Problems on Bayes Theorem

Exercise 1

Consider a test to find out the presence of a disease (for example TBC, HIV, etc.). Use symbol P to identify the fact that the test is positive (we deduce that the patient has the disease), and symbol N to identify the fact that the test is negative (we deduce that the patient does not have the disease). Use symbol D_y to identify the fact that the patient really has the disease, and symbol D_n to identify that the patient really does not have the disease. We know that

- $P(T_p|D_y) = 0.9$ (the test is positive in 90% of the cases given that the patient has the disease, this is the true positive case, and $P(T_p|D_y)$ is the test sensitivity)
- $P(T_p|D_n) = 0.05$ (the test is positive in 5% of the cases given that the patient does not have the disease, this is the false positive case)
- $P(D_y) = 0.1$

Evaluate:

1. the probability that the test on a randomly chosen person is positive
2. the probability that a person really has the disease, given that the test is positive
3. the probability that a person really does not have the disease, given that the test is positive
4. the probability that a person really does not have the disease, given that the test is negative
5. the probability that a person really has the disease, given that the test is negative
6. the probability that the test fails

Exercise 2

In a pharmaceutical factory, 3 lines (A, B, C) are used to produce a given blister of pills. Production line A outputs 55% of the blisters and 2% of them is defective (for example one pill is missing). Production line B outputs 30% of the blisters and 3% of them is defective. Production line C outputs the remaining 15% of blisters and 6% of them is defective.

What is the probability that a randomly chosen blister is defective? What is the probability that a defective blister (chosen at random) comes from production line C?