EE 5630: Optimal Control - Assignment 1

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Problem 2-1.

The states for the mixing process from **Problem 1-6** are given by

$$\dot{v}_1(t) \begin{cases} m(t) - \dfrac{v_1(t)\,k}{\alpha_1}[h_1(t) \,-\, h_2(t)], & \text{if } h_1(t) \leq h_2(t) \\ m(t) + \dfrac{v_2(t)\,k}{\alpha_2}[h_2(t) \,-\, h_1(t)], & \text{if } h_2(t) > h_1(t) \end{cases} \tag{1}$$

and

$$\dot{v}_2(t) \begin{cases} -\dfrac{v_1(t)\,k}{\alpha_1}[h_1(t) \,-\, h_2(t)], & \text{if } h_1(t) \leq h_2(t) \\ \dfrac{v_2(t)\,k}{\alpha_2}[h_2(t) \,-\, h_1(t)], & \text{if } h_2(t) > h_1(t). \end{cases} \tag{2}$$

a)

The type of problem defined here is a *tracking problem*. Here, the $v_2(t)$ state is to be kept as close to M ft^3 as possible. Therefore, a performace measure that can be used is

$$J = \int_{t_0}^{t_f} [v_2(t) \,-\, M]^2 dt \tag{3}$$

where t_0 and t_f are the initial and final times, respectively, and $t_f \,-\, t_0 = 1$ day.

b)

A set of physically realistic state and control constraints are

$$0 \leq h_1(t) \leq H_1, \tag{4}$$
$$0 \leq h_2(t) \leq H_2, \tag{5}$$
$$0 \leq w_1(t) \leq W_1, \tag{6}$$
$$0 \leq w_2(t) \leq W_2, \tag{7}$$
$$0 \leq m(t) \leq M \tag{8}$$

where

- H_1 and H_2 are the maximum heights of tanks 1 and 2, respectively
- W_1 and W_2 are the maximum rates of water entering tanks 1 and 2, respectively

Problem 2-2.

This type of problem is classified as a *terminal control problem* in which a parameter is being *maximized* in which the final total volume of dye in tank 2 is to be as close to N ft^3 as possible.

a)

A performance measure that can be used is

$$J = -v_2(t_f)\,. \tag{9}$$

A minus sign is being used because the quantity $v_2(t_f)$ is being *maximized.*

b)

A set of physically realizable state and control constraints are

$$\begin{aligned}
0 \;\le\; h_1(t) \;&\le H_1, &(10)\\
0 \;\le\; h_2(t) \;&\le H_2, &(11)\\
0 \;\le\; w_1(t) \;&\le W_1, &(12)\\
0 \;\le\; w_2(t) \;&\le W_2, &(13)\\
\int_{t_0}^{t_f} m(t)\,dt \;&\le N &(14)
\end{aligned}$$

where

- H_1 and H_2 are the maximum heights of tanks 1 and 2, respectively
- W_1 and W_2 are the maximum rates of water entering tanks 1 and 2, respectively
- where t_0 and t_f are the initial and final times, respectively, and $t_f - t_0 = 1$ day

Problem 2-3.