Assignment 3

Instructor: Dr. Shaurya Agarwal EE5600, Spring 2018

April 12, 2018

Instructions:

• Please complete all the questions and prepare a hard copy.

• Submit the assignment: April, 19 2017

• No scanned copies. No late submissions please. Please write clearly.

Question 1: The ability to balance actively is a key ingredient in the mobility of a device that hops and runs on the one springy leg, as shown in figure. The control of attitude of device uses a gyroscope and a feedback such that u(t) = Kx(t), where

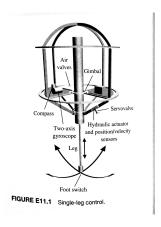
$$K = \begin{bmatrix} -k & 0 \\ 0 & -2k \end{bmatrix} and$$

$$\dot{x}\left(t\right) = Ax\left(t\right) + Bu\left(t\right)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; B = I$$

Determine K so that response of the system is critically damped? (Use $\zeta = 1$)



Question 2: A system is described by the equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 2 \end{bmatrix} x(t)$$

determine controllabilty and observability.

Question 3: A system is described by the equations

$$\dot{x}\left(t\right) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x\left(t\right) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u\left(t\right)$$

$$y\left(t\right) = \begin{bmatrix} 1 & 0 \end{bmatrix} x\left(t\right)$$

determine controllabilty and observability.

Question 4: Hydraulic power actuators were used to drive the dinosaurs of the movie Jurassic park, the motions of the large monsters required high power actuators requiring 1200 watts. One specific limb motion has dynamics represented by

$$\dot{x}\left(t\right) = \begin{bmatrix} -4 & 0\\ 1 & -1 \end{bmatrix} x\left(t\right) + \begin{bmatrix} 1\\ 0 \end{bmatrix} u\left(t\right)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

We want to place the closed loop poles at $S = -1 \pm j3$. Determine the required state variable feedback using any method (ackermans formula or coefficient comparison). Assume that complete state vector is available for feedback.

Question 5: Consider

$$\dot{x}\left(t\right) = AX\left(t\right) + BU\left(t\right)$$

$$y\left(t\right) = CX\left(t\right) + DU\left(t\right)$$

Where;

$$A = \begin{bmatrix} 1 & 4 \\ -5 & -10 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & -4 \end{bmatrix}; D = \begin{bmatrix} 0 \end{bmatrix}$$

- a) Comment on observability.
- b) Design full state observer placing poles at $S_{1,2} = -1$.
- c) Plot the response of the estimations error $e(t) = x(t) \hat{x}(t)$ with initial estimation error $e(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}'$ using Matlab and attach your result.

Question 6: A system is described by the equations

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ -5 & -20 \end{bmatrix} x(t) + \begin{bmatrix} 100 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Determine observer gains to place the observer poles at $S_{1,2} = -5$.