Optimal Control of California Sea Lions Predation on Salmon

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Final Project Presentation - EE5630

Outline of the Presentation

- Background
- Problem Formulation
- Simulation Result
- Conclusion

Background

Salmon Run

- Salmon species swim to upper levels of river to spawn
- Spawn is the release of eggs into the water
- Salmon runs at the Columbia River are the perfect opportunity for predators such as sea lions to hunt salmon

Figure: Fish Ladder (Source: https://en.wikipedia.org/wiki/Fish_ladder)

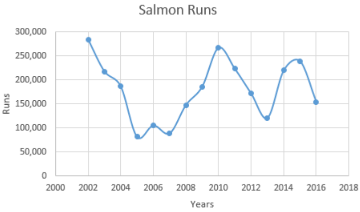
California Sea Lions Pose a Threat to Pacific Salmon

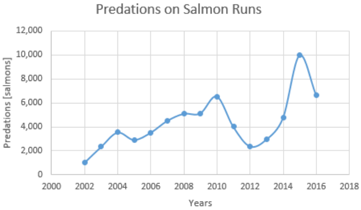
- California Sea Lions typically migrate north to Columbia River for food
- Population of salmon species is threatened by California Sea Lions
- Pacific Northwestern states receive Federal authorization to lethally eliminate California Sea Lions
- Since 2002, data of predation by California Sea Lions during salmon run at Bonneville Dam have been recorded

Figure: Sea Lion predation on Salmon (Source: http://www.dfw.state.or.us/fish/SeaLion/)

Data on Salmon Runs in Bonneville Dam, Columbia River (2002-2016)

- **Left**: Salmon Run Counts, **Right**: Predations during Salmon Runs





- Data obtained from Washington State, Department of Fish and Wildlife
- Data recorded each year during months of January through May

Problem Formulation

Predator-Prey System

- As predator consumes more prey, prey population decreases
- However, predator population begins to decrease due to increasing prey shortage
- Eventually, prey population begins to rise back up due to less predators
- Rinse and repeat

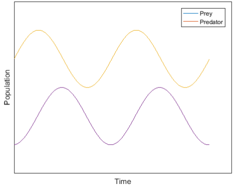


Figure: Population Dynamics of Predator and Prey

Dynamics of a Predator-Prey System

- The prey and predator dynamics are described by a pair of 1st-order, non-linear, differential equations known as the Lotka-Volterra Equations

Lotka-Volterra Equations

$$\frac{dx(t)}{dt} = \alpha x(t) - \beta x(t)y(t)$$
$$\frac{dy(t)}{dt} = \delta x(t)y(t) - \gamma y(t)$$

- x = prey population, α = prey growth rate, β = predation rate
- y = predator population, δ = pedator growth rate, γ = predator mortality rate

Modified Predator-Prey Dynamics

- Account for prey carrying capacity (max limit of population)
- Account for preservation of salmon

Modified Lotka-Volterra Equations

$$\frac{dx(t)}{dt} = \alpha x\left(1 - \frac{x(t)}{K}\right) - \beta(1 - u(t))x(t)y(t)$$
$$\frac{dy(t)}{dt} = \delta x(t)y(t) - \gamma y(t)$$

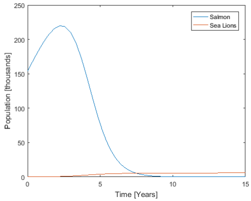
- K = carrying capacity
- $u(t)$ = Elimination efforts of California Sea Lions for salmon preservation

Examination of Uncontrolled Populations

- Current control is non-optimal, ”kill-on-sight” approach
- Initial populations: 154,000 Salmons & 149 California Sea Lions
- Salmons die out in about 10 years

No Preservation Effort, $u(t) = 0$

$$\frac{dx}{dt} = \alpha x(t)(1 - \frac{x(t)}{K}) - \beta x(t)y(t)$$
$$\frac{dy}{dt} = \delta x(t)y(t) - \gamma y(t)$$



States Equations and Cost Function

- Objective is to find optimal control to maintain or increase salmon population and possibly eliminate the least amount of sea lions possible

State Equations

$$\frac{dx}{dt} = 0.468x(1 - \frac{x(t)}{450}) - 0.317(1 - u(t))x(t)y(t)$$
$$\frac{dy}{dt} = 0.00361x(t)y(t) - (-5)y(t)$$

Cost Function

$$J = k_1 x(T) + k_2 y(T) - \int_{t_0}^{T} \frac{u(t)^2}{2} dt$$

- $\alpha = 0.468, \beta = 0.317, K = 450, \delta = 0.00361, \gamma = -5, k_1 = 1, k_2 = varies$

How shall we solve this?

- Given that this is a non-linear system, let's use Pontryagin Minimization Principle

Hamiltonian, Co-states, and Control Law

Hamiltonian

$$H = -\frac{1}{2}u^2 + p_1[\alpha x(1-\frac{x}{K}) - \beta(1-uxy)] + p_2[\delta xy - \alpha y]$$

Co-states

- $\frac{dp_1^*}{dt} = -\frac{\partial H}{\partial x} = p_1^*(-\alpha + \frac{2\alpha}{K}x^* + \beta y^* - \beta y^* u^*) - p_2^* \delta y$
- $\frac{dp_2^*}{dt} = -\frac{\partial H}{\partial y} = p_2^*(-\delta x^* + \gamma) + p_1^* \beta(1-u^*)x$

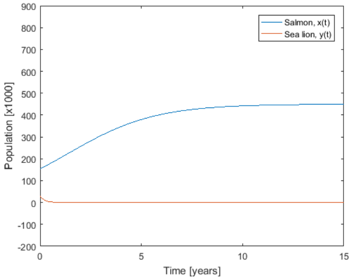
Control Law

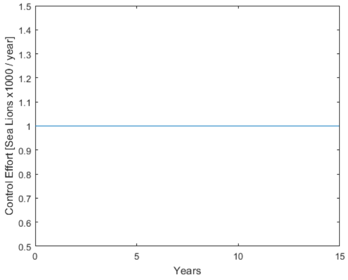
$$\frac{\partial H}{\partial u} = 0 = -u^*(t) + \beta p_1^*(t) x^*(t) y^*(t)$$
$$\rightarrow u^* = \beta p_1^* x^* y^*$$

Optimal Control Solution

State Trajectories Using Steepest Descent Method

- **Left**: Population Trajectories, **Right**: Control Effort, $u^*(t)$

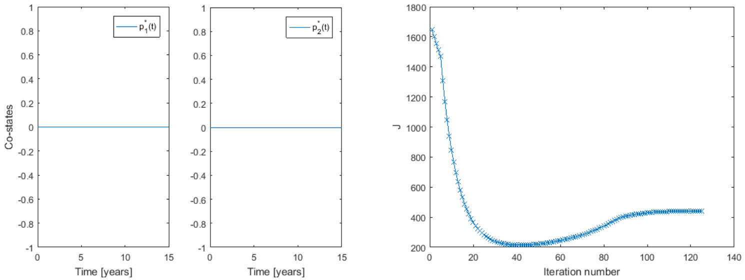




- Weights: $k_1 = 1$, $k_2 = 50$
- Initial populations: 154,000 Salmon and 30,000 sea lions
- Salmon species reach their carrying capacity in about 10 years
- All California Sea Lions removed in about 2 years

Co-state Trajectories and Cost

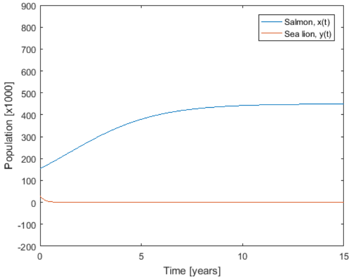
- **Left**: Co-state Trajectories, **Right**: Cost, J

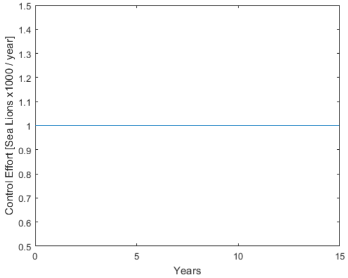


- Weights: $k_1 = 1$, $k_2 = 50$
- Constant $u^*(t)$ may suggest 0 valued co-states

State Trajectories Using Steepest Descent Method

- **Left**: Population Trajectories, **Right**: Control Effort, $u^*(t)$

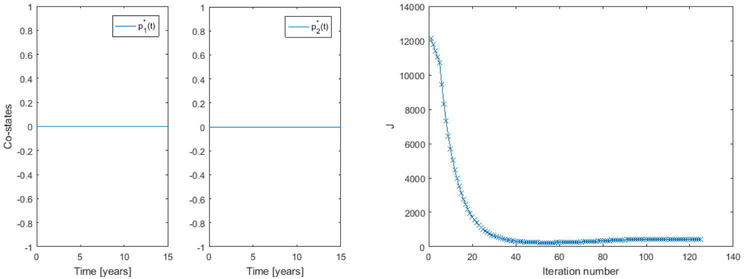




- Weights: $k_1 = 1$, $k_2 = 400$
- Initial populations: 154,000 Salmon and 30,000 sea lions
- Population trajectories and control effort remain the same

Co-state Trajectories and Cost

- **Left**: Co-state Trajectories, **Right**: Cost, J



- Weights: $k_1 = 1$, $k_2 = 400$
- Co-states remain the same
- Cost reaches steady state smoother due to heavier weight applied to final predator population

Conclusions

- Control effort removes all California Sea Lions in 2 years and increases Salmon population to carrying-capacity
- Weight applied to final predator population influences cost that define measure of control effort
- Constant zero-valued co-states need further investigation

Limitations

- In reality, it is expected that predator-prey dynamics would occur if salmon population increases
- Constant removal of 1,000 California Sea Lions per year probably not practical given increasing predator population
- Data used in this research is rough estimate (ex. impossible to know exact salmon population; just too many)

Possible Implementations to Consider

- Stranded sea lions
- Poaching of sea lions
- Fishing

Thank you!