# Assignment-1

Instructor: Dr. Shaurya Agarwal EE-5630

## February 2, 2018

### Instructions:

- Total Marks: **60**. Total Questions: 10
- Please write CLEARLY.
- Complete all the questions, Scan and Upload on Moodle
- All the assignment submissions will be digital, and NO hard copy will be allowed.
- Students can either use a traditional scanner or even a smartphone based **scanner app** such as tiny scanner, cam scanner or any other application.
- Final submission should be a **single pdf** with all the assignment questions in order, and all sub-parts of a question solved together.
- Due Date: 14<sup>th</sup> Feb, by 11:55pm. Submissions will close on Moodle after this date.
- All submissions will be graded on moodle and comments provided on original submissions. Students should check back and read the comments for feedback.

Question 1: Read example 2.2-1, and summarize your learning briefly.

Following problems are from textbook:

Question 2: 2.1

Question 3: 2.2

Question 4: 2.4

Question 5: 2.5

Question 6: 2.6

#### REFERENCES

- M-1 Merriam, C. W., III, and F. J. Ellert, "Synthesis of Feedback Controls Using Optimization Theory—An Example," *IEEE Trans. Automatic Control* (1963), 89–103.
- S-2 Schultz, D. G., and J. L. Melsa, State Functions and Linear Control Systems. New York: McGraw-Hill, Inc., 1968.

### **PROBLEMS**

- **2-1.** Refer to the chemical mixing process of Problem 1-6. The amount of dye in tank 2,  $v_2(t)$ , is to be maintained as closely as possible to M ft<sup>3</sup> during a one-day interval.
  - (a) What would you suggest as a performance measure to be minimized?
  - (b) Determine a set of physically realistic state and control constraints.
- 2-2. Repeat Problem 2-1 if the objective is to maximize the amount of dye in tank 2 at the end of one day. It is specified that the total volume of dye that enters tank 1 in the one-day period cannot be more than N ft<sup>3</sup>.
- 2-3. An unmanned roving vehicle has been proposed as part of the Mariner Mars exploration series of space missions. The roving vehicle is designed to navigate on the Martian surface and transmit television pictures and other scientific data to earth. Suppose that the rover is to be driven by a dc motor supplied from rechargeable storage batteries; a simplified model is shown in Fig. 2-P3.

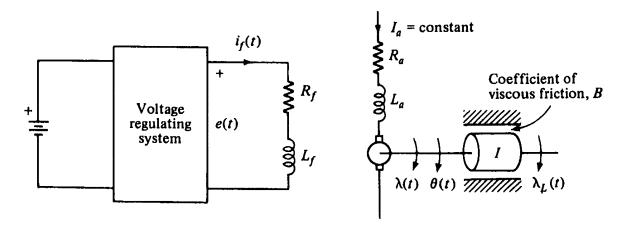


Figure 2-P3

The output of the voltage regulating system is the control signal e(t). The developed torque is  $\lambda(t) = K_t i_f(t)$ , where  $K_t$  is a known constant;  $\lambda_L(t)$  is the load torque caused by hills on the Martian surface. The vehicle's speed is to

deviate as little as possible from 5 mph without requiring excessive energy output from the voltage regulating system (to prolong the life of the batteries). Let  $i_f(t)$  and  $\dot{\theta}(t)$  be state variables.

- (a) Write state equations for the motor-load combination.
- (b) Determine a physically reasonable set of state and control constraints.
- (c) Suggest a performance measure if:
  - (i)  $L_f = 0$ .
  - (ii)  $L_f \neq 0$ .
- 2-4. Refer to the simplified spacecraft model used in Example 2.2-1. Suppose that the objective is to change the spacecraft attitude from an arbitrary initial value to an angle of  $+15^{\circ} \pm 0.1^{\circ}$  with respect to the reference axis shown in Fig. 2-3. This maneuver is to be accomplished in 30 sec with minimum fuel expenditure.
  - (a) Determine the state and control constraints.
  - (b) Suggest an appropriate performance measure.
- 2-5. Repeat Problem 2-4 if the maneuver is to be accomplished in minimum time.
- **2-6.** Figure 2-P6 shows a rocket that is to be approximated by a particle of instantaneous mass m(t). The instantaneous velocity is v(t), T(t) is the thrust, and

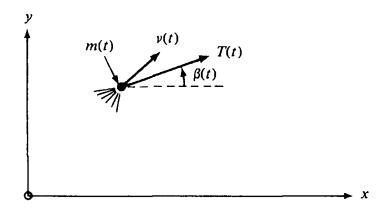


Figure 2-P6

 $\beta(t)$  is the thrust angle. If we assume no aerodynamic or gravitational forces, and if we select  $x_1 \triangleq x$ ,  $x_2 \triangleq \dot{x}$ ,  $x_3 \triangleq y$ ,  $x_4 \triangleq \dot{y}$ ,  $x_5 \triangleq m$ ,  $u_1 \triangleq T$ ,  $u_2 \triangleq \beta$ , the state equations are

$$\dot{x}_1(t) = x_2(t) 
\dot{x}_2(t) = \frac{\left[u_1(t)\cos u_2(t)\right]}{x_5(t)} 
\dot{x}_3(t) = x_4(t) 
\dot{x}_4(t) = \frac{\left[u_1(t)\sin u_2(t)\right]}{x_5(t)} 
\dot{x}_5(t) = -\frac{1}{c}u_1(t),$$

where c is a constant of proportionality. The rocket starts from rest at the point x = 0, y = 0.

- (a) Determine a set of physically reasonable state and control constraints.
- (b) Suggest a performance measure, and any additional constraints imposed, if the objective is to make  $y(t_f) = 3$  mi and maximize  $x(t_f)$ ;  $t_f$  is specified.
- (c) Suggest a performance measure, and any additional constraints imposed, if it is desired to reach the point x = 500 mi, y = 3 mi in 2.5 min with maximum possible vehicle mass.