Solutions Assignment-3

E11.1 The system is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{u} = \mathbf{K}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{K} = \begin{bmatrix} -k & 0 \\ 0 & -2k \end{bmatrix} .$$

Then, with $\mathbf{u} = \mathbf{K}\mathbf{x}$, we have

$$\dot{\mathbf{x}} = \begin{bmatrix} -k & 1 \\ -1 & -2k \end{bmatrix} \mathbf{x} .$$

The characteristic equation is

$$\det[s\mathbf{I} - \mathbf{A}] = \det \begin{bmatrix} s+k & -1 \\ 1 & s+2k \end{bmatrix} = s^2 + 3ks + 2k^2 + 1$$
$$= s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 .$$

Solving for k where $\omega_n^2 = 2k^2 + 1$ and $\zeta = 1$ (critical damping) yields

$$k = 2.$$

E11.3 The controllability matrix is

$$\mathbf{P}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & -10 \end{bmatrix} ,$$

and $\det \mathbf{P}_c = -4 \neq 0$, therefore the system is controllable. The observability matrix is

$$\mathbf{P}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & -10 \end{bmatrix} ,$$

and $\det \mathbf{P}_o = 0$; therefore the system is unobservable.

E11.6 The controllability matrix is

$$\mathbf{P}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} ,$$

and $\det \mathbf{P}_c \neq 0$; therefore the system is controllable. The observability matrix is

$$\mathbf{P}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ,$$

and $\det \mathbf{P}_o \neq 0$; therefore the system is observable.

P11.16 Let

$$u = -\mathbf{K}\mathbf{x} \ .$$

Then, Ackermann's formula is

$$\mathbf{K} = [0, 0, ..., 1]\mathbf{P}_c^{-1} q(\mathbf{A})$$

where $q(s)$ is the desired characteristic polynomial, which in this case is

$$q(s) = s^2 + 2s + 10 \ .$$

A state-space representation of the limb motion dynamics is

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 0 \\ 1 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \ .$$

The controllability matrix is

$$\mathbf{P}_c = [\mathbf{B} \ \ \mathbf{AB}] = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

and

$$\mathbf{P}_c^{-1} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \ .$$

Also, we have

$$q(\mathbf{A}) = \mathbf{A}^2 + 2\mathbf{A} + 10\mathbf{I} = \begin{bmatrix} 18 & 0 \\ -3 & 9 \end{bmatrix} \ .$$

Using Ackermann's formula, we have

$$\mathbf{K} = [-3 \quad 9] \ .$$

P11.25 The observability matrix is

$$\mathbf{P}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 21 & -36 \end{bmatrix} ,$$

and $\det \mathbf{P}_o = 48 \neq 0$; therefore the system is completely observable. The desired poles of the observer are $s_{1,2} = -1$. This implies that the desired characteristic polynomial is

$$p_d(s) = s^2 + 2s + 1 .$$

The actual characteristic polynomial is

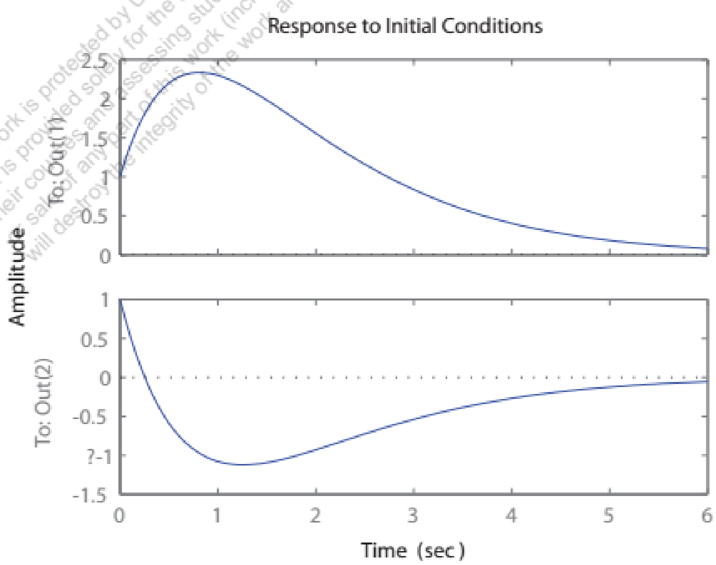
$$\det |\lambda\mathbf{I} - (\mathbf{A} - \mathbf{LC})| = \det \begin{vmatrix} \lambda - 1 + L_1 & -4 - 4L_1 \\ 5 + L_2 & \lambda - 10 - 4L_2 \end{vmatrix}$$

$$= \lambda^2 + (L_1 - 4L_2 - 11)\lambda + 10L_1 + 8L_2 + 30 = 0 .$$

Solving for L_1 and L_2 yields

$$\mathbf{L} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} -0.25 \\ -3.3125 \end{bmatrix} .$$

Checking we find that $\det(\lambda\mathbf{I} - (\mathbf{A} - \mathbf{LC})) = s^2 + 2s + 1$. The response of the estimation error is shown in Figure P11.25, where $\mathbf{e}(0) = [\ 1 \quad 1\ ]^T$.



P11.27 The observability matrix is

$$\mathbf{P}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} .$$

The $\det \mathbf{P}_o = 0$, hence the system is not completely observable. So, we cannot find an observer gain matrix that places the observer poles at the desired locations.