Optimal Control of Information Spread in Politics

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Outline of the Presentation

- Introduction
- Background and Motivation
- Problem Formulation
- Dynamics and Cost Function
- Analysis and Simulation
- Conclusion

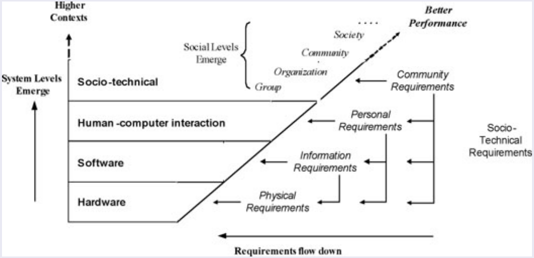
Introduction

Socio-technical Systems: Examples

Socio-technical Systems

- Socio-technical Systems (STS) are social systems that emerge from a technical base.
- For STS, not only must the technical system be controlled, but also the human elements that run, use, or drive the system.

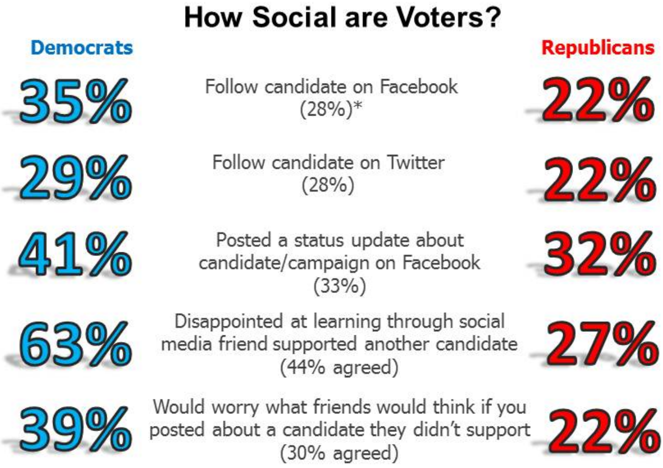
General System Levels:



Socio-technical Systems: Characteristics

- Emergent Properties: the group has properties that the individual does not possess
- Non-deterministic: the same input yields a different output due to the human elements
- Complex Relationships: the extent to which the system meets its objective does not just depend on the physical system itself
- Two main elements to control: the system itself and the humans who use it

Background and Motivation



Social Media Information and Politics

- Political advertising through social media has seen a massive increase in recent years as traditional political advertising methods diminish.
- Both the 2008 and 2016 Presidential elections were greatly impacted by the use of social media to influence voters.

Information Spread Theory: The SIR Model

- Spreaders: (S) Those who spread the information
- Ignorants: (I) Those who do not know the information
- Stiflers: (R) Those who know the information, but have stopped spreading it
- Pairwise interactions between these three ”classes” dictate the evolution of the information spread.
- Interactions happen VERY quickly in social media.

Problem Formulation

The Maki-Thomson Model

- MT Model developed by American mathematicians Daniel Maki and Maynard Thompson in 1973
- Developed by simplifying early attempts to apply disease spread models to information spread
- Lays out rules for the spread of information
- Modern applications span a wide range of areas:
    - Advertising
    - Stock Market Manipulation
    - Emergency Alerts
    - Politics
    - Social Movements

MT Rules of Information Spread

Class Interaction

1. $S+I \rightarrow 2S$
2. $S+S \rightarrow S+R$
3. $S+R \rightarrow 2R$
4. $S+I+R=1$

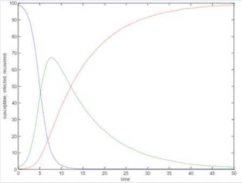
Result

1. When a spreader interacts with an ignorant, the ignorant will become a spreader.
2. When two spreaders interact, one of them will become a stifler.
3. When an spreader interacts with a stifler, the spreader will lose interest in spreading the information and become a stifler.
4. Conservation of individuals.

Information Spread Evolution

- Beginning with a single Spreader:
- Ignorants (blue) decrease sharply as everyone learns the information
- Spreaders (green) increase, reaching a peak and then begin to become stiflers.
- Stiflers (red) dominate after the information is well-known.

Population Over Time



Politics, Social Networks, and Optimal Control

- Information spread naturally evolves on its own, based on the strength, truth, or interest of the topic.
- Spread can be manipulated using the proper control
- In politics, a simple and effective control mechanism is social media advertisement/tweets/posts, which effectively cost money.
- Ideally, the control should be done to make best use of a limited budget and optimize the number of people who know and care about the political information on voting day.

Social Media vs Traditional Media Advertising

- Speed - social media propagates much faster
- Spread - social media has a larger reach
- Less limited - maximum allowable control is almost unlimited (no need for magazine space, TV time blocks, etc)
- Main practical limit is budget constraints.

Parameters

Parameter	Meaning
$i(t)$	ignorants at time t
$s(t)$	spreaders at time t
$r(t)$	stiflers at time t
$\beta(t)$	spreading rate at time t
γ	stifling rate at time t
T	voting day
$u(t)$	control (rate of advertising)
$c(u(t))$	cost from advertising at time t
B	budget
$b(t)$	total resources spent at time t

Dynamics of the System

Using the MT Model rules, three state equations can be formed:

Agent States

$$\dot{i} = -\beta(t)i(t)s(t)$$
$$\dot{s} = \beta(t)i(t)s(t) - \gamma s(t)[s(t) + r(t)]$$
$$\dot{r} = \gamma s(t)[s(t) + r(t)]$$

where β is the spreading rate and γ is the stifling rate.

Reduced System Dynamics

From the conservation of individuals rule $S+I+R=1$ the dynamics can be simplified to two state variable equations:

Reduced Dynamics

$$\dot{i}(t) = -\beta(t)i(t)s(t) - u(t)i(t)$$
$$\dot{s}(t) = [\beta(t)+\gamma]i(t)s(t) - \gamma s(t) + u(t)i(t) + \alpha u(t)[1-i(t)-s(t)]$$
$$s(0) = s_0,\ i(0) = 1-s_0$$

where $u(t)$ is the state control, representing the amount of social media tweets, posts, articles and advertisements put out.

Value Function

The objective for the control law is to minimize the number of ignorants i at voting day T, while staying within budget B.

Value Function

$\min_{u \varepsilon U}[J = i(T)]$,

subject to the constraints of the system dynamics $\dot{i}, \dot{s}$ and the isometric cost constraint $\dot{b}$ associated with a fixed budget:

Budget Constraint

$\int_0^T c(u(t))dt = B$
$\dot{b}(t) = c(u(t)) = u^2(t)$
$b(0) = 0, b(t) = B$

Optimal Control Approach using Pontryagin

Defining the Hamiltonian

$$H = \rho_i(t)[\beta(t)i(t)s(t) - u(t)i(t)] + \rho_s(t)[\beta(t) + \gamma]i(t)s(t) - \gamma s(t) + u(t)i(t) + \alpha u(t)[1 - i(t) - s(t)] + \rho_b(t)[c(u(t))]$$

Co-state Equations

Co-state for Ignorants

$$\dot{\rho_i}^*(t) = -\frac{\partial H}{\partial i(t)} =$$
$$\rho_i^*(t)[\beta(t)s^*(t) + u^*(t)] - \rho_s^*(t)[\beta(t)s^*(t) + \gamma s^*(t) + u^*(t) - \alpha u^*(t)]$$

Co-state for Spreaders

$$\dot{\rho_s}^*(t) = -\frac{\partial H}{\partial s(t)} =$$
$$\rho_i^*(t)[\beta(t)i^*(t)] - \rho_s^*(t)[\beta(t)i^*(t) + \gamma i^*(t) - \gamma - \alpha u^*(t)]$$

Co-state for the Budget

$$\dot{\rho_b}^*(t) = -\frac{\partial H}{\partial b(t)} = 0$$

Analytical Solution

Using the Hamiltonian Minimization Principle:

Hamiltonian Minimization

$$\frac{\partial H}{\partial u(t)} = -\rho_i^*(t)i^*(t) + \rho_s^*(t)i^*(t) + \rho_s^*(t)\alpha[1 - i^*(t) - s^*(t)] + \rho_b^*(t)c(u^*(t)) = 0$$

Optimal Control Law

$$u^*(t) = c'^{-1}\frac{\rho_i^*(t)i^*(t) - \rho_s^*(t)i^*(t) - \rho_s^*(t)\alpha[1 - i^*(t) - s^*(t)]}{\rho_b^*(t)}$$

Results

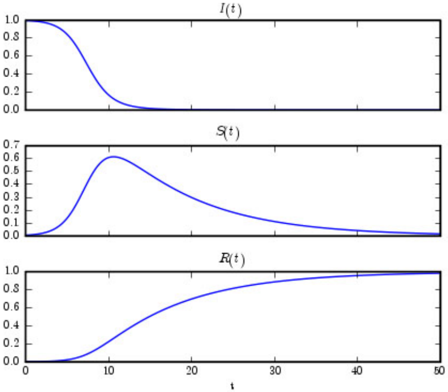


Figure: State Trajectories of Ignorants, Spreaders, and Stiflers at $\beta = 0.8$, $\gamma = 0.1$

Results

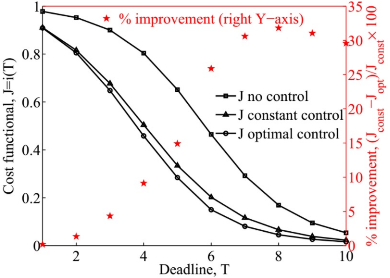


Figure: Numerical Results for Cost vs. Voting Deadline

Limitations of the work

- Stochastic elements are largely overlooked in this analysis
- Social networks have high complexity and are modeled essentially as fast-paced and isolated populations

Conclusion

- The optimal control strategy is more effective than no control or static control, particularly as the voting deadline grows farther away from the start time.
- Generally, without strict control upper bounds in social media ads, it is best to apply control spending early as it eventually drives agents away from ignorant and toward stifling states.
- High spread rate values, achieved by better ads, posts, and tweets, lead to ultimately more final information spread as agents spend more time as spreaders.

Future Work

- Maximizing the "effect" of spreaders before they become stiflers
  - Examine and mitigate "fake news" spread
- Integrate modern data science and machine learning to rumor spread models to deal with limitations on our knowledge of social media populations

Thank you!