Optimal Control of Fisheries

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Final Project Presentation - EE5630

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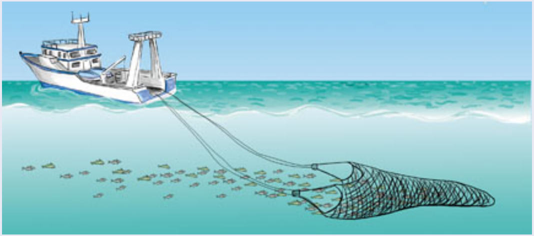
Outline of the Presentation

- Introduction
- Background and Motivation
- Problem Formulation
- Dynamics and Cost Function
- Analysis and Simulation
- Conclusion

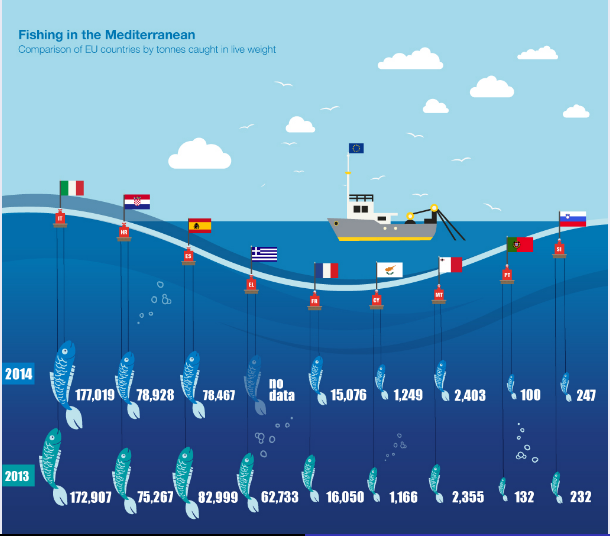
Introduction

Harvesting Fish



- In 2009, Americans consumed a total of 4.8 billion pounds
- 15.8 pounds of fish and shellfish per person

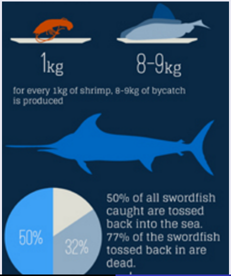
Fishing Demand in Europe



Overfishing

- Overfishing occurs when fish are caught faster than nature can normally supply.
- From 1950 - 2011, fishing stocks have seen a decrease of 90 percent.
- By Catch: Sea creatures incidentally caught in the net.

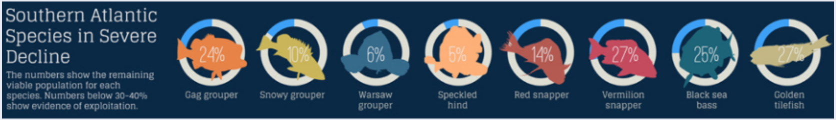
Overfishing



Overfishing

- As a result of overfishing, various fish species are declining.

Overfishing



Considerations

Considerations

1. Population of Fish
2. Cost of Fish
3. Profits for Fishermen
4. Control of Fishing
5. Time for Fishing

Problem Formulation

Dynamics of the System

We can consider a model for a single population of fish.

Dynamics

$$dx = Kx(t)(M - x(t)) - u(t)x(t) \tag{1}$$

where $x(t)$, is population level of fish and $u(t)$ is the harvesting control.

Cost Function

For the model previous dynamics, the object of the control law is to maximize net profit for the fishermen for the given x_0 and time t_0.

Cost Function

$$J_= \left\{ \int_0^T \exp(-\delta t)(p_1 u(t)x(t) - p_2(x(t)u(t))^2 - c_1 u(t))dt \right\}, \qquad (2)$$

where, p_1 , p_2 and c_1 represent profit from sales of fishing, diminishing returns when there is a large amount of fish to sell and cost of fishing. The value function then becomes:

Value Function

$$\mathscr{V}(x,t) = \inf_{u(\cdot)} J_{x,t}[u(\cdot)]. \qquad (3)$$

Dynamics of the System - Baltic Sea

We can consider a model for multiple fish population.

Dynamics

$$\dot{x}_i(t) = \varepsilon_i x_i(t)\left(1 - \frac{x_i(t)}{K_i}\right) - \sum_{j=1}^{m} \gamma_{ij} \frac{x_i(t)}{K_i}\frac{x_j(t)}{K_j} - u_i(t) r_i d \cdot \frac{x_i(t)}{K_i}$$

where $x(t)$, is population level of fish and $u(t)$ is the harvesting control, ε is growth coefficients, γ is the phagos coefficent, d is the number of days, r is the catch proportionality and K is a given constant number.

Cost Function

For the model previous dynamics, the object of the control law is to maximize net profit for the fishermen for the given x_0 and time t_0.

Cost Function

$$J(u) = \int_0^T \left\{ \sum_{i=1}^{m} p_i u_i(t) r_i d \cdot \frac{x_i(t)}{K_i} - cd \cdot \sum_{i=1}^{m} u_i(t) \right\} e^{-\delta t} \rightarrow \max_{u(\cdot)}$$

where, $p1$ is profits from fishing, $x(t)$ is population level of fish and $u(t)$ is the harvesting control, $e^(- \delta t)$ is the discount factor, d is the number of days, r is the catch proportionality and K is a given constant number.

Method

To simulate, use the Pontryagin method with Steepest descent method is used to find the solution.

Dynamics

$$dx = Kx(t)(M - x(t)) - u(t)x(t) \tag{4}$$

Cost Function

$$J_{=}\left\{\int_0^T x^2(t) - u^2(t)dt\right\}, \tag{5}$$

Method

To simulate, use the Pontryagin method with Steepest descent method is used to find the solution.

State Equation

$$dx = Kx(t)(M - x(t)) - u(t)x(t) \tag{6}$$

Costate Equation

$$dp = 2x + p(t)(KM - 2Kx(t) - u(t)); \tag{7}$$

Partial derivative of H with respect to U

$$\frac{dH}{du} = 2u(t) + p(t)x(t); \tag{8}$$

Expectations

- The cost function is to rise exponentially, then settle.
- Population/Control should have an overshoot, then settle.
- Note: Following Figures was from the paper ”Some Applications of Optimal Control in Sustainable Fishing in the Baltic Sea”

Expectations

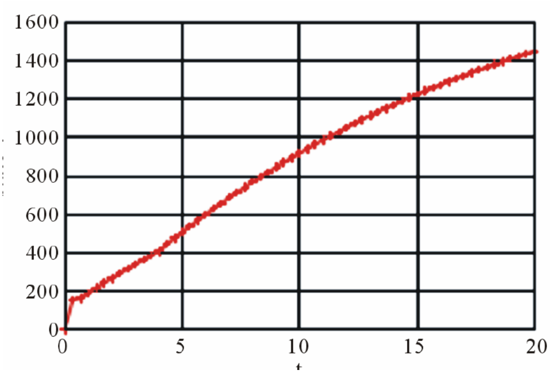


Figure: Expectations for cost [1]

[1] Copyright 2011 SciRes. Some Applications of Optimal Control in Sustainable Fishing in the Baltic Sea

Expectations

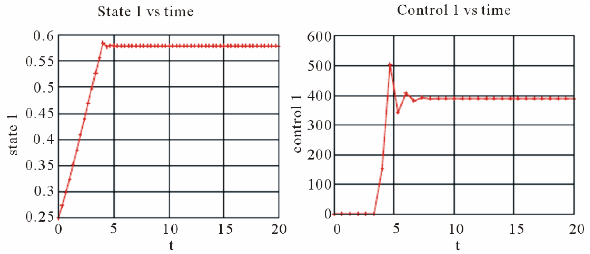


Figure: Expectations for states[2]

[2]Copyright 2011 SciRes. Some Applications of Optimal Control in Sustainable Fishing in the Baltic Sea

Simulation Results

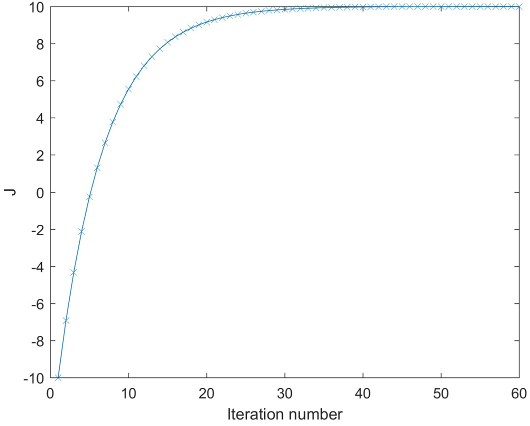


Figure: Numerical Results for Inverse Problem-1

Simulation Results

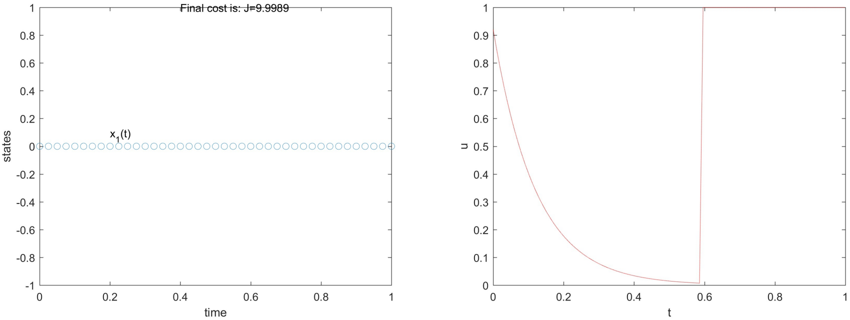


Figure: Numerical Results for Inverse Problem-1

Conclusion

- Although I did not get great results, I understand what I am supposed to look for.
- Learned that everything can be written as a optimal control problem.

Future Work

- Try using multiple states
- Try using the steady state method
- Finalize code to work properly

Thank you!