

# ADTs Stack and Queue

# After studying this chapter, you should be able to

- Describe a stack and its operations at a logical level
- Demonstrate the effect of stack operations using a particular implementation of a stack
- Implement the Stack ADT in two array-based implementations and a linked implementation
- Describe the structure of a queue and its operations at a logical level
- Demonstrate the effect of queue operations using a particular implementation of a queue
- Implement the Queue ADT using both an array-based implementation and a linked implementation
- Use inheritance to create a Counted Queue ADT

# Stacks: Logical Level

- **Stack:** An ADT where items are added and removed only from the top of the structure; this behavior is called **LIFO** (**L**ast **I**n, **F**irst **O**ut)
- Items are “ordered” by when they were added to the stack
- Like lists, stacks store homogeneous items

# Stack Operations

- **Push:** Adds an item to the top of the stack
- **Pop:** Removes the top item from the stack
- **Top:** Returns the item at the top of the stack but does not remove it
- **IsEmpty:** Returns true if the stack has no items
- **IsFull:** Stacks are logically unbounded, but implementations may be bounded

# Stacks: Application Level

Stacks have many applications in software engineering. Some examples:

- Tracking the function calls in a program
- Performing syntax analysis on a program
- Traversing structures like trees and graphs
- Some programming languages are entirely stack-based, such as Forth

Stacks are useful in situations where we must process nested components.

# Stacks: Implementation Level

- Since elements are homogeneous (the same type), an array-based approach can be used
- We can put elements into sequential slots in the array, placing the first element pushed into the first array position, the second element pushed into the second array position, and so on.

# Definitions of Stack Operations

- In the Stack ADT, `top` indicates which element is on top. Thus the class constructor sets (`top == -1`) rather than `0`. `IsEmpty` should compare `top` with `-1`, and `IsFull` should compare `top` with `(MAX_ITEMS - 1)`

```
StackType::StackType()
{
    top = -1;
}

bool StackType::IsEmpty() const
{
    return (top == -1);
}

bool StackType::IsFull() const
{
    return (top == MAX_ITEMS-1);
}
```

# Definitions of Stack Operations (cont.)

- We write the algorithm to **Push** an item on the top of the stack, **Pop** an item from the top of the stack, and return a copy of the top item. Push must increment top and store the new item into `items[top]`. If the stack is already full when we invoke **Push**, the resulting condition is called **stack overflow**. The overflow causes an exception to be thrown; thus we can use a **try/catch** statement to handle the overflow.

## Push

```
if stack is full
    throw an exception FullStack
else
    increment top
    Set items[top] to newItem
```

```
void StackType::Push(ItemType newItem)
{
    if( IsFull() )
        throw FullStack();
    top++;
    items[top] = newItem;
}
```



# Definitions of Stack Operations (cont.)

```
void StackType::Pop()
{
    if( IsEmpty() )
        throw EmptyStack();
    top--;
}
```

```
ItemType StackType::Top()
{
    if (IsEmpty())
        throw EmptyStack();
    return items[top];
}
```

- Pop is essentially the reverse of Push. We decrement **top**. If the stack is empty when we invoke **Pop** or **Top**, a **stack underflow** results. As with the Push function, the specifications for the operations say to throw an exception in this event.

```
class StackType
{
public:
    StackType();
    // Class constructor.
    bool IsFull() const;
    // Function: Determines whether the stack is full.
    // Pre: Stack has been initialized.
    // Post: Function value = (stack is full)
    bool IsEmpty() const;
    // Function: Determines whether the stack is empty.
    // Pre: Stack has been initialized.
    // Post: Function value = (stack is empty)
    void Push(ItemType item);
    // Function: Adds newItem to the top of the stack.
    // Pre: Stack has been initialized.
    // Post: If (stack is full), FullStack exception is thrown;
    //        otherwise, newItem is at the top of the stack.
    void Pop();
    // Function: Removes top item from the stack.
    // Pre: Stack has been initialized.
    // Post: If (stack is empty), EmptyStack exception is thrown;
    //        otherwise, top element has been removed from stack.
    ItemType Top();
    // Function: Returns a copy of top item on the stack.
    // Pre: Stack has been initialized.
    // Post: If (stack is empty), EmptyStack exception is thrown;
    //        otherwise, top element has been removed from stack.

private:
    int top;
    ItemType items[MAX_ITEMS];
};
```

# Implementing a Stack as a Linked Structure

- Instead of using array, we can implement a stack using linked list structure.

# Linked-List Implementation: Push

```
void StackType::Push(ItemType newItem)
// Adds newItem to the top of the stack.
// Pre: Stack has been initialized.
// Post: If stack is full, FullStack exception is thrown;
//       else newItem is at the top of the stack.
{
    if (IsFull())
        throw FullStack();
    else
    {
        NodeType* location;
        location = new NodeType;
        location->info = newItem;
        location->next = topPtr;
        topPtr = location;
    }
}
```

- Create a new node for the new element
- Have it point to topPtr as its next element
- Update topPtr to point to the new node
- Additionally, must throw an exception if the stack is full when Push is called.

# Linked-List Implementation: Pop

- Make a tempPtr that copies topPtr
- Update topPtr to point to the next node on the stack
- Delete the node pointed to by tempPtr
- If the stack is empty when Pop is called, an exception should be thrown.

```
void StackType::Pop()
// Removes top item from Stack and returns it in item.
// Pre: Stack has been initialized.
// Post: If stack is empty, EmptyStack exception is thrown;
//       else top element has been removed.
{
    if (IsEmpty())
        throw EmptyStack();
    else
    {
        NodeType* tempPtr;
        tempPtr = topPtr;
        topPtr = topPtr->next;
        delete tempPtr;
    }
}
```

# Other Stack Functions

- **Top:** Returns `topPtr->info`
- **IsEmpty:** Returns `topPtr == NULL`
- **IsFull:** Attempts to allocate a new node, using a try-catch block to handle the `bad_alloc` exception; if an exception is thrown, returns true
- **Constructor:** Initializes `topPtr` to `NULL`
- **Destructor:** Walks the stack and deallocates every node

# Big-O Comparison of Stack Implementations

- So which is better? It depends on the situation. The linked implementation certainly gives more flexibility, and in applications where the number of stack items can vary greatly, it wastes less space when the stack is small.
- When the stack size is unpredictable, the linked implementation is preferable, because size is largely irrelevant.
- The array implementation executes faster because it does not incur the run-time overhead of the new and delete operations. If the size is small and it can be sure that it does not need to exceed the declared stack size, the array-based is a good choice.

	Static Array	Linked
Class constructor	$O(1)$	$O(1)$
IsFull	$O(1)$	$O(1)$
IsEmpty	$O(1)$	$O(1)$
Push	$O(1)$	$O(1)$
Pop	$O(1)$	$O(1)$
destructor	NA	$O(N)$

# Queues



# Queues: Logical Level

- **Queue:** An ADT in which elements are added to the rear and removed from the front; this behavior is called **FIFO** (**F**irst **I**n, **F**irst **O**ut)
- Items are homogeneous, like in stacks and lists
- Example: A line of people at a cash register

# Queue Operations

- **Enqueue:** Add an item to the end of the queue
- **Dequeue:** Removes the item at the front of the queue and returns it
- **IsEmpty:** Returns true if the queue is empty
- **IsFull:** Returns true if the queue is full
- **MakeEmpty:** Removes all items from the queue

# Queues: Application Level

Like stacks, queues are used in various ways by the OS and other systems:

- Scheduling jobs on the processor
- Buffering data between processes or other systems

# Queues: Implementation Level

- Several implementations are possible
- As before, we'll start with an array-based implementation

# Fixed-Front Queue

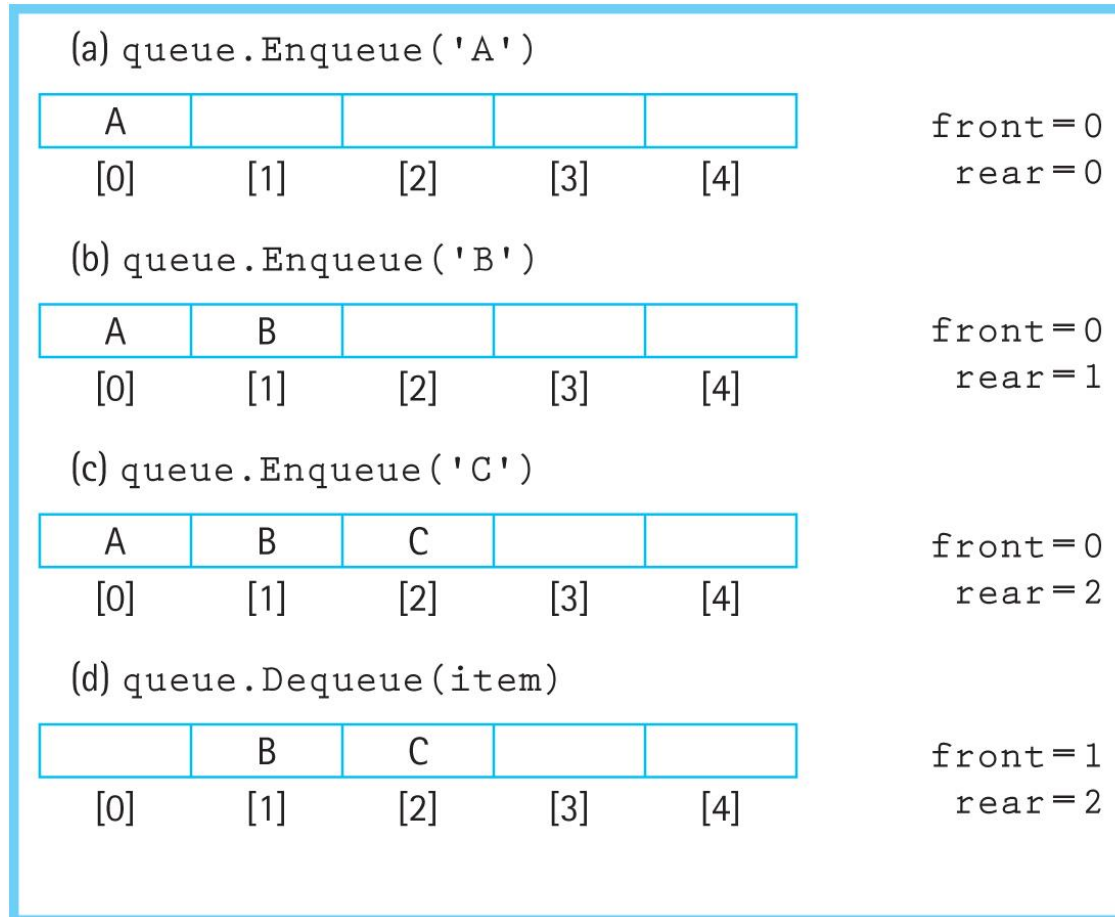
- Array-based implementation where index 0 is always the front of the queue
- Enqueue fills in the first empty slot, second and so on ...
- Dequeue empties the first slot and moves all subsequent elements up
- Copying elements like this is inefficient

# Floating Queue

- We keep track of the index of the front as well as the rear, we can let both ends of the queue float in the array.
- The Enqueue operation has the same effect as in fix queue operation; they add elements to subsequent slots in the array and increment the index of the rear indicator.
- The Dequeue operation is simpler, however. Instead of moving elements up to the beginning of the array, it merely increments the front indicator to the next slot.

# Floating Queue

Both the front and end of the queue float in the array



# Floating Queue

What happens when we reach the end of the array but the queue isn't full?

We treat the array as a circular structure by using **rear = (rear + 1) % maxQue**:

```
queue.Enqueue('L')
```

(a) Rear is at the bottom of the array



front = 3  
rear = 4

(b) Using the array as a circular structure, we can wrap the queue around to the top of the array



front = 3  
rear = 0

Wrapping the queue elements around



# Floating Queue: IsFull and IsEmpty

(a) Initial conditions

		A		
[0]	[1]	[2]	[3]	[4]

front=1  
rear=2

(b) `queue.Dequeue(item)`

[0]	[1]	[2]	[3]	[4]

front=2  
rear=2

C	D	reserved	A	B
[0]	[1]	[2]	[3]	[4]

front=2  
rear=1

How do we know if the queue is empty or full?

- One approach: Keep track of the number of elements in the queue
- Another approach: Make front point to the space *before* the actual first element
  - If `front == rear`, the queue is empty
  - The space indicated by front is reserved

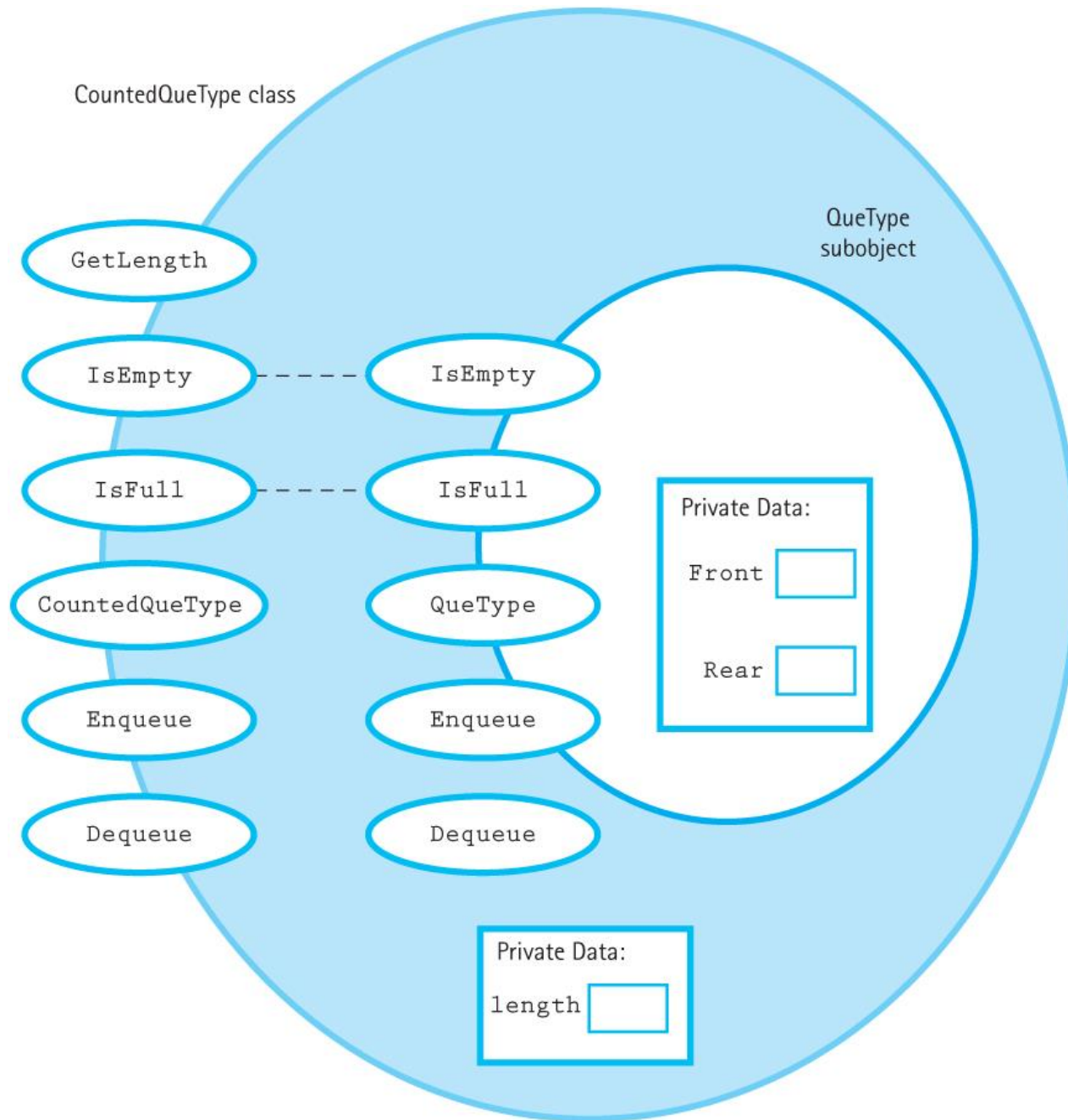
# Comparing Array Implementations

- The fixed-front queue has a less efficient Dequeue implementation
- The floating circular queue is more complex but has a more efficient Dequeue
- All other operations are  $O(1)$

# Counted Queue

- We may want a count of items in the queue
- Instead of making an entirely new class, we'll instead derive CountedQueueType from QueueType
- Logically, it needs a field for the count, a method to return the count, a new constructor that initializes the count to 0, and slightly modified Enqueue and Dequeue that change the count
- **Deriving the class** lets us avoid most of the work

# Counted Queue Class Diagram



Class interface diagram for  
`CountedQueueType` class

# Queue Inheritance

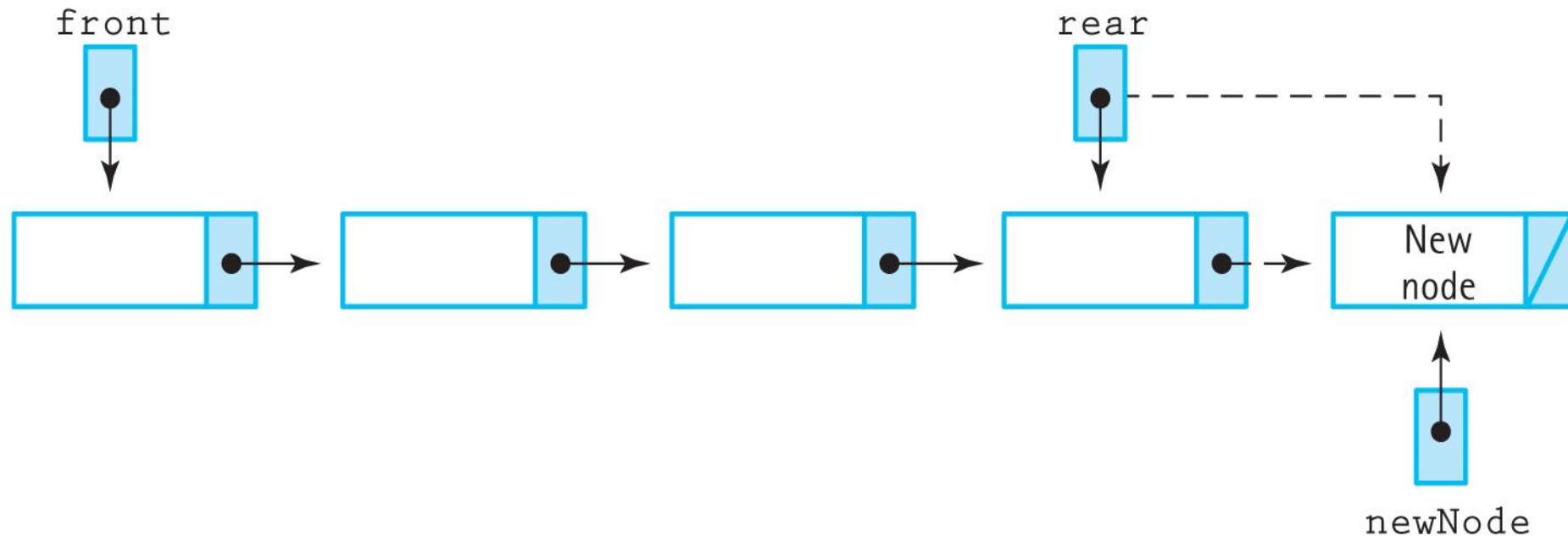
- CountedQueueType is the **derived class** or subclass
  - QueueType is the **base class** or superclass
  - CountedQueueType can't access QueueType's private members, but can call the public methods
- 
- CountedQueueType's Enqueue and Dequeue call QueueType's methods and then modify length
  - CountedQueueType uses QueueType's IsFull and IsEmpty method instead of writing new ones

# Linked-List Based Queues

- Conceptually similar to the circular queue
- Keep two pointers, tracking the front and rear of the queue

# Linked-List Queue: Enqueue Operation

- Algorithm:
  - Allocated new node
  - Update rear->next to point to new node
  - Update rear to point to new node
  - If queue was empty, also update front



# Alternate Enqueue – a bad design

- Could try to implement Enqueue the same as the stack's Push by reversing the rear and front pointers
- But this makes Dequeue impossible to implement because we can't go back up the queue

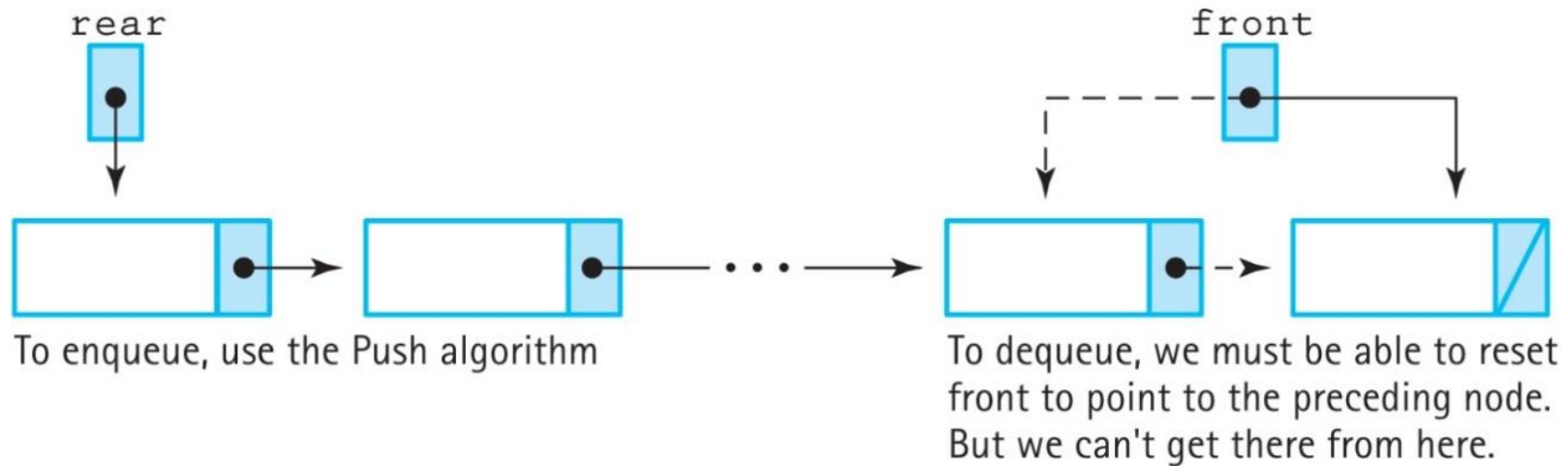
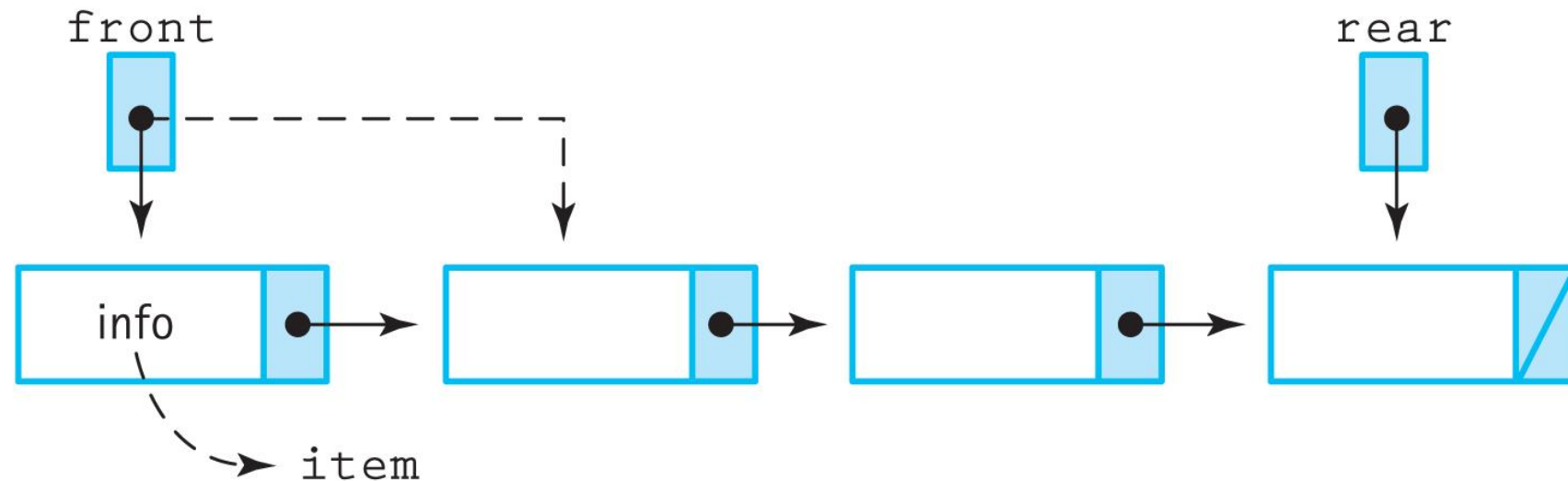


Figure 5.18 A bad queue design



# Linked-List Queue: Dequeue Operation

- Similar to the linked list stack's Pop:
  - Use a temp pointer to track the front item
  - Advance the front pointer to front->next
  - If front is now NULL, set rear to NULL (queue is empty)



# Other Queue Operations

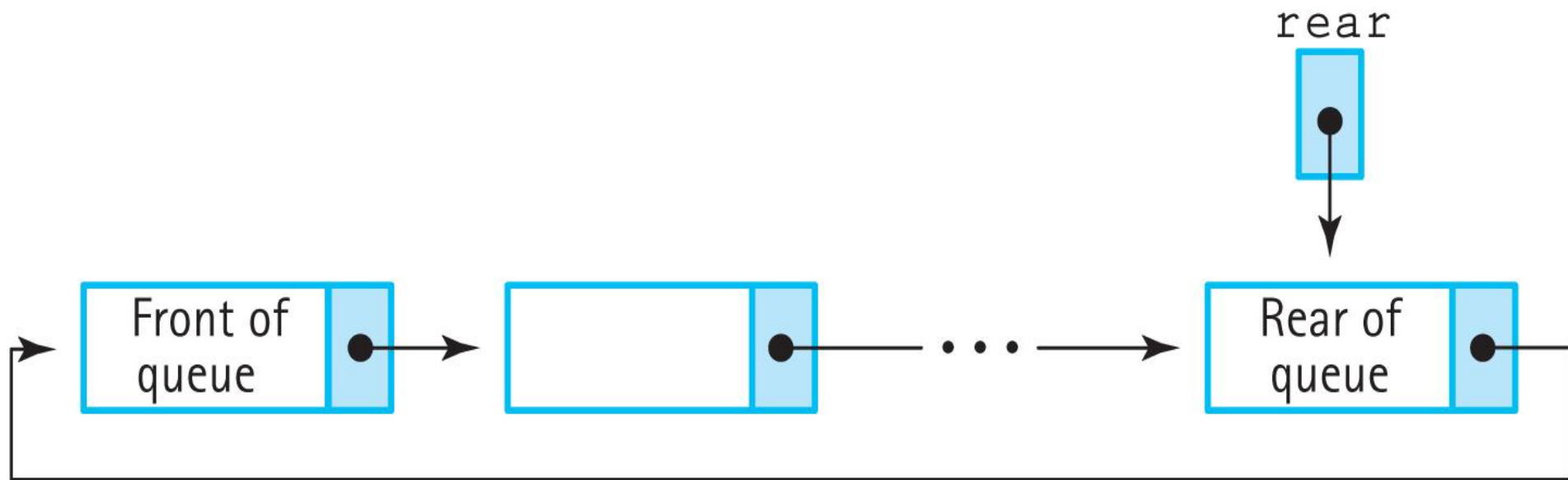
- **IsFull:** Attempts to allocate a new node
- **IsEmpty:** Checks if front is NULL
- **MakeEmpty:** Walks the linked structure and deallocates the nodes
- **Destructor:** Calls MakeEmpty

# Circular Linked Queue

- Is it possible to have only one pointer in the QueType class?
  - Only front: Can access the rear by walking the links, which is  $O(N)$
  - Only rear: Can't access the front because the links only point towards the rear
- Yes, by implementing a circular linked list

# Circular Linked Queue

- The class only has a pointer to the rear of the queue
- Instead of pointing to NULL, the last node's next points to the front of the queue



# Comparing Queue Implementations

- All operations are  $O(1)$ , except the linked list-based MakeEmpty and destructor are  $O(N)$
- The linked list-based queue has memory overhead

	Dynamic Array Implementation	Linked Implementation
Class constructor	$O(1)$	$O(1)$
MakeEmpty	$O(1)$	$O(N)$
IsFull	$O(1)$	$O(1)$
IsEmpty	$O(1)$	$O(1)$
Enqueue	$O(1)$	$O(1)$
Dequeue	$O(1)$	$O(1)$
Destructor	$O(1)$	$O(N)$

# Comparing Queue Implementations

- The array-based queue requires a fixed amount of memory no matter how many items are actually in the queue.
- The linked list-based queue consumes more memory as more elements are added.
- For cases with few items or large items, linked list-based queues can be more efficient.

The End!