}$

$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 1$

$= \frac{9}{4} + \frac{3}{2} + 1$

$$= \frac{9+6-4}{4}$$

$$= \frac{11}{4}$$

Now,

$$f\left(\frac{2}{3}\right) - f\left(\frac{3}{2}\right)$$

$$= \frac{1}{9} - \frac{11}{4}$$

$$= \frac{4-99}{36}$$

$$= -\frac{95}{36}$$

6. Define domain of the function and find Df for $f(x) = 2x - 3\ Rf = \{-3, 1, 0\}$

→ for $Rf = -3$

$$f(x) = 2x - 3\ Rf$$

$$\therefore 2x - 3 = -3$$

$$\therefore x = \frac{-3+3}{2}$$

$$\therefore x = 0$$

for $Rf = 1$

$$f(x) = 2x - 3\ Rf$$

$$\therefore 2x - 3 = 1$$

$$\therefore x = \frac{1+3}{2}$$

$$\therefore x = \frac{4}{2}$$

$$\therefore x = 2$$

for $R_f = 0$

$f(x) = 2x - 3R_f$

$\therefore 2x - 3 = 0$

$\therefore 2x = 3$

$\therefore x = \frac{3}{2}$

$D_f = \{0, 2, \frac{3}{2}\}$

Q: 2

A. In usual notations prove that $A - (B \cup C) = (A - B) \cap (A - C)$

→. $x \in A - (B \cup C)$

$x \in A$ but $x \notin (B \cup C)$

$x \in A$ but $x \notin B$ and $x \notin C$

$x \in A$ but $x \notin B$ and $x \in A$ but $x \notin C$

$x \in A - B$ and $x \in A - C$

$x \in (A - B) \cap (A - C)$

$\therefore A - (B \cup C) = (A - B) \cap (A - C)$

OR

A. In usual notations prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

→. let $(x, y) \in A \times (B \cup C)$

$x \in A$ and $y \in (B \cup C)$

$x \in A$ and $(y \in B$ or $y \in C)$

$(x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$(x, y) \in A \times B$ or $(x, y) \in A \times C$

$(x, y) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

B.

1. let $U = \{x \mid 2 \leq x < 14, x \in N\}$, $A = \{z \mid 3 \leq z \leq 8, z \in N\}$ and $B = \{y = 2n+1, y \leq 12, N \in n \cup \{0\}\}$ then find
(i) A'
(ii) B'
(iii) $(A \cup B)'$

→. $U = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
$A = \{3, 4, 5, 6, 7, 8\}$
$B = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$

Now,
$A' = U - A$
$= \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\} - \{3, 4, 5, 6, 7, 8\}$
$A' = \{9, 10, 11, 12, 13\}$

$B' = U - B$
$= \{4, 6, 8, 10, 12\}$

$(A \cup B)' = \{1, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15, 17, 19, 21, 23, 25\}'$

$(A \cup B)' = \{10, 12\}$

2. If $A = \{x \leq 3 ; x \in N\}$, $B = \{x : -1 \leq x \leq 2 ; x \in Z\}$ and $C = \{x : x^2 - 5x + 6 ; x \in R\}$ verify Demorgan's law for intersection considering $U = R$

→. $A = \{1, 2, 3\}$

$B = \{-1, 0, 1, 2\}$

$C = \{2, 3\}$

De morgan's law for intersection.

$(A \cap B)' = A' \cup B'$

Now

$(A \cap B)' = \{1, 2, 3\} \cap \{-1, 0, 1, 2\}$

$= \{1, 2\}'$

~~$(A \cap B)' = U - (A \cap B)$~~

~~$= \{2, 3\} - \{1, 2\}$~~

$=$.

$\therefore (A \cap B)' = R - \{1, 2\}$ ——— ① (LHS)

Now,

$A' = R - \{1, 2, 3\}$

$B' = R - \{-1, 0, 1, 2\}$

$A' \cup B' = R - \{1, 2\}$ ——— ② (RHS)

from result ① and ②

$(A \cap B)' = A' \cup B'$

3. If $A = \{x \leq 4 ; x \in N\}$, $B = \{x : x^2 \leq 4 ; x \in Z\}$ and $C = \{x : -2 \leq x \leq 3 ; x \in N\}$ then verify that $A - (B \cap C) = (A - B) \cup (A - C)$

→. $A = \{1, 2, 3, 4\}$
$B = \{-2, -1, 0, 1, 2\}$
$C = \{1, 2, 3\}$

Now,
$(B \cap C) = \{-2, -1, 0, 1, 2\} \cap \{1, 2, 3\}$
$= \{1, 2\}$

$A - (B \cap C) = \{1, 2, 3, 4\} - \{1, 2\}$
$= \{3, 4\}$ ——— (1) LHS

Now,
$(A - B) = \{1, 2, 3, 4\} - \{-2, -1, 0, 1, 2\}$
$= \{3, 4\}$

$(A - C) = \{1, 2, 3, 4\} - \{1, 2, 3\}$
$= \{3\}$

$\therefore (A - B) \cup (A - C) = \{3, 4\}$ ——— (2) RHS

$\therefore A - (B \cap C) = (A - B) \cup (A - C)$

4. In a class of 42 students, each play at least one of the three games cricket ; Hockey and football. It is found that 14 play cricket, 20 play Hockey and 24 play football, 3 play both cricket and football, 2 play both Hockey

and football. None play all the three games. find the number of students who play cricket but not Hockey.

→. c = set of students who play cricket
H = set of students who play hockey
f = set of students who play football.

$n(C) = 14$

$n(H) = 20$

$n(f) = 24$

$n(C \cap f) = 3$

$n(H \cap f) = 2$

$n(C \cap H \cap f) = 0$

$n(C \cup H \cup f) = 42$

$n(C - H) = (?)$

$$n(C \cup H \cup f) = n(C) + n(H) + n(f) - n(C \cap f) - n(H \cap f) - n(C \cap f) + n(C \cap H \cap f)$$

$42 = 14 + 20 + 24 - 3 - 2 - n(C \cap H) + 0$

$42 = 58 - 3 - 2 - n(C \cap H) + 0$

$42 = 53 - n(C \cap H)$

$n(C \cap H) = 53 - 42$

$n(C \cap H) = 11$

$n(C - H) = n(C) - n(C \cap H)$

$= 14 - 11$

$n(C - H) = 3$

→. 3 play cricket but not play hockey.

(34)

Q:3

A. If $f(x) = x^2 (x-1)^2$, $x \in R$ then prove that $f(x+1) - f(x) = 4x^3$.

→. $f(x) = x^2 (x-1)^2 = x^2 (x^2 - 2x + 1)$

$= x^4 - 2x^3 + x^2$

$f(x+1) = (x+1)^2 (x+1-1)^2$

$= (x^2 + 2x + 1) x^2$

$= x^4 + 2x^3 + x^2$

Now,

$f(x+1) - f(x)$

$= \cancel{x^4} + 2x^3 + \cancel{x^2} - \cancel{x^4} + 2x^3 - \cancel{x^2}$

$= 4x^3$

OR

A. The demand function of a commodity is $d = f(P) = 1605 - 5P^2$; find demand when price is Rs. 5, 6 and 8 respectively. At what price the demand will be zero?

→. $d = f(P) = 1605 - 5P^2$

Demand, when price is Rs. 5

$d = f(P) = 1605 - 5(5)^2$

$= 1605 - 125$

$= 1480$

vision

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Demand, when price is RS. 6 :-

$$d = f(P) = 1605 - 5(6)^2$$
$$= 1605 - 180$$
$$= 1425$$

Demand, when price is RS. 8 :-

$$d = f(P) = 1605 - 5(8)^2$$
$$= 1605 - 320$$
$$= 1285$$

→. when demand will be zero.

$$d = f(P) = 1605 - 5P^2$$

$$\therefore 0 = 1605 - 5P^2$$
$$\therefore +5P^2 = +1605$$
$$\therefore P^2 = \frac{1605}{5}$$
$$\therefore P^2 = 321$$
$$\therefore P = \sqrt{321}$$
$$\therefore P = 17.91 \text{ RS.}$$
$$\therefore P \approx 18 \text{ RS.}$$

→. At RS. 18 demand will be zero.

B. Attempt any two :-

1. If $f(x) = \frac{1}{x} + \frac{2}{x-3}$; $x \in R - \{0, 3\}$ then find $f(\frac{1}{3}) - f(-3) + f(2)$.

$\rightarrow$ $f(\frac{1}{3}) = \frac{1}{3} + \frac{2}{\frac{1}{3} - 3}$

$= 3 + \frac{2}{1 - \frac{9}{3}}$

$= 3 + \frac{6}{-8}$

$= \frac{-24 + 6}{8}$

$f(\frac{1}{3}) = -\frac{18}{8}$

$f(-3) = \frac{1}{(-3)} + \frac{2}{-6}$

$= \frac{-2 - 2}{6}$

$= -\frac{4}{6}$

$f(-3) = -\frac{2}{3}$

$f(2) = \frac{1}{2} + \frac{2}{2-3}$

$= \frac{1 - 4}{2}$

$f(2) = \frac{-3}{2}$

Now,

$f(\frac{1}{3}) \cdot f(-3) - f(2)$

$$= -\frac{18}{8} + \frac{2}{3} + \frac{3}{2}$$

$$= \frac{-108 + 32 + 72}{48}$$

$$= -\frac{4}{48}$$

$$= -\frac{1}{12}$$

$f(\frac{1}{3}) - f(-3) - f(2) = -\frac{1}{12}$

2. fixed cost of a factory producing particular types of bag is Rs. 9000 and the variable cost per bag is Rs. 110 If the selling price per bag is Rs. 240 then find profit function.

→ selling price per bag = Rs. 240

let x be the number of bags then

$R(x) = 240x$

total cost $C(x) = 9000 + 110x$

Now,

profit function :

$P(x) = R(x) - C(x)$

$= 240x - 9000 - 110x$

$P(x) = 130x - 9000$

3. If $f(x) = \dfrac{ax+b}{(x-a}$ then prove that $x = f(y)$

→. $f(x) = y = (ax - b)\ (x - a)$

$y \times c(x - a) = ax - b$

$(cxy - ax) = (ay - b)$

$x\ (cy - a) = ay - b$

$x = (ay - b) / (cy - a)$

$\therefore f(y) = x = \dfrac{(ay + b)}{(cy - a)}$

4. If $f(x) = x^3$ and $g(x) = 3x^2 - 2x$, $x \in \{0, 1, 2\}$ are the functions equal?

→. for $x = 0$

$f(0) = 0^3$ $\quad g(0) = 3(0)^2 - 2(0) = 0$

$\therefore f(0) = g(0)$

for $x = 1$

$f(1) = (1)^3$ $\quad g(1) = 3(1)^2 - 2(1)$

$= 1$ $\quad = \cancel{3}\ 1$

for $x = 2$

$f(2) = (2)^3$ $\quad \cancel{f} \cdot g(2) = 3(2)^2 - 2(2)$

$= 8$ $\quad = 8$

∴ Both function are equal.