

Linear Systems Analysis

Joshua Saunders

February 2018

1 Linear Algebra Review

This is a review of linear algebra terms and concepts.

1.1 Terms

A review of linear algebra terms.

Consider an n -dimensional linear space \mathbb{R}^n . Every vector in \mathbb{R}^n is an n -tuple of reals:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{1}$$

1.1.1 Linear Independence

Let V be a set of vectors

$$V = \{X_1, X_2, \dots, X_n\} \in \mathbb{R}^n \tag{2}$$

whose element vectors are *linearly dependent* if

$$\exists \alpha_1, \alpha_2, \dots, \alpha_n \neq 0 \tag{3}$$

such that

$$\alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_n X_n = 0 \quad (4)$$

This means that a set of vectors V is linearly dependent if they can be composed of some combination of each other. We can take this definition and turn it around to find that a set of vectors V is *linearly independent* if Equation 4 is satisfied only when $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$.

1.1.2 Basis

A set of n linearly independent vectors in \mathbb{R}^n is a *basis* if every vector in \mathbb{R}^n can be expressed as a unique combination of this set (i.e., span the space). **Note:** in \mathbb{R}^n , any set of n linear independent vectors can be used as a basis.

1.1.3 Basis and Representation

Let $Q = \{q_1, q_2, \dots, q_n\}$ be a set of linearly independent vectors in \mathbb{R}^n . Now, any vector, $X \in \mathbb{R}^n$, can be expressed as

$$X = \alpha_1 q_1 + \alpha_2 q_2 + \cdots + \alpha_n q_n \quad (5)$$

such that $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$.

Assume that

$$Q = \{q_1, q_2, \dots, q_n\} \in \mathbb{R}^n \times \mathbb{R}^n, \quad \text{then} \quad (6)$$

$$X = Q[\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n]^T \quad (7)$$

$$= Q\bar{X} \quad (8)$$

where $\bar{X} = [\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n]^T$ and is the *representation* of X with respect to the basis Q .

Question: Consider svector $X, q_1, q_2 \in \mathbb{R}^2$ such that

$$X = [1 \quad 3]^T \tag{9}$$

$$q_1 = [3 \quad 1]^T \tag{10}$$

$$q_2 = [2 \quad 2]^T \tag{11}$$

a) Do q_1 and q_2 form a basis in \mathbb{R}^2 ?

b) If so, find the representatino of X with respect to the basis formed by q_1 and q_2 .