EE 5630: Optimal Control - Assignment 1

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Question 1.

Example 2.2-1 describes a process in which a performance measure and associated weights can be determined for controlling the attitude, $\theta(t)$, of a manned spacecraft using gas expulsion system, shown in Figure 1. The objective of the control system is to maintain the attitude of the spacecraft at $\theta(t) = 0$ and to do with with small accelerations.

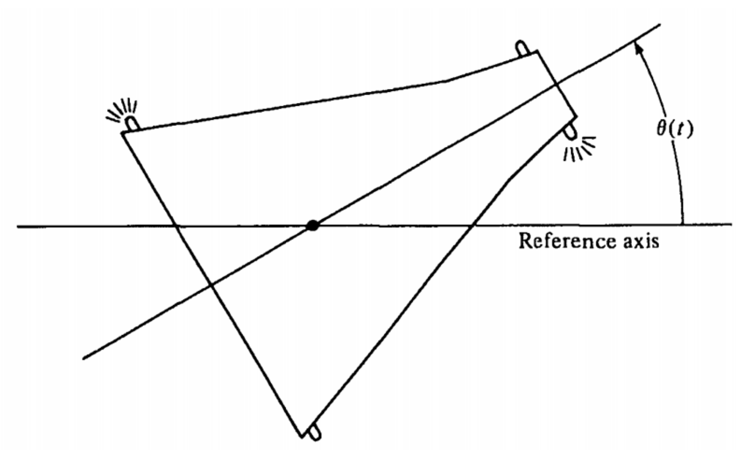

Figure 1: Attitude control of a spacecraft [1]

The dynamics of the system are given by the differential equation given in Equation 1 and the performance measure is given in Equation 2.

$$I\ddot{\theta}(t) = \lambda(t) \tag{1}$$

and

$$J = \int_0^\infty [q_{11}x_1^2(t) + q_{22}x_2^2(t) + Ru^2(t)]\, dt \tag{2}$$

where I is angular moment of inertia and $\lambda(t)$ is the torque produced by the gas jets. where $u(t) = \frac{1}{I}\lambda(t)$. The state space equations are

$$\dot{x_1}(t) = x_2(t) \tag{3}$$
$$\dot{x_2}(t) = u(t) \tag{4}$$

Problem 2-1.

The states for the mixing process from **Problem 1-6** are given by

$$\dot{v}_1(t) \begin{cases} m(t) - \dfrac{v_1(t)\,k}{\alpha_1}[h_1(t) \;-\; h_2(t)], & \text{if } h_1(t) \le h_2(t) \\ m(t) + \dfrac{v_2(t)\,k}{\alpha_2}[h_2(t) \;-\; h_1(t)], & \text{if } h_2(t) > h_1(t) \end{cases} \tag{5}$$

and

$$\dot{v}_2(t) \begin{cases} -\dfrac{v_1(t)\,k}{\alpha_1}[h_1(t) \;-\; h_2(t)], & \text{if } h_1(t) \le h_2(t) \\ \dfrac{v_2(t)\,k}{\alpha_2}[h_2(t) \;-\; h_1(t)], & \text{if } h_2(t) > h_1(t). \end{cases} \tag{6}$$

a)

The type of problem defined here is a *tracking problem*. Here, the $v_2(t)$ state is to be kept as close to M ft^3 as possible. Therefore, a performace measure that can be used is

$$J = \int_{t_0}^{t_f} [v_2(t) \;-\; M]^2 dt \tag{7}$$

where t_0 and t_f are the initial and final times, respectively, and $t_f \;-\; t_0 = 1$ day.

b)

A set of physically realistic state and control constraints are

$$0 \le h_1(t) \le H_1, \tag{8}$$
$$0 \le h_2(t) \le H_2, \tag{9}$$
$$0 \le w_1(t) \le W_1, \tag{10}$$
$$0 \le w_2(t) \le W_2, \tag{11}$$
$$0 \le m(t) \le M \tag{12}$$

where

- H_1 and H_2 are the maximum heights of tanks 1 and 2, respectively
- W_1 and W_2 are the maximum rates of water entering tanks 1 and 2, respectively

Problem 2-2.

This type of problem is classified as a *terminal control problem* in which a parameter is being *maximized* in which the final total volume of dye in tank 2 is to be as close to N ft^3 as possible.

a)

A performance measure that can be used is

$$J = -v_2(t_f)\,. \tag{13}$$

A minus sign is being used because the quantity $v_2(t_f)$ is being *maximized*.

b)

A set of physically realizable state and control constraints are

$$0 \le h_1(t) \le H_1, \tag{14}$$
$$0 \le h_2(t) \le H_2, \tag{15}$$
$$0 \le w_1(t) \le W_1, \tag{16}$$
$$0 \le w_2(t) \le W_2, \tag{17}$$
$$\int_{t_0}^{t_f} m(t)\,dt \le N \tag{18}$$

where

- H_1 and H_2 are the maximum heights of tanks 1 and 2, respectively
- W_1 and W_2 are the maximum rates of water entering tanks 1 and 2, respectively
- where t_0 and t_f are the initial and final times, respectively, and $t_f - t_0 = 1$ day

Problem 2-3.

References

[1] D. E. Kirk, *Optimal control theory: an introduction.* Dover Publications, 1998.