Assignment-3

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EE-5630, Spring 2018

March 26, 2018

Instructions:

- Please complete all the questions and prepare a pdf copy.
- Please include all the required codes, charts and plots.

Question 1: Find the extremal for the following functional:

$$J = \int_0^1 [x^2(t) + \dot{x}^2(t)]dt; \quad x(0) = 0, \quad x(1) = 1$$

Question 2: Find the extremal for the following functional:

$$J = \int_0^1 [x(t) + \dot{x}(t) + \frac{1}{2}\dot{x}^2(t) + x(t)\dot{x}(t)]dt; \quad x(0) = 1/2, \quad x(1) = free$$

Question 3: Determine the necessary conditions that must be satisfied by the extremals for the functional:

$$J(w) = \int_{t_0}^{t_f} [w_3^2(t) + 5]dt;$$

and the differential constraints that must be satisfied are:

$$\dot{w_1}(t) = w_2(t) \tag{1}$$
$$\dot{w_2}(t) = w_3(t) \tag{2}$$

Question 4: Find the optimal control that transfers the system

$$\dot{x_1}(t) = x_2(t) \tag{3}$$
$$\dot{x_2}(t) = u(t) \tag{4}$$

from any initial state (x_0) to the origin in minimum time. The constraint on the control is as follows:

$$-1 \leq u(t) \leq +1$$

(**Note:** This is a special class of **minimum time problem** with linear stationary dynamics and linear regulator (control). The solution of these type of problems is **Bang-Bang Control** and can be obtained analytically! Bang-bang control arises in linear minimum-time problems with constrained input magnitude.

The resulting optimal control for these problems needs only take on two values, which are the extreme values of the control.)

Question 5: Find the optimal control that transfers the system

$$\dot{x}(t) = u(t) \tag{5}$$

from any initial state (x_0) to the origin such that the following cost is minimized:

$$J(u) = \int_0^{t_f} |u(t)|dt,$$

where t_f is free and the constraint on the control is as follows:

$$-1 \leq u(t) \leq +1$$

(**Note:** This is a special class of **minimum fuel/control problem** with linear stationary dynamics and linear regulator (control). The solution of these type of problems is **Bang-Off-Bang Control** and can be obtained analytically!)

Question 6: The dynamics of a system are given as:

$$\dot{x_1}(t) = x_2(t) \tag{6}$$
$$\dot{x_2}(t) = -x_2(t) + u(t) \tag{7}$$

and the cost function to be minimized is

$$J = \frac{1}{2}\int_0^2 u^2(t)dt,$$

Optimal feedback solution is to be found by using Pontryagin's Minimization Principle. Admissible states and controls are not bounded. $X(0) = [0 \;\; 0]'$ and $X(2) = [5 \;\; 2]'$.
(a) Find the necessary conditions that must be satisfied. (Obtain state equations and co-state equations)
(b) Try to solve analytically using the necessary conditions and boundary values.
(c) Develop a MATLAB code to solve the new ODE system symbolically using *syms* and *dsolve* functions.
(d) Compare the results obtained in part (b) and (c) by plotting relevant state trajectories.

Question 7: Given the system dynamics of a continuous stirred tank chemical reactor:

$$\dot{x_1}(t) = -2[x_1(t) + 0.25] + [x_2(t) + 0.5]exp\left[\frac{25x_1(t)}{x_1(t)+2}\right] - [x_1(t) + 0.25]u(t) \tag{8}$$
$$\dot{x_2}(t) = 0.5 - x_2(t) - [x_2(t) + 0.5]exp\left[\frac{25x_1(t)}{x_1(t)+2}\right] \tag{9}$$

with initial conditons $X(0) = [0.05 \;\; 0]'$ and the cost function to minimize is

$$J = \int_0^{0.78} [x_1^2(t) + x_2^2(t) + 0.1u^2(t)]dt;$$

Using the state and co-state equations obtained in Example 6.2-2 (Kirk):
a) Solve this boundary value problem using steepest descent algorithm in MATLAB.
b) Solve this boundary value problem using the MATLAB function *bvp4c*.
c) Which method do you like better and why?