2-1

(a) $J = \int_0^{1\,\text{day}} [v_2(t) - M]^2 \, dt$, or

$$J = \int_0^{1\,\text{day}} |v_2(t) - M| \, dt.$$

(b) state constraints :

(i) $\left.\begin{array}{l} 0 \le h_1(t) \le H_{1\,max} \\ 0 \le h_2(t) \le H_{2\,max} \end{array}\right\}$ H_{1max} and $H_{2\,max}$ are the depths of the tanks

<u>OR</u>,

(ii) $\left.\begin{array}{l} 0 \le v_1(t) \le V_{1max} \\ 0 \le v_2(t) \le V_{2\,max} \end{array}\right\}$ V_{1max} and $V_{2\,max}$ are the capacities of the tanks.

Notice that $V_{1_{max}} = \alpha_1 H_{1_{max}}$, $V_{2max} = \alpha_2 H_{2max}$, and that satisfaction of the constraints (i) implies satisfaction of the constraints (ii) and vice-versa; therefore, satisfaction of (i) or (ii) for all $t \in [0, 1]$ is sufficient.

2-1 (b) (cont.)

Control constraints:

$$\left.\begin{array}{ll} 0 \le w_1(t) \le W_{1max} & \\ 0 \le m(t) \le M_{max} & \text{for all} \\ 0 \le w_2(t) \le W_{2max} & t \in [0, T] \end{array}\right\} \begin{array}{l} W_{1max}, W_{2max}, M_{max} \\ \text{determined by} \\ \text{maximum flow} \\ \text{rates.} \end{array}$$

2-2

(a) $J = -v_2(t_f)$; $t_f = 1$ day.

The minus sign converts the maximization problem to a *minimization* problem.

(b) Same as 2-1, but with the additional control constraint

$$\int_0^{1\,\text{day}} m(t)\,dt \le N.$$

2-4

(a) State constraints:

$14.9° \le \theta(30) \le 15.1°$ end point constraint.

control constraints:

$|u(t)| \le U_{max}$ limited thrust available.

(b) $J = \int_0^{30} |u(t)|\, dt$.

Rate of fuel expenditure is proportional to $|u(t)|$.

2-5

(a) State constraints:

$14.9° \le \theta(t_f) \le 15.1°$

Control constraints:

$|u(t)| \le U_{max}$

There might also be a constraint on the total amount of fuel available to perform the maneuver, if so, this constraint would be

$$\int_0^{t_f} |u(t)|\, dt \le M,$$

where M is a specified real number.

(b) $J = \int_0^{t_f} dt$ t_f is free --

the first time the constraint

$14.9° \le \theta(t_f) \le 15.1°$

is satisfied.

2-6

(a) (Inherent physical) constraints:

state -- $0 \le x_1(t)$ assuming surface of the earth at zero elevation and a

2-6 (cont.)

flat earth approximation.

$M_{min} \le x_5(t) \le m(t_0)$;

this is a fuel-expended constraint and could alternatively be expressed in terms of an integral involving the thrust.

control -- $-\pi \le u_2(t) \le \pi$

limation on thrust angle

$0 \le u_1(t) \le T_{max}$.

(b) $J = -x_1(t_f)$ (J to be minimized).

An additional state constraint imposed by the problem statement is

$y(t_f) = x_3(t_f) = 3$ miles.

(c) $J = \int_0^{2.5} u_1(t)\,dt$, or $J = -x_5(t_f)$.

Additional state constraints imposed:

$x_1(t_f) = 500$ miles

$x_3(t_f) = 3$ miles.