

# EE 5600: Linear Systems Analysis - Assignment 1

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**Question 1.**

$$x_1 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

**a)**

First norm:

$$\|x_1\|_1 = \sum_1^3 x_{1i} \tag{1}$$

$$= 2 - 3 - 1 \tag{2}$$

$$= \mathbf{-2} \tag{3}$$

and

$$\|x_2\|_1 = \sum_1^3 x_{2i} \tag{4}$$

$$= 1 + 1 - 1 \tag{5}$$

$$= \mathbf{1} \tag{6}$$

**b)**

Second norm:

$$\|x_1\|_2 = \sqrt{\sum_1^3 x_{1i}^2} \tag{7}$$

$$= \sqrt{2^2 + (-3)^2 + (-1)^2} \tag{8}$$

$$= \mathbf{\sqrt{14}} \tag{9}$$

and

$$||x_2||_2 = \sqrt{\sum_1^3 x_{2i}^2} \quad (10)$$

$$= \sqrt{1^2 + 1^2 + (-1)^2} \quad (11)$$

$$= \sqrt{3} \quad (12)$$

**c)**

Infinite norm:

$$||x_1||_\infty = \max(x_1) \quad (13)$$

$$= \mathbf{2} \quad (14)$$

and

$$||x_2||_\infty = \max(x_2) \quad (15)$$

$$= \mathbf{1} \quad (16)$$

**Question 2.** Find two orthonormal vectors that span the same space as the two vectors,  $x_1$  and  $x_2$ , in Problem 1.

Equation 17 shows is that the vectors  $x_1$  and  $x_2$  are orthogonal. Because  $x_1$  and  $x_2$  are orthogonal, they only need to be normalized, as shown below in Equations 18 to 19.

$$x_1^T \cdot x_2 = x_2^T \cdot x_1 = \mathbf{0} \quad (17)$$

The normalization process, where  $u_1$  and  $u_2$  are the normalized versions of  $x_1$  and  $x_2$ , respectively:

$$\begin{aligned} u_1 &= \frac{x_1}{\|x_1\|} \\ &= \frac{x_1}{\sqrt{2^2 + (-3)^2 + (-1)^2}} \\ &= \frac{x_1}{\sqrt{4 + 9 + 1}} \\ &= \frac{x_1}{\sqrt{14}} \end{aligned} \quad (18)$$

$$\begin{aligned} \mathbf{u}_1 &= \begin{bmatrix} \frac{2}{\sqrt{14}} \\ \frac{-1}{\sqrt{14}} \\ \frac{-3}{\sqrt{14}} \end{bmatrix} \\ u_2 &= \frac{x_2}{\|x_2\|} \\ &= \frac{x_2}{\sqrt{1^2 + 1^2 + (-1)^2}} \\ &= \frac{x_2}{\sqrt{1 + 1 + 1}} \\ &= \frac{x_2}{\sqrt{3}} \end{aligned}$$

$$\mathbf{u}_2 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \end{bmatrix} \quad (19)$$

**Question 3.**

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**A<sub>1</sub>:**

By examination, it can be seen that the matrix  $A_1$  has two linearly independent columns. Therefore, the **rank of A<sub>1</sub> is 2**. There are four columns in  $A_1$  and its rank is two, therefore **A<sub>1</sub>'s nullity is  $4 - 2 = 2$** .

**A<sub>2</sub>:**

Matrix  $A_2$  can be transformed into an upper triangle using a sequence of elementary transformations as demonstrated by [1] and is given by Equation 20. According to [1], the rank of an upper triangular matrix is equal to the number of nonzero rows. The matrix  $A_{2ref}$  has three nonzero rows and therefore it and  $A_2$  have a **rank of 3**. The **nullity of A<sub>2</sub> is 0**.

$$A_2 \longrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A_{2ref} \quad (20)$$

**A<sub>3</sub>:**

By examination, it can be seen that the matrix  $A_3$  has three linearly independent columns. Therefore, the **rank of A<sub>3</sub> is 3**. There are four columns in  $A_3$  and its rank is three, therefore **A<sub>3</sub>'s nullity is  $4 - 3 = 1$** .

## References

- [1] C.-T. Chen, *Linear system theory and design*. Oxford University Press, Inc., 1998.