Linear Systems Analysis

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1 Linear Algebra Review

This is a review of linear algebra terms and concepts.

1.1 Terms

A review of linear algebra terms.

Consider an n-dimensional linear space $\mathbb{R}^n$. Every vector in $\mathbb{R}^n$ is an n-tuple of reals:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{1}$$

1.1.1 Linear Independence

Let V be a set of vectors

$$V = \{X_1, X_2, \ldots, X_n\} \in \mathbb{R}^n \tag{2}$$

whose element vectors are *linearly dependent* if

$$\exists\, \alpha_1, \alpha_2, \ldots, \alpha_n \neq 0 \tag{3}$$

such that

$$\alpha_1 X_1 + \alpha_2 X_2 + \ldots + \alpha_n X_n = 0 \tag{4}$$

This means that a set of vectors V is linearly dependent if they can be composed of some combination of each other. We can take this definition and turn it around to find that a set of vectors V is *linearly independent* if Equation 4 is satisfied only when $\alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$.

1.1.2 Basis

A set of n linearly independent vectors in $\mathbb{R}^n$ is a *basis* if every vector in $\mathbb{R}^n$ can be expressed as a unique combination of this set (i.e., span the space). **Note:** in $\mathbb{R}^n$, any set of n linear independent vectors can be used as a basis.

1.1.3 Basis and Representation

Let $Q = \{q_1, q_2, \ldots, q_n\}$ be a set of linearly independent vectors in $\mathbb{R}^n$. Now, any vector, $X \in \mathbb{R}^n$, can be expressed as

$$X = \alpha_1 q_1 + \alpha_2 q_2 + \ldots + \alpha_n q_n \tag{5}$$

such that $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{R}$.

References