Assignment-2

Instructor: Dr. Shaurya Agarwal
EE-5630, Spring 2018

Instructions:

- Please complete all the questions and prepare a single pdf copy. Upload on moodle.
- Submit the assignment by 28^{th} **Feb**, 2017 (Wednesday).
- No late submissions please. Please write clearly.
- Matlab codes should be properly commented, and complemented by explanation where necessary.

Question 1: Given the system dynamics

$$\dot{x_1}(t) = x_2(t) \tag{1}$$
$$\dot{x_2}(t) = -x_1(t) + x_2(t) + u(t) \tag{2}$$

and the cost function as

$$J = \int_0^T \frac{1}{2}[q_1 x_1^2(t) + q_2 x_2^2(t) + u^2(t)]dt; \qquad q_1, q_2 > 0.$$

Find $U^*(t)$ expressed as a function of $X(t)$, t, and J_X^* for the given system.

Question 2: A system has the following first order linear dynamics

$$\dot{x}(t) = -10x(t) + u(t)$$

which needs to be controlled while minimizing the following cost function:

$$J = \frac{1}{2}x^2(T) + \int_0^T [\frac{1}{4}x^2(t) + \frac{1}{2}u^2(t)]dt;$$

The admissible state and control trajectories are not constrained by any boundaries. Find the optimal control law using HJB equation for $T = 5$ and assuming the solution of HJB PDE to be of the quadratic form. You can leave your answers in terms for K(t).

Question 3: The dynamics of a nonlinear scalar system is:

$$\dot{x}(t) = x(t)u(t); \qquad x(0) = 1;$$

and the cost function to be minimized is

$$J = x^2(1) + \int_0^1 [x(t)u(t)]^2 dt,$$

Find the optimal feedback solution by solving the corresponding HJB equation. [Hint: First, find the HJB partial differential equation in terms of J_X, J_t. Then using boundary conditions show that the PDE admits a solution that is quadratic in x. Finally integrate the ODE in $K(t)$ to get the feedback solution.]

Question 4: Given the system dynamics of a plant:

$$\dot{x_1}(t) = x_2(t) \tag{3}$$
$$\dot{x_2}(t) = -x_1(t) - 2x_2(t) + u(t) \tag{4}$$

and the cost function to minimize is

$$J = 10x_1^2(T) + \int_0^T \frac{1}{2}[x_1^2(t) + 2x_2^2(t) + u^2(t)]dt;$$

Final time $T = 10$ and the admissible state and control trajectories are not constrained by any boundaries. Find the optimal control law by solving the Riccati Equation numerically.
a) Identify A, B, Q, R, H
b) Find and plot $K(t)$ using Riccati Equation
c) Find and plot optimal control law
d) Plot the trajectories of states x_1 and x_2 using control in part c)
(Hint: Identify A, B, Q, R, H etc and then use ODE45 command in matlab to solve the ODE in $K(t)$)

Question 5: Derive the Hamilton-Jacobi-Bellman partial differential equation.