Attitude and Optimal Control for CubeSat

Student: Joshua Baltazar, Danny Clemons

Advisor: Dr. S. Agarwal

Final Project Presentation - EE5630

CAL STATE LA
CALIFORNIA STATE UNIVERSITY, LOS ANGELES

Outline of the Presentation

- Introduction
- Background and Motivation
- Problem Formulation
- Optimal Control Solution
- Conclusion

Introduction

Background

- The CubeSat standard was created by Cal Poly SLO and Stanfor's Space Systems Development Lab in 1999.
- Facilitate access to space for university students.
- CubeSat developers include educational institutions, private firms, and government organizations

Background

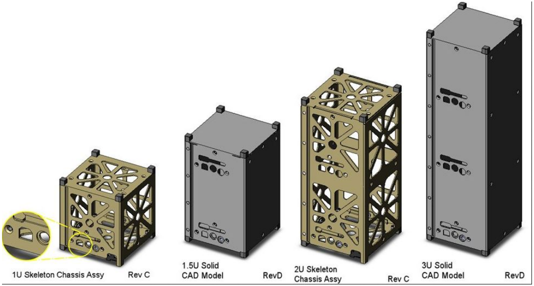
Mission Types

1. Monitoring
2. Improving capabilities
3. Biological Research
4. Imagery

Examples

1. CubeSat QB50 measurements in the lower thermosphere (90350 km) and re-entry research
2. Quakesat measures magnetic signals hoping to ID earthquake onset
3. KSAT2 climatology satellite with RF water vapor sensor for improved prediction of rain and tornado.

Background


1U Skeleton Chassis Assy
Rev C
1.5U Solid
CAD Model
RevD
2U Skeleton
Chassis Assy
Rev C
3U Solid
CAD Model
RevD


ULA
America's Ride to Space
ULA CubeSat Program
Applications Open for Atlas V Launch Slots

Background:Previous Work

- Develle, M.J. Developed optimal controller that minimized torque using B-Spline Prey Motion
- Kedare, S.S. Reduced computational cost for on-board propagation
- Tudor, Z. Developed detumbling algorithm as the cubesat exits it's pod
- Brathen, G. Further developed Tudor's algorithm and implemtented an LQR controller

Problem Formulation

System Dynamics

Governing Equations

- Newton's Universal Gravitation Law

$$F = \frac{Gm_1 m_2}{r^2} \tag{1}$$

- Torque Law

$$\tau^b = I\dot{\omega}^b_{ib} + \omega^b_{ib} x I \omega^b_{ib} \tag{2}$$

- Satellite Kinematics

$$\dot{\eta} = -\frac{1}{2}\varepsilon^T \omega^b_{ob} \tag{3}$$

$$\dot{\varepsilon} = \frac{1}{2}(\eta I_{3x3} + S(t))\omega^b_{ob} \tag{4}$$

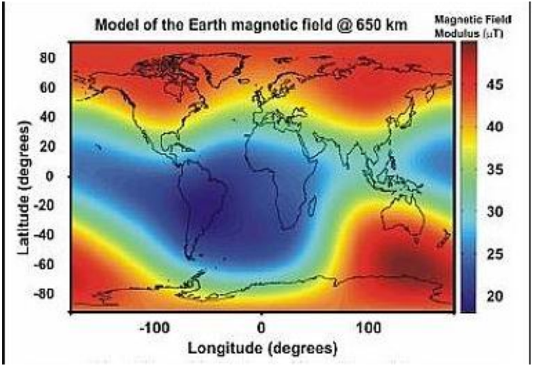
- Geomagnetic Field Model

$$B = -\nabla V \tag{5}$$

Actuators: Magnetorquers

- Runs a current through a coil which generates a magnetic dipole that interacts with the geomagnetic field.
- Magnetic field of the magnetorquer will try and align itself with the earth's geomagnetic field
- Used for low Earth orbit missions(2000km from earth's surface)

International Geomagentic Reference Field(IGRF)



- Magnetic Field

$$B = \nabla V(R_c, \lambda', \theta) \tag{6}$$

$$B = \nabla V(R_c, \lambda', \theta) \tag{7}$$

System Dynamics

Fundamental Diagram

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -4\sigma_x\omega_o^2 & 0 & 0 & 0 & 0 & (1-\sigma_x)\omega_o \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -3\sigma_y\omega_o^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -(1-\sigma_z)\omega_o & 0 & 0 & -\sigma_z\omega_o^2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{B_z^o(t)}{2I_x} & -\frac{B_y^o(t)}{2I_x} \\ 0 & 0 & 0 \\ -\frac{B_z^o(t)}{2I_y} & 0 & \frac{B_x^o(t)}{2I_y} \\ 0 & 0 & 0 \\ \frac{B_y^o(t)}{2I_z} & -\frac{B_x^o(t)}{2I_z} & 0 \end{bmatrix}$$

Parameters

- Angular Velocity ω_o
- Intertial difference σ_i

Cost Function

- General LQR Cost Function

$$J = \frac{1}{2}\int_0^\infty [\boldsymbol{x}^{\boldsymbol{T}}(\boldsymbol{t})Q\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{u}^{\boldsymbol{T}}(\boldsymbol{t})R\boldsymbol{u}(\boldsymbol{t})]dt \tag{8}$$

- Cost Function with Weight Matrices as Identity Matrix

$$J = \frac{1}{2}\int_0^\infty [x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + u_1^2 + u_2^2 + u_3^2]dt \tag{9}$$

Optimal Control Approach: HJB

Hamilton Jacobi Bellman Equation

$$0 = J_t^* + H^* \tag{10}$$

with the Hamiltonian defined as

$$H = g + J_x^* a \tag{11}$$

Analytical Treatment of HJB

Assuming the solution $J^* = 1/2X^T(t)P(t)X(t)$ for (**??**) we get the Differential Ricatti Equation

HJB

$$-\dot{P}(t) = P(t)A + A^T P(t) + 1 - P(t)B(t)B^T(t)P(t)$$
$$p(T) = 0. \tag{12}$$

Afte the ODE is solved, the control law can be determined by evaluating $\frac{\partial H}{\partial u}$

Necessary conditions to find u^*

$$\frac{\partial H}{\partial u_1} = 0, \frac{\partial H}{\partial u_2} = 0, \frac{\partial H}{\partial u_3} = 0 \tag{13}$$

Analytical Treatment of HJB

Optimal Control Laws

$$\begin{aligned}
u_1^* &= \frac{J_{x_4^*}}{2}\left(\frac{B_z^o(t)}{2I_y}\right) - \frac{J_{x_6^*}}{2}\left(\frac{B_y^o(t)}{2I_z}\right) \\
u_2^* &= \frac{J_{x_6^*}}{2}\left(\frac{B_x^o(t)}{2I_z}\right) - \frac{J_{x_2^*}}{2}\left(\frac{B_z^o(t)}{2I_y}\right) \\
u_3^* &= \frac{J_{x_2^*}}{2}\left(\frac{B_y^o(t)}{2I_x}\right) - \frac{J_{x_4^*}}{2}\left(\frac{B_x^o(t)}{2I_y}\right)
\end{aligned} \tag{14}$$

Pontryagin's Maximum Principle

- General LQR Cost Function

$$J = \frac{1}{2}\int_0^\infty [\boldsymbol{x}^{\boldsymbol{T}}(\boldsymbol{t})Q\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{u}^{\boldsymbol{T}}(\boldsymbol{t})R\boldsymbol{u}(\boldsymbol{t})]dt \tag{15}$$

- Cost Function with Weight Matrices as Identity Matrix

$$J = \frac{1}{2}\int_0^\infty [x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + u_1^2 + u_2^2 + u_3^2]dt \tag{16}$$

Pontryagin's Maximum Principle

- General Hamiltonian

$$H = g + p^T[a] \tag{17}$$

- Hamiltonian

$$\begin{aligned} H = {} & \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + u_1^2 + u_2^2 + u_3^2) + p1[x_2] \\ & + p2[-4\sigma_x \omega_o^2 x_1 + (1-\sigma_x)\omega_o x_6 + \frac{B_z^o(t)}{2I_x}u_2 - \frac{B_y^o(t)}{2I_x}u_3 \\ & + p3[x_4] + p4[-3\sigma_y \omega_o^2 x3 - \frac{-B_z^o(t)}{2I_y}u_1 + \frac{B_x^o(t)}{2I_y}u^3] + p5[x6] \\ & p6[-1(1-\sigma_z)w_o x_2 - \sigma_z \omega_o^2 x_5 + \frac{B_y^o(t)}{2I_z}u_1 - \frac{B_x^o(t)}{2I_z}u_2] \end{aligned} \tag{18}$$

Pontryagin's Maximum Principle

- General State Equation

$$\dot{x}^* = \frac{\partial H}{\partial p} \tag{19}$$

Returns State Equations

Pontryagin's Maximum Principle

- State Equation

$$\dot{x}_1^* = x_2 \tag{20}$$

$$\dot{x}_2^* = -4\sigma_x\omega_o^2 x_1 + (1-\sigma_x)\omega_o x_6 + \frac{B_z^o(t)}{2I_x}u_2 - \frac{B_y^o(t)}{2I_x}u_3 \tag{21}$$

$$\dot{x}_3^* = x_4 \tag{22}$$

$$\dot{x}_4^* = -3\sigma_y\omega_o^2 x3 - \frac{-B_z^o(t)}{2I_y}u_1 + \frac{B_x^o(t)}{2I_y}u^3 \tag{23}$$

$$\dot{x}_5^* = x_6 \tag{24}$$

$$\dot{x}_6^* = -1(1-\sigma_z)w_o x_2 - \sigma_z\omega_o^2 x_5 + \frac{B_y^o(t)}{2I_z}u_1 - \frac{B_x^o(t)}{2I_z}u_2$$

Pontryagin's Maximum Principle

- General Co-State Equation

$$\dot{p}^* = \frac{\partial H}{\partial x} \tag{26}$$

Returns Lagrange Equations

Pontryagin's Maximum Principle

- Co-State Equation

$$\dot{p}_1^* = -x_1 + 4\sigma_x \omega_o^2 p_2 \tag{27}$$

$$\dot{p}_2^* = -x_2 - p_1 + (1-\sigma_z) w_o p_6 \tag{28}$$

$$\dot{p}_3^* = -x_3 + 3\sigma_y \omega_o^2 p_4 \tag{29}$$

$$\dot{p}_4^* = -x_4 - p_3 \tag{30}$$

$$\dot{p}_5^* = -x_5 + \sigma_z \omega^2 p_6 \tag{31}$$

$$\dot{p}_6^* = -x_6 - (1-\sigma_x) \omega_o p_2 \tag{32}$$

Pontryagin's Maximum Principle

- Control Law with unbounded u

$$\frac{\partial H}{\partial u} = 0 \tag{33}$$

- Control Law with bounded u

$$H(x^*(t), p^*(t), u(t), t) \geq H(x^*(t), p^*(t), u^*(t), t) \tag{34}$$

All hamiltonians must be greater than the hamiltonian with the u^*

Pontryagin's Maximum Principle

- Control Law with unbounded u

$$\frac{\partial H}{\partial u_1} = 0 \tag{35}$$

$$u_1^* = \frac{B_z^o}{2I_y} p_4 - \frac{B_y^o}{2I_z} p_6 \tag{36}$$

- Control Law with bounded u

$$u_1^* = \begin{cases} .25; & -\frac{B_z^o}{2I_y} p_4 + \frac{B_y^o}{2I_z} p_6 < 1 \\ \frac{B_z^o}{2I_y} p_4 - \frac{B_y^o}{2I_z} p_6; & 0 \leq -\frac{B_z^o}{2I_y} p_4 + \frac{B_y^o}{2I_z} p_6 \leq .25 \\ . - 25; & -\frac{B_z^o}{2I_y} p_4 + \frac{B_y^o}{2I_z} p_6 > 1 \end{cases} \tag{37}$$

Pontryagin's Maximum Principle

- Control Law with unbounded u

$$\frac{\partial H}{\partial u_2} = 0 \tag{38}$$

$$u_2^* = -\frac{B_z^o}{2I_x} p_2 + \frac{B_x^o}{2I_z} p_6 \tag{39}$$

- Control Law with bounded u

$$u_2^* = \begin{cases} .25; & \frac{B_z^o}{2I_x} p_2 - \frac{B_x^o}{2I_z} p_6 < 1 \\ -\frac{B_z^o}{2I_x} p_2 + \frac{B_x^o}{2I_z} p_6; & 0 \leq \frac{B_z^o}{2I_x} p_2 - \frac{B_x^o}{2I_z} p_6 \leq .25 \\ . - 25; & \frac{B_z^o}{2I_x} p_2 - \frac{B_x^o}{2I_z} p_6 > 1 \end{cases} \tag{40}$$

Pontryagin's Maximum Principle

- Control Law with unbounded u

$$\frac{\partial H}{\partial u_3} = 0 \tag{41}$$

$$u_3^* = \frac{B_y^o}{2I_x} p_2 - \frac{B_x^o}{2I_y} p_4 \tag{42}$$

- Control Law with bounded u

$$u_3^* = \begin{cases} .25; & -\frac{B_y^o}{2I_x} p_2 + \frac{B_x^o}{2I_y} p_4 < 1 \\ \frac{B_y^o}{2I_x} p_2 - \frac{B_x^o}{2I_y} p_4; & 0 \leq \frac{B_z^o}{2I_x} p_2 - \frac{B_x^o}{2I_z} p_6 \leq .25 \\ .-25; & -\frac{B_y^o}{2I_x} p_2 + \frac{B_x^o}{2I_y} p_4 > 1 \end{cases} \tag{43}$$

Current Direction

- Determine the magnetic field $B(t)$
- Numerically solve the control law using both HJB and Pontryagin
- Analyze the effect of weight matrices on the control law and tune them accordingly

Conclusion

- Introduced background and motivation CubeSat
- Presented state dynamics for the system
- Analytically solved HJB equation
- Analytically solved Pontryagin Equation

Thank you!