EE 5600: Linear Systems Analysis - Assignment 1

Joshua Saunders

February 10, 2018

Question 1.

$$x_1 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

a)
First norm:

$$\begin{aligned} ||x_1||_1 &= \sum_1^3 x_{1i} && (1) \\ &= 2 - 3 - 1 && (2) \\ &= \mathbf{-2} && (3) \end{aligned}$$

and

$$\begin{aligned} ||x_2||_1 &= \sum_1^3 x_{2i} && (4) \\ &= 1 + 1 - 1 && (5) \\ &= \mathbf{1} && (6) \end{aligned}$$

b)
Second norm:

$$\begin{aligned} ||x_1||_2 &= \sqrt{\sum_1^3 x_{1i}^2} && (7) \\ &= \sqrt{2^2 + (-3)^2 + (-1)^2} && (8) \\ &= \mathbf{\sqrt{14}} && (9) \end{aligned}$$

and

$$||x_2||_2 = \sqrt{\sum_1^3 x_{2i}^2} \tag{10}$$
$$= \sqrt{1^2 + 1^2 + (-1)^2} \tag{11}$$
$$= \boldsymbol{\sqrt{3}} \tag{12}$$

c)
Infinite norm:

$$||x_1||_\infty = max(x_1) \tag{13}$$
$$= \mathbf{2} \tag{14}$$

and

$$||x_2||_\infty = max(x_2) \tag{15}$$
$$= \mathbf{1} \tag{16}$$

Question 2. Find two orthonormal vectors that span the same space as two vectors in problem 1.

Question 3.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$