Introduction to Dynamic Programming

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February 2018

1 Introduction

After a performance measure for a system has been chosen, we must next choose a method to minimize it. One technique is *dynamic programming*.

1.1 Principle of Optimality

According to Bellman [1],

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitue an optimal policy with regard to the state resulting from the first decision.

Figure 1 shows a multistage decision process. The first decision leads from a to b with a cost of J_{ab} and the remaining decisions from b to e yield cost J_{be}.

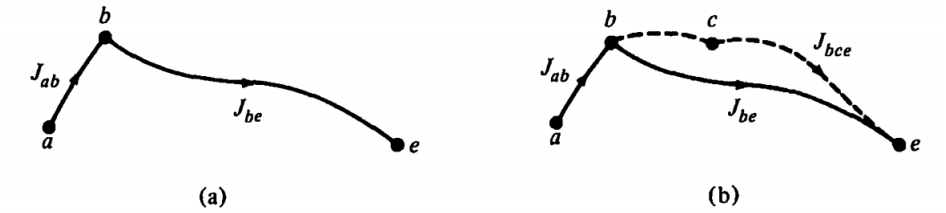

Figure 1: (a) Optimal path from a to e (b) Two possible optimal paths from b to e [2]

The total cost from a to e is the sum of the costs, and if the cost is the minimum then the minimum cost is given in Equation 1. If the path a-b-e is an optimal path from a to e, then by the Prinicple of Optimality the path b-e is also optimal. A proof by contradiction is given on pg. 54 of [1].

$$J^*_{ae} = J_{ab} + J_{be} \tag{1}$$

References

[1] R. E. Bellman and S. E. Dreyfus, *Applied Dynamic Programming.* Princeton University Press, 2015.

[2] D. E. Kirk, *Optimal control theory: an introduction.* Dover Publications, 1998.