Optimal Control

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1 State Space Review

The state space representation of a linear time-invariant system has the following form:

$$\dot{X}(t) = A(t)\,X(t) \,+\, B(t)\,U(t) \tag{1}$$
$$Y(t) = C(t)\,X(t) \,+\, D(t)\,U(t) \tag{2}$$

.

Equations 1 and 2 are referred to as the **state differential equation** and **state output equation**, respectively.

1.1 Example: Mass-Spring-Damper

A classic example of a linear system is the Mass-Spring-Damper (MSD) system, as shown in Figure 1. The differential equation for the MSD is

$$\ddot{y}(t) \;=\; -\frac{k}{m}y(t) - \frac{b}{m}\dot{y}(t) + \frac{1}{m}r(t) \tag{3}$$

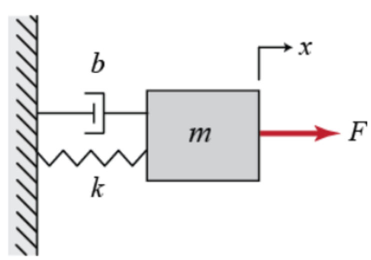

Figure 1: Linear mass-spring-damper system

By examination of Equation 1.1, we can see that the system has two states (the number of states is equal to the order, n, of the differential equation, which in this case is 2). We'll call these states x_1 and x_2 and define them in the following way:

$$
\begin{aligned}
x_1 &= y \\
\dot{x_1} &= \dot{y} = x_2 \qquad (4) \\
x_2 &= \dot{y} = \dot{x_1} \\
\dot{x_2} &= \ddot{y} = -\frac{k}{m}x_1 - \frac{b}{m} + \frac{1}{m}u(t) \qquad (5)
\end{aligned}
$$

where $u(t) = r(t)$. The states, x_1 and x_2, are defined by Equations 4 and 5 which give the position and velocity of the mass, m.