CONTROL STRATEGIES FOR MITIGATING TRAFFIC SHOCK WAVES UTILIZING CONNECTED AND AUTONOMOUS VEHICLES

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Abstract

In this research paper, we aim to develop and test various control algorithms to mitigate shock waves in traffic streams consisting of connected and autonomous vehicles (CAVs). We first explain the formation of shock-waves with the help of mathematical modeling and numerical simulation. Further, we formulate an optimal control problem to mitigate shock waves on a circular track consisting of CAVs. We solve the optimal control problem using Pontryagin's minimization principle. We use entropy to measure the effectiveness of CAVs to reduce shock waves (stop-and-go) waves on a circular track, thereby increasing throughput and reducing emissions. An optimal control law is also developed to minimize the error of the headway between vehicles in a platoon.

Acknowledgments

Acknowledge people here

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Part I

Motivation

1 Why Traffic Congestion Matters

Transportation has played an integral role in human society from the very beginning, allowing humans to colonize different geographical regions, trade amongst different civilizations, and to spread ideas. Initially, humans walked and carried items on their backs and heads. Beasts of burden such as mules or oxen were then used which allowed for more goods to be transported at a higher speed. Then, roads were built and people and goods were carried by wheeled carts. As transportation systems became more complex and advanced, people and goods were able to be transported further and at greater speeds.

Transportation has played an integral role in human society from the very beginning, allowing humans to colonize different geographical regions, trade amongst different civilizations, and to spread ideas. Transport by land started by foot with people walking from place to place carrying goods on their backs and heads. By the early twentieth century, automobiles began to be mass produced in America. Initially they were seen as toys for the rich.

Traffic congestion, air and noise pollution, and safety are common problems that many cities face today. It is estimated that 50% of the total population is living in cities, and by 2050, this percentage is projected to grow over 70%, i.e. over six billion people will live in cities and urban areas. In 2014, U.S. drivers drove over 2 trillion vehicle miles, with congestion costing over 5 billion dollars on average for very large urban areas and over 191 million dollars on average in small urban areas. In 2014, there was a reported 5.9 million accidents by passenger vehicles and the costs and accident rates are increasing on average across the nation [21].

The transportation engineering and planning communities are witnessing the emergence of a new generation of traffic systems, also known as connected and autonomous vehicles (CAVs). Recent advancements in CAVs are expected to transform how people use transportation. There is a growing interest in the research community to exploit the available vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication technologies in CAVs, to improve the efficiency and safety of traffic flow while reducing emissions. It is estimated that integration of CAVs will reduce about 40% of fatal traffic accidents [15]. Aside from passenger and road safety, researchers also envision that CAV technology will reduce congestion and fuel consumption [7].

The ability to communicate between vehicles improves the perception of the driving environment and the quality of driving related decisions. V2V communications provide detailed information about vehicle movements and operation decisions (e.g. speed and acceleration), while V2I communications provide detailed information on road and weather conditions. The CAV is also able to obtain information from adjacent vehicles and predict the traffic stream parameters ahead of them (e.g., shockwave formations). The availability of V2I communications provides information on breakdown formations (e.g. lane closure or crash), changes in the speed limit, work zone conditions, and additional information [29]. User experiences are important for smoother transition to next generation technologies. Bansal and Kockelman performed a national survey to understand the future vehicle preferences related to CAV technology [3]. As noted by the respondents, traffic sign recognition and left turn assist was of no interest, as about 46% of users were not willing to pay for the technologies to be added to their vehicles. However, respondents were very interested in a blind-spot monitoring system. As a result, many manufacturers implemented this technology in their vehicles. About 54.4% of the respondents perceived CAVs as a useful advancement in the near future. Furthermore, 50.4% of the respondents were comfortable with their vehicle transmitting information to other vehicles and 62.3% were willing to trust technology companies. Most users show signs of accepting the changes, and as CAV technology continues to advance, it will provide a positive impact on the future of mobility.

Part II

Background

2 Trends in Automation

Like many other industries, the automotive industry is benefiting from the increasing capabilities and especially shrinking footprint of computers. Processing power is getting cheaper and because of this very powerful computers are able to fit into smaller volumes including the cramped spaces inside of motor vehicles. Not only are computers more powerful, smaller, and cheaper than ever before so are sensors. Lidar, radar, and cameras are (relatively) cheap, powerful, and have high resolution. Powerful computers and sensors in novel objects is fertile ground for innovation. Large automakers and smaller startups (many funded by large companies and/or investors) are in a race to create the first autonomous vehicle.

2.1 Current Capabilities

SAE International breaks down autonomous vehicles into six automation levels. These levels are given in Table 1 and are sourced from [2]. The levels of automation range from the human driver being completely in control 100% of the time (Level 0) to an Advanced Driver System (ADS) is fully in control with no human intervention necessary (Level 5). A great example of advancing automation in the automotive industry is Tesla.

Currently, Tesla's Autopilot (which is one of the most advanced ADS on the market) is at Level 2 which means that Tesla vehicles are capable of steering within a lane, maintaining a speed relative to other vehicles, assisting lane changes, parking, and summoning to and from garage. Tesla will continue to update the Autopilot on its vehicles (which Tesla will the Enhanced Autopilot), steadily adding new features such as automatic lane keeping, speed matching, lane changing, freeway exiting, parking (when near a parking space). Like many other car-makers, Tesla plans to eventually have its cars be self-driving [35, 34].

2.2 Research and Testing

There are many companies that are competing in the race to the elusive self-driving car. Some of the biggest names are Uber, Waymo (formerly known as the Google self-driving car project), NVIDIA Corporation, and major car manufacturers such as Honda, GM Cruise LLC, and Tesla [20]. Because of the secretive nature of industrial research it's difficult to determine just how far along each company is towards achieving full autonomy for self-driving cars. However, one way to determine how far along companies are towards realizing a fully autonomous (Level 5) car is to examine documents that they are required to release. One such document is the Autonomous Vehicle Disengagement Report that the California Department of Motor Vehicles requires as a part of the

Automation Level	Who does what, when
0	The human driver does all the driving.
1	An advanced driver assistance system (ADAS) on the vehicle can sometimes assist the human driver with either steering or braking/accelerating, but not both simultaneously.
2	An ADAS on the vehicle can itself actually control both steering and braking/accelerating simultaneously under some circumstances. The human driver must continue to pay full attention (monitor the driving environment) at all times and perform the rest of the driving task.
3	An Automated Driving System (ADS) on the vehicle can itself perform all aspects of the driving task under some circumstances. In those circumstances, the human driver must be ready to take back control at any time when the ADS requests the human driver to do so. In all other circumstances, the human driver performs the driving task.
4	An ADS on the vehicle can itself perform all driving tasks and monitor the driving environment essentially, do all the driving in certain circumstances. The human need not pay attention in those circumstances.
5	An Automated Driving System (ADS) on the vehicle can do all the driving in all circumstances. The human occupants are just passengers and need never be involved in driving.

Table 1: SAE International automation levels

licensing requirements for performing field tests on California roads under the Autonomous Vehicle Testing Program.

Autonomous Vehicle Disengagement Reports contain the number of miles driven autonomously on public roads in California per month and disengagement events and related information. During field tests of autonomous vehicles, a driver (test operator) must be in the driver's seat of the vehicle [17]. A disengagement event is

> "a deactivation of the autonomous mode when a failure of the autonomous technology is detected or when the safe operation of the vehicle requires that the autonomous vehicle test driver disengage

> the autonomous mode and take immediate manual control of the vehicle, or in the case of driverless vehicles, when the safety of the vehicle, the occupants of the vehicle, or the public requires that the autonomous technology be deactivated." [19]

The information related to this event is what initiated the event (driver or autonomous system), the location of the event, and a description of the event [18]. A very simple (although perhaps a bit naive) method is to examine all of the Autonomous Vehicle Disengagement Reports for each company for every year. By doing so, a picture of each company's progress begins to emerge.

Here, put a plot of miles driven per year for the top 5 (?) testers and a plot for miles per disengagement as a way to judge progress

It's interesting to note that Tesla is not included in 2017. In that year's report Tesla states that it has not had any disengagement events. This is not because Tesla is not performing any autonomous tests, it's just not performing any autonomous tests *as defined by California law.* Tesla is performing autonomous tests via "simulation, in laboratories, on test tracks, and on public roads in various locations around the world" [32]. In addition, it's testing its control algorithms against the human drivers of its vehicles out in field and thus considering its hundres of thousands of customers' vehicles part of its test fleet. By doing this Tesla has been able to acquire billions of test miles in a very short time. This is accomplished by the vehicles running in "shadow mode" which allows Tesla to collect telemetry data such as braking and acceleration and Autopilot data and then uses this to develop and test its control algorithms safely as none of the algorithms actuate the vehicle [33, 32].

2.3 Integration of AI

According to Chen et al, in much of industry, mediated perception techniques for vision-based autonomous driving systems are used [6]. Mediated perception techniques use subcomponents to recognize traffic data such as other vehicles, lanes, traffic signs and signals, and pedestrians. This data is then combined to create a representation of the traffic scenario such as splines for lane detections and bounding boxes for vehicle detections. An AI-based control system can then take in this representation, determine appropriate trajectories, and actuate the vehicle. Chen et al. describe a few problems with this technique, however.

First, the representation that is created can be too "rich" meaning that not all of the information in the representation is necessary for the AI-based controller. Second, AI-controllers don't directly use this representation (i.e., it has to be transformed to some other representation that the controller *can* use) [6]. These transformations from splines and bounding boxes into useful information can create noise and errors that add more uncertainty to the mix. Third, structured details from the traffic scene such as straight edges in buildings can cause false positives for other vehicles.

Another technique is behavior reflex approach [6]. In this technique, sensory input (e.g., radar, lidar, and images) is mapped to driving actions (e.g., steering angles and accelerations or braking) using neural networks. Although straightforward, this technique has flaws. First, driving conditions are never the same. Even if the same cars are in the same location at the same time of day, people have different moods and intentions and may exhibit different a behavior. Second, the neural network must process an entire image and determine the relevant parts which is difficult for the network.

2.4 Obstacles and Limitations

A hurdle for obtaining level 5 autonomy is that of data. Training neural networks takes a lot of data, which is why we see manufacturers performing extensive field tests. Also, it is necessary to prove that these vehicles are safe and currently the way to prove this is through real-world tests. The RAND Corporation estimates that in order for autonomous vehicles to match the safety rate of human drivers (about 1 fatality in 100 million miles driven) autonomous vehicles need to have in the range of billions to hundreds of billions of test miles driven, which could take hundreds (or even thousands) of years depending on the size of the test fleet and how much better it is desired for the autonomous vehicles to be than human drivers [13]. This is why Tesla is running tests using its customers' vehicles as its own test fleet. The more vehicles it has, the faster Tesla can improve and prove its technology.

Another limitation of the current implementation of autonomy in industry is the lack of coordination of autonomous vehicles and sharing of information. As far publications go, no car companies are talking about if and how their vehicles will share information with infrastructure (V2I) and each other (V2V). Some companies, such as Tesla and Waymo, are discussing about eventually having an autonomous ride-hailing service [4, 31]. However, the lack of sharing information about the vehicles and coordination with other vehicles on the road (including private vehicles) can prove to be problematic for the larger picture. Connected Autonomous Vehicles (CAVs) can increase the throughput and efficiency of roads and highways, but a necessity is the sharing of information between vehicles on the road due to faster response times and the shorter distances (headways) needed between vehicles [24]. If this information is not shared then it will be difficult to glean these efficiencies and much of the improvements of autonomous vehicles will not be realized.

3 Overview of Traffic Flow Theory

Traffic may be modeled using different levels of models such as macroscopic, microscopic, mesoscopic, and aggregation and disaggregation levels [36]. Macroscopic models describe the aggregate behavior of traffic, similar to how hydrodynamic models describe the aggregate behavior of particles in liquids and gases. Microscopic models on the other hand describe the interactions and reactions of single vehicles with each other similar to multi-particle systems. Mesoscopic models are hybrids of macroscopic and microscopic models. Finally, aggregation and disaggregation is the use of microscopic quantities to describe macroscopic ones (aggregation) and using macroscopic quantities to describe microscopic ones (disaggregation).

The macroscopic quantities that are of interest are the speed (V), density (ρ), and flow (Q) of the vehicles. There are several different definitions of speed, but essentially speed is the distance traveled per unit time. Density is defined as the number of vehicles per unit length of road. Finally, flow is defined as the number of vehicles passing through a certain point in space per unit time. Because these macroscopic quantities. These quantities are related to each other through two charts, combined called the fundamental diagram.

Insert an example of the fundamental diagram. Explain them

3.1 Shock Waves

A more mathematical treatment of traffic shock waves will be given later on, however, traffic shock waves can be understood through the straightforward idea of positive feedback [9]. In general, drivers try to keep a safe distance between themselves and the vehicle ahead of them. If they drive at velocity v and have a response T, then the minimum safe separation distance, d_{min}, is defined by

$$d_{\min} = Tv \tag{1}$$

If all drivers are driving at a safe distance from one another, then the density, ρ, is defined by

$$\rho = \frac{1}{\ell + Tv} \tag{2}$$

and the throughput, r, is given by

$$r = \frac{v}{\ell + Tv} \tag{3}$$

Instabilities, in the form of waves in density and speed, grow in amplitude and may grow until the speed of the vehicles approach zero [9]. This phenomena can understood through positive feedback. Lower vehicle speeds cause higher traffic densities, which in turn cause lower speeds. On the other hand, higher speeds cause lower traffic densities, which cause higher speeds and so on (of course taking into account speed limits and safety considerations). Therefore, we can see that the system is amplifying the characteristics of traffic (vehicle speed and traffic density) rather than dampening them. This is characteristic of a system with positive feedback. In addition to the system having positive feedback, after a certain density, drivers braking can cause the vehicles behind them to brake, propagating the wave backwards through the roadway.

Horn states that the root of the problem stems from the drivers' feedback and the flow of information. Drivers tend to perform actions that benefit themselves, but tend to be detrimental to other drivers. The flow of information on the road from vehicle to vehicle is unidirectional from the front vehicle to the back. Each vehicle adjusts its acceleration based on the relative position and velocity of the vehicle immediately ahead of it. The deceleration coupled with the backwards, unidirectional flow of information creates an effect that is similar to a shock wave is which also moves backwards through traffic [9].

3.2 Vehicle Platoons

A vehicle platoon consists of a string of vehicles that travel along the road, acting as one single unit, with each car following one another closely at normal highway speeds, as shown in Fig. 1 [5, 14]. The 1^{st} car is the leader and the subsequent cars are followers. Vehicle platooning is a promising strategy that can lead to significant fuel savings and emission reduction [11].

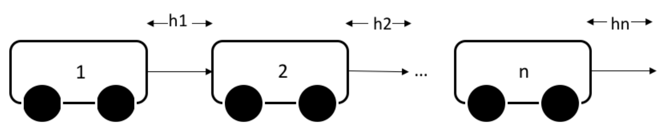

Figure 1: An n-vehicle platoon formation

Platoons are an ideal remedy to driving situations for peak traffic conditions such as at intersections and highways. Traffic congestion is caused by a cascade of driver decelerations down a roadway which, in turn, causes an increase in density of vehicles causing more driver decelerations, and so on. These cascaded decelerations cause a shock wave to move backwards through traffic that can be sustained even after the cause of the shock wave has disappeared. By sharing information throughout and coordinating vehicles within a platoon, these shock waves can be reduced. This increases highway capacity by a factor of 2 or 3, relieves congestion, reduces travel times, and decreases fuel consumption [5].

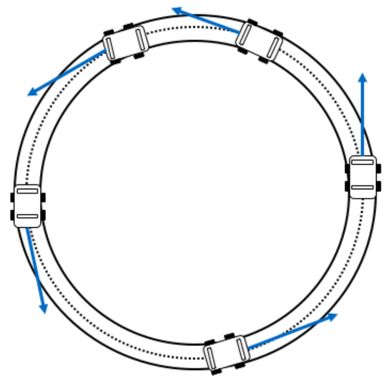

Figure 2: Slot car race track with five vehicles. The arrows indicate the instantaneous velocity vector.

3.3 CAVs

4 Mathematical Background

4.1 Modeling Traffic

Here, traffic will be modeled using a macroscopic model using a hydrodynamic flow-density relation. Before the model is developed it is important define the parameters that will be used, which are given in Table 2 below. It is also important to note that each of the parameters in Table 2 have different forms (such as average, effective, and total).

Parameter	Description
$\rho(x,t)$	Traffic density
$Q(x,t)$	Traffic flow
$V(x,t)$	Lane-averaged, or effective speed
I	Number of lanes
n	Number of vehicles

Table 2: Model parameters for macroscopic model of traffic flow

4.2 Macroscopic Models

Traffic Flow Characteristics

Density

Density, ρ, is defined as the number of vehicles per length of road and can either be for

- a single lane: $\rho_i(x,t)$ where $i = 1, \ldots, n$ and n is the number of lanes
- total density over all lanes of a length of road: $\rho_{tot}(x,t) = \sum_{i=1}^{I} \rho_i(x,t)$
- lane-averaged density: $\rho(x,t) = {}^{\rho_{tot}(x,t)}/_{I}$

Velocity

Velocity, V, is defined in terms as a lane-averaged (or, effective) speed and is given by

$$V(x,t) = \sum_{i=1}^{I} \frac{\rho_i(x,t)}{\rho_{tot}(x,t)} V_i(x,t) \tag{4}$$

where $V_i(x,t)$ is the local speed of lane i.

Hydrodynamic Flow Relation

Flow, Q, is defined as the number of vehicles per unit time and is the product between density and velocity, as shown below

$$Q(x,y) = \rho(x,t) V(x,t) \tag{5}$$

Flow can be given in terms of a single lane, all lanes, or as a lane-average (Equation 5). Many times the flow in terms of all lanes will be used as the conservation of vehicles holds only for the total density, ρ_{tot}, and not for each lane individually (vehicles can change lanes afterall). It should be noted that this relation holds for all types of macroscopic models, but not for all types of roads. In other words, the geometry of the road matters but not the model. For example, on- and off-ramps and a change in the number of lanes will change Equation 5.

Continuity Equation

In physics, the continuity equation is a consequence of the conservation of matter that relates the density and velocity of a fluid [30]. Its analog in electromagnetism is the conservation of electric charge [30]. In traffic flow, there is an analog to the conservation of matter: the *conservation of vehicles.* On a certain length of road, the number of vehicles is conserved and given by

$$n\left(t\right)=\int_{x}^{x+\Delta x}\rho_{tot}\left(x',t\right)dx'\approx\rho_{tot}\left(x,t\right)\Delta x \tag{6}$$

For a road section with no on- or off-ramps and no change in number of lanes (i.e., a homogeneous road section) of length Δx, the continuity equation can be derived as follows (as given in [36])

$$\begin{aligned}\frac{dn}{dt}&=Q_{in}\left(x,t\right)-Q_{out}\left(x,t\right)\\&=Q_{tot}\left(x,t\right)-Q_{tot}\left(x+\Delta x,t\right)\\&=\frac{\partial}{\partial t}\left(\rho_{tot}\,\Delta x\right)\end{aligned} \tag{7}$$

Rewriting Equation 7 gives

$$\begin{aligned}\frac{\partial\rho_{tot}}{\partial t}&=\frac{1}{\Delta x}\frac{dn}{dt}\\&=-\left(\frac{Q_{tot}\left(x+\Delta x,t\right)-Q_{tot}\left(x,t\right)}{\Delta x}\right)\end{aligned} \tag{8}$$

and

$$
\begin{aligned}
\lim_{\Delta x \to 0} \frac{\partial \rho_{tot}}{\partial t} &= -\frac{\partial Q_{tot}(x,t)}{\partial x} \\
\frac{\partial \rho_{tot}}{\partial t} &\approx -\frac{\partial Q_{tot}(x,t)}{\partial x}
\end{aligned}
\tag{9}
$$

Finally, rewriting Equation 9 we get

$$
\frac{\partial \rho_{tot}(x,t)}{\partial t} + \frac{\partial Q_{tot}(x,t)}{\partial x} = 0 \tag{10}
$$

We can use Equation 5 (the hydrodynamic flow relation) to rewrite Equation 10 (and using the fact that $\frac{\partial Q_{tot}}{\partial x} = \frac{\partial \rho_{tot} V}{\partial x}$) and arrive at the continuity equation

$$
\boxed{\frac{\partial \rho_{tot}}{\partial t} + \frac{\partial \rho_{tot} V}{\partial x} = 0 \qquad \text{or} \qquad \frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial x} = 0} \tag{11}
$$

4.3 Lighthill-Whitham-Richards Models

Lighthill-Whitham-Richards (LWR) models are first-order models, i.e., they don't include acceleration, and they only are the same except for the form of their respective fundamental diagrams and mathematical representation which is determined by the modeling of the flow and speed only [36]. LWR models give flow and velocity in a functional form

$$
\begin{aligned}
Q(x,t) =& Q_e(\rho(x,t)) \\
V(x,t) =& V_e(\rho(x,t))
\end{aligned}
\tag{12}
$$

The equations in 12 assume that local flow (Q_i) or speed (V_i) are always in equilibrium with respect to the actual density, ρ. It also assumes that the flow and local velocity changes instantaneously to follow the density.

Using the continuity equation, Equation 10, the LWR model can be defined as

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{dQ_e(\rho)}{d\rho}\frac{\partial \rho}{\partial x} = 0} \tag{13}$$

if we recognize that $\frac{\partial Q_e(\rho)}{\partial x} = \frac{dQ_e(\rho)}{d\rho}\frac{\partial \rho}{\partial x}$ due to the chain rule. Equation 13 can also be written as

$$\boxed{\frac{\partial \rho}{\partial t} + \left(V_e + \rho\frac{dV_e}{d\rho}\right)\frac{\partial \rho}{\partial x} = 0} \tag{14}$$

which is the LWR model.

Fundamental Diagram

The fundamental diagram is determined by fitting empirical data, but its basic shape is determined by the definitions given earlier.

Show fundamental diagram...

4.4 Microscopic Models

Microscopic models take into account interactions between vehicles and the reactions of drivers. Many first-order models have the following (typical kinematic model) variables for each i^{th} vehicle

- l_i: length of the vehicle
- x_i: position
- v_i: velocity
- a_i: acceleration

  Some models also include the headway (bumper-to-bumper distance of the i^{th} and $i+1^{th}$vehicles), h_i, as well.

Traffic Flow Characteristics

Mesoscopic models can be used to relate microscopic parameters to macroscopic ones such as lane-average speed, density, and flow.

4.5 Theory of Traffic Shock Waves

The formation of traffic shock waves can be explained using the LWR model, although with some limitation. Let's define the initial density of the traffic flow to be $\rho(x,0) = \rho_0(x)$ where $\rho(x,t) = \rho_0(x - \widetilde{c}t)$ and $\widetilde{c}$ is the propagation velocity of the density waves. We can see that $\frac{\partial\rho}{\partial t} = -\widetilde{c}\rho_0'(x - \widetilde{c}t)$ and $\frac{\partial\rho}{\partial x} = \rho_0'(x - \widetilde{c}t)$ which, when combined with Equation 13 (the LWR model) leads to

$$-\widetilde{c}\,\rho_0'(x - \widetilde{c}t) + \frac{dQ_e}{d\rho}\rho_0'(x - \widetilde{c}t) = 0$$

or

$$\boxed{\widetilde{c}(\rho) = \frac{dQ_e}{d\rho} = \frac{d(\rho V_e(\rho))}{d\rho}} \tag{15}$$

This shows that density waves propagate at a velocity dependent on the change in the flow of traffic. This important result will be used to help explain the formation of traffic shock waves later. If we examine the density wave propagation from the point of view of driver we get

$$\begin{aligned}\widetilde{c}_{\mathrm{rel}}(\rho) &= \widetilde{c}(\rho) - V \\ &= \widetilde{c}(\rho) - V_e(\rho) \\ &= \rho V_e'(\rho)\end{aligned} \tag{16}$$

$V_e'(\rho) < 0$, thus, from the driver's perspective when $\widetilde{c}_{\mathrm{rel}}(\rho) \leq 0$ the density variations propagate backwards.

Insert image for use in derivation below

For a single-lane highway, let $n = \rho_1 x_{12} + \rho_2 (L - x_{12})$ be the number of vehicles passing through a sufficiently small section of road. Then,

$$\begin{aligned}
\frac{dn}{dt} &= \frac{d(\rho_1 x_{12})}{dt} + \frac{d(\rho_2 (L - x_{12}))}{dt} \\
&= \rho_1 \frac{dx_{12}}{dt} - \rho_2 \frac{dx_{12}}{dt} \\
&= (\rho_1 - \rho_2) c_{12} \\
&= Q_1 - Q_2 \\
&= Q_e(\rho_1) - Q_e(\rho_2) \\
\therefore \quad (\rho_1 - \rho_2) c_{12} &= Q_e(\rho_1) - Q_e(\rho_2)
\end{aligned}$$

where $c_{12} = \frac{dx_{12}}{dt}$. Finally, we arrive at

$$\boxed{c_{12} = \frac{Q_e(\rho_1) - Q_e(\rho_2)}{\rho_1 - \rho_2}} \tag{17}$$

A great property of the LWR model is that velocities can be determined directly from the fundamental diagram.

- $\widetilde{c}(\rho) = Q_e'(\rho)$: propagation velocity density variations (slope of the fundamental diagram)
- c_{12}: propagation velocity of shock wave fronts (slope of secant line of connecting points of the fundamental diagram)
- $V_e(\rho) = \frac{Q_e(\rho)}{\rho}$: vehicle speed (slope of secant line from origin to corresponding point on fundamental diagram)

Show the fundamental diagram here and different parts of it. Explain where free and congested traffic is and above points in list.

4.6 Driver Models

The following is a discussion on two different microscopic models.

Human Driver Model

To mimic the traffic shock waves to be controlled, a human driver model is used. The dynamics of the human driver is from [10, 23] and is given by

$$\begin{aligned} \dot{h}_i(t) &= v_{i+1}(t) - v_i(t) \\ \dot{v}_i(t) &= \alpha_i \left(V_i\left(h_i(t)\right) - v_i(t)\right) \\ &\quad + \beta_i \left(v_{i+1}(t) - v_i(t)\right) \end{aligned} \tag{18}$$

where $V_i(h)$ is the distance-dependent velocity, or range policy, given by

$$V_i(h) = \begin{cases} 0 & \text{if } 0 \leq h_i \leq h_{stop} \\ \frac{v_{max}}{2} A\left(h_i(t)\right) & \text{if } h_{stop} < h_i < h_{go} \\ v_{max} & \text{if } h_{go} \leq h_i \end{cases} \tag{19}$$

where $A\left(h_i(t)\right) = \left(1 - \cos\left(\pi \frac{h_i - h_{stop}}{h_{go} - h_{stop}}\right)\right)$.

In the $v_i(t)$ term of 18, the α_i term can be thought of how much the human driver prefers to match the speed given by 19 and the β_i term can be thought of how much the human driver prefers to match the vehicle immediately ahead of it.

Intelligent Driver Model

Treiber and Kesting describe the Intelligent Driver Model (IDM) as "probably the simplest complete and accident-free model producing acceleration profiles and a plausible behavior in essentially all single-lane traffic situations" [36]. The IDM defines a safe headway, s_0, and time gap, T, respective to the lead vehicle, has soft braking maneuvers, and smooth transitions between driving modes. The mathematical model for the IDM is

$$\begin{aligned}
\dot{x}_i &= v_i \\
\dot{v}_i &= a\left[1-\left(\frac{v}{v_0}\right)^{\delta}-\left(\frac{s^*\left(v,\Delta v\right)}{s}\right)^2\right] \\
s^*\left(v,\Delta v\right) &= s_0+max\left(0,vT+\frac{v\Delta v}{2\sqrt{ab}}\right)
\end{aligned} \tag{20}$$

Treiber and Kesting break s^* into two components: the equilibrium term s_0+vT and the dynamical term $\frac{v\Delta v}{2\sqrt{ab}}$. The dynamical term takes care of the situation when the vehicle is approaching the lead vehicle

Parameter	Description
v_0	Desired speed
T	Time gap
s_0	Desired headway
δ	Acceleration exponent
a	Acceleration
b	Comfortable deceleration

Table 3: Model parameters for the Intelligent Driver Model

Figs. 5 a) and b) show the headways and speeds of three vehicles during simulated traffic shock waves, respectively.

4.7 Cellular Automata

Cellular automata are simple objects that have cells, grid, and neighborhood [26]. Cellular automata "evolve" by a simple set of rules. Each generation of the cellular automaton updates its state by examining the previous state (generation) and using a simple rule, each cell in the automaton is updated. Each cell represents a position on the road and a 0 represents the lack of a vehicle and a 1 represents a vehicle. A simplistic model of traffic using a cellular automaton is Rule 184 [36]. Rules are numbered 0 to 255 and are transformed into binary then a simple truth table is used to define the evolution of the automaton. In the new generation, each cell examines its and its neighbors' previous states and, based on the rule's truth table, is either occupied (1) or empty (0). The truth table for Rule 184 is given in Table below.

Previous states	New state
000	0
001	0
010	0
011	1
100	1
101	1
110	0
111	1

Table 4: Truth table for Rule 184. The left column is the cell's previous state (center number) and its neighbors states.

Using the simple rule set above with a random initial state, a surprisingly complex simulation of traffic is created and shown in Figure 3. It is important to note that each row in the figure is one generation of the traffic simulation. Therefore, one row after another is another time step in the simulation.

Figure 3: Cellular automaton model of traffic simulation using Rule 184

4.8 Connected Cruise Control Model

Consider the system shown in Fig. 2. A chain of n vehicles, i.e., a platoon of vehicles, traveling on a circular track. The vehicles implement a Connected Cruise Control (CCC) algorithm given in [10]. To simplify the model, only drivers with identical (homogeneous) characteristics are considered. In the CCC algorithm, headway and velocity information is shared between all vehicles via a V2V and V2I communication scheme. The dynamics of the CCC vehicles are given by

$$\begin{aligned} \dot{h}_i &= v_{i+1}(t) - v_i(t) \\ \dot{v}_i &= u_i(t) \end{aligned} \tag{21}$$

where h_i is the headway (bumper-to-bumper distance) between the i^{th} and $i + 1^{th}$ vehicles, u_i is the control action [10].

Part III

Literature Review

Traffic research with vehicle platoons extends back to at least the 1950s with major development taking off in the 1990s a [1, 8]. Currently, there has been an interest in research regarding modeling lane changing, control of platoons at intersections, and string stability of platoons. Much work with vehicle platoons has been completed in recent years. There are two different types of methods of coordinating vehicles that have been proposed in current research: centralized and decentralized coordination. In decentralized approaches, information is shared from vehicle to vehicle and in centralized approaches information is coordinated through a global coordinator. For example, in highway merging problems decentralized approaches handle each vehicle as an autonomous entity that aims to maximize efficiency. On the other hand, in centralized approaches vehicles are controlled by a global agent [25].

The efficiency of traffic flow was quantified by Stern et. al. Experiments were conducted on a circular track as it represents an infinitely long straight track making it simple to identify stop and go waves (i.e., shock waves). In the experiments, consisting of more than twenty non-autonomous vehicles, uniform speed could not be maintained. With the introduction of an autonomous vehicle at the front of the formation the vehicles reached the uniform, desired speed. [27].

Kachroo et. al. used Mean Field Games and Radon measure with connected and normal vehicles to analyze hetergeneous traffic on a circular track. The connected vehicles were treated as discrete agents which could be used to control the overall traffic streams and entropy was used as a measure of traffic flow efficiency. Their analysis included the effect of various penetration levels of CAVs in traffic stream, their placement, and various sizes of circular track. It was shown that the use of connected vehicles improved the efficiency by lowering the entropy [12].

Part IV

Hardware

Field trials for testing of communication protocols and control algorithms require large-scale setup, posing safety issues for those driving on roads as well as pedestrians, and requires funding to conduct the experimentation. Alternate to actual field setups are simulation studies using virtual micro traffic simulators, or designing a macro simulator in coding environment [12]. Nevertheless, these simulations are associated with shortcomings in terms of actual modeling and conditions. There is a strong need to evaluate potential CAV applications via small-scale testbeds.

Conducting vehicle platooning experiments can be quite expensive and can often require a collaboration or sponsorship from industry. For example, Lidström et. al. collaborated with the Volvo Car Corporation during Grand Cooperative Driving Challenge (GCDC) which took place in the Netherlands in 2011. In their collaboration with the Volvo corporation, Lidström et. al. were lent a recent-generation Volvo S60 with a sensor package including radar, lidar, and a camera which are part of the production vehicle [16]. This setup is cost prohibitive for small-scall research. The GCDC also takes place on a public road which adds safety as a factor.

5 Raspberry Pi (Controller)

5.1 Why it was chosen

5.2 What I'm trying to do

5.3 What was accomplished

6 Electronics/Sensors

Pictures

Track

Part V

Software

7 Robot Operating System

Part VI

Control

8 Optimal Control versus Classical Control

Classical control theory concerns itself with the instantaneous control of a system (plant) within design parameters such as rise time, overshoot, etc. Frequency response methods such as Bode plots and root locus are the bread and butter of designing classical control systems [22]. Although these control systems do satisfy the design parameters they are by no means optimal. In fact, the controllers cannot "see" what's ahead of them or "anticipate" what's coming next as far as state and control action is concerned. This is where optimal controllers come into play.

Optimal controllers have a long and storied history involving some of the greatest minds of mathematics, physics, and engineering spanning over three centuries and ignoring arbitrary national borders [28]. Optimal control provide a guarantee of optimality through well-developed and rigorously proven theorems. Using a performance measure (or cost function) engineers are able to give preference to certain behaviors of the controller and to shape and mold it into giving many different trajectories and thus behaviors.

9 Problem Formulation

The goal is to remove the shock waves created by human drivers in traffic. In order to do so, we must find the optimal control actions that will move the vehicles along the optimal state trajectories so that there is the suppression of the shock waves.

9.1 Modeling

A CCC vehicle, a vehicle in which information is transmitted and shared between other vehicles and sources , is considered. Recall that the platoon of vehicles is on an circular track and each vehicle starts from rest and it is desired to have each vehicle attain its safe headway, h_{safe}, and the maximum possible speed (the speed limit), v_{max}, within time T. Therefore, the initial conditions for the system are given below

$$h_i(0) = h_0^i$$

$$v_i(0) = 0$$

$$h_i(T) = h_{safe}$$

$$v_i(T) = v_{max}$$

9.2 Pontryagin Method for Optimal Control

In this section, a controller shall be developed using the Pontryagin method. The cost function is given by

$$J = \frac{1}{2}\int_0^T \sum_{i=1}^{n} \left(e_i^2(t) + r_i u_i^2(t)\right)\, dt \tag{22}$$

where $e_i(t)$ is the error function and r_i is the weighting constant such that $r_i > 0$. The cost function is designed such that the error and control action are minimized resulting in lower error and smooth accelerations.

The constraints are given such that the control action must be between some predetermined values $U_{min} \leq u(t) \leq U_{max}$ such that U_{min} is a negative real number and U_{max} is a positive real number. There are also constraints on the states such that the vehicles cannot go backwards (negative velocity) and they cannot exceed the speed limit, $0 \leq v_i(t) \leq v_{max}$.

There are various error functions that can be used. Three in particular are of interest for platooning with the testbed due to the variety of behaviors that they encourage and are given in (23). The first error function, $e_i(t) = h_i(t) - h_{isafe}$, penalizes deviations from the desired safety distance and is the error function that will be examined in this paper. The second error function, $e_i(t) = h_{i+1}(t) - h_i(t)$, penalizes differences between headways of consecutive vehicles essentially forcing an even spacing between every car in the platoon. Finally, $e_i(t) = v_{i+1}(t) - v_i(t)$, penalizes deviations in speed between the i^{th} and $i+1^{th}$ vehicles.

$$\begin{aligned} e_i(t) =& h_i(t) - h_{isafe} \\ e_i(t) =& h_{i+1}(t) - h_i(t) \\ e_i(t) =& v_{i+1}(t) - v_i(t) \end{aligned} \tag{23}$$

Pontryagin's minimization principle is applied to arrive at an analytical solution of (22). Using the error function $e_i(t) = h_i(t) - h_{isafe}$, the Hamiltonian is defined as

$$\begin{aligned} \mathcal{H} =& \sum_{i=1}^{n-1} \Big\{ \Big[(h_i(t) - h_{isafe})^2 + r_i u_i^2(t) \Big] \\ &+ p_1^i(t)\left[v_{i+1}(t) - v_i(t)\right]^2 + p_2^i(t)\, u_i(t) \Big\} \\ &+ (h_n(t) - h_{nsafe})^2 + r_n u_n^2(t) \\ &+ p_1^n(t)\left[v_1(t) - v_n(t)\right]^2 + p_2^n(t)\, u_n(t) \end{aligned} \tag{24}$$

To find the costate equations, first we differentiate (24) part with respect to h_i and v_i, take the negative. This gives us

$$\begin{aligned}\frac{\partial \mathcal{H}_i}{\partial h_i} =& \dot{p}_1^{i*}(t) = -\left(h_i(t) - h_{safe}^i\right) \\ \frac{\partial \mathcal{H}_i}{\partial v_i} =& \dot{p}_2^{i*}(t) = pi_1^*(t)\end{aligned} \tag{25}$$

We now solve the resulting differential equations

$$\begin{aligned}p_1^{i*}(t) =& -\left(h_i(t) - h_{safe}^i\right)t + a \\ p_2^{i*}(t) =& -\frac{1}{2}\left(h_i(t) - h_{safe}^i\right)t^2 + at + b\end{aligned} \tag{26}$$

where a and b are constants of integration. Differentiating (24) partially with respect to the control action variable $u_i(t)$ yields

$$\frac{\partial \mathcal{H}_i}{\partial u_i} = r_i u_i(t) + p_2^{i*}(t) \tag{27}$$

For the necessary condition, we shall minimize (24) which leads to

$$\begin{aligned}\frac{\partial \mathcal{H}_i}{\partial u_i} =& 0 \\ u_i(t) =& -\frac{1}{r_i}p_2^{i*}(t)\end{aligned} \tag{28}$$

Combining (26) and (28) we get

$$u_i(t) = -\frac{1}{r_i}\left[\frac{1}{2}\left(h_i(t) - h_{safe}^i\right)t^2 - at - b\right] \tag{29}$$

Because $u_i(t)$ is constrained to be $U_{min}^i \leq u_i(t) \leq U_{max}^i$ and $p_2^{i*}(t)$ has no such constraints, clearly (29) cannot always hold. Therefore, it is necessary to find the conditions in which the relationship does indeed hold and, if it doesn't, what to apply as $u_i(t)$. Luckily, this is rather straightforward. If $p_2^{i*}(t)$ goes beyond the allowable limits for $u_i(t)$, then either the maximum or minimum values for $u_i(t)$ can be used, otherwise the relationship described in (29) holds, as shown below in (30)

$$u_i(t) = \begin{cases} U_{max} & \text{if } p_2^{i*}(t) < r_i\, U_{min}^i \\ -\frac{1}{r_i} p_2^{i*}(t) & \text{if } r_i\, U_{min} \leq p_2^{i*}(t) \leq r_i\, U_{min}^i \\ U_{min}^i & \text{if } r_i\, U_{min}^i < p_2^{i*}(t) \end{cases} \tag{30}$$

9.3 Variational Approach

9.4 Linear Quadratic Tracking Controller

Using the same model as before, a Linear Quadratic Tracking (LQT) controller will be developed. The system (Equation 21) is linear, the cost function (Equation 22) is quadratic, and the system is to track a trajectory. In this section, an LQT controller shall be developed using the Pontryagin method.

10 Simulation

To determine the effectiveness of the controllers mentioned in 9.2 and 9.3, simulations were created using Matlab. First, a simulation of three vehicles on a circular track was created in order to simulate traffic shock waves. Then the Pontryagin and LQT controllers were simulated.

10.1 Traffic Shock Wave with Three Vehicles

Traffic shock waves were simulated using three human drivers (as described by 4.6) on a circular track with a 70 meter radius.

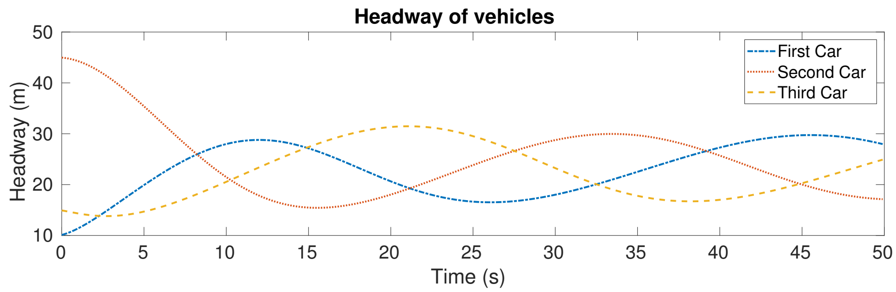

Figure 4: Headways of platooning vehicles

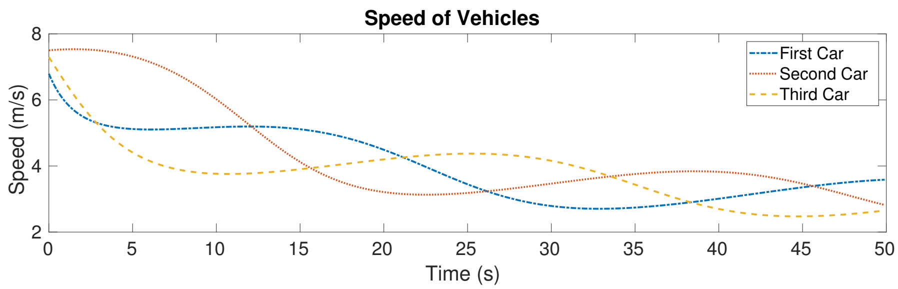

Figure 5: Velocities of platooning vehicles

11 Results

12 Discussion

12.1 Pontryagin Controller

12.2 LQT Controller

As seen in Figure 8, the controller succeeds at removing the

Part VII

Conclusion

Appendix

13 Table of Symbols

Table 5: Symbols used

SYMBOL	MEANING	UNITS
α_i	Preference of the i^{th} vehicle to follow range policy	second^{-1}
β_i	Preference of the i^{th} vehicle to match lead vehicle velocity	second^{-1}
e_i	Error function of the i^{th} vehicle	depends
h_i	Headway of the i^{th} vehicle	meters
h_{go}	Safety distance for the i^{th} car	meters
h_{safe}	Safety distance for the i^{th} vehicle	meters
h_{stop}	Safety distance for the i^{th} vehicle	meters
l_{ring}	Length of the ring	meters
n	Number of vehicles	-
r_i	Weighting on control action of i^{th} vehicle	-
u_i	Control action of i^{th} vehicle	meters/second^2
t_f	Final time	seconds
T_i	Delay time of i^{th} vehicle	seconds
U_{max}	Maximum control action of the i^{th} vehicle	meters/second^2
U_{min}	Minimum control action of the i^{th} vehicle	meters/second^2
v_i	Instantaneous velocity of the i^{th} vehicle	meters/second
V_i	Headway-based velocity of the i^{th} vehicle (range policy)	meters/second
v_0^i	Initial velocity of the i^{th} vehicle	meters/second
v_{max}	Maximum velocity (speed limit)	meters/second

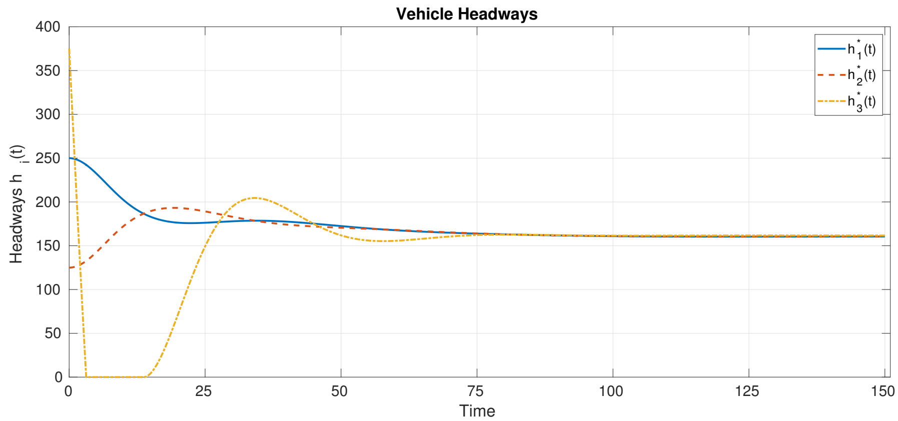

Figure 6: Headways of platooning vehicles

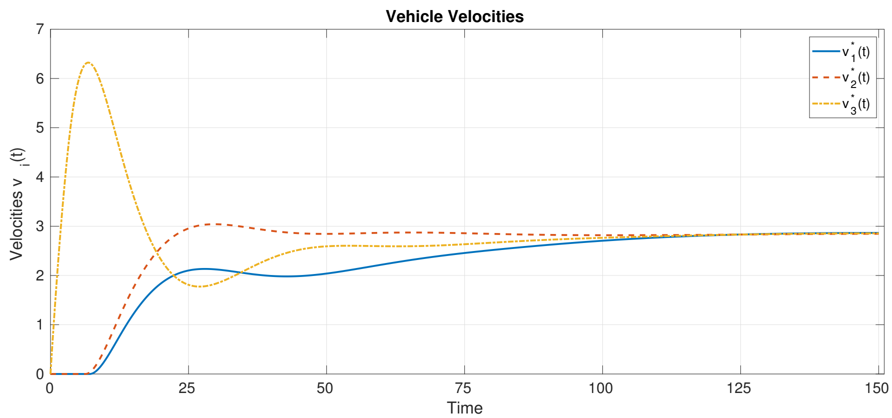

Figure 7: Velocities of platooning vehicles

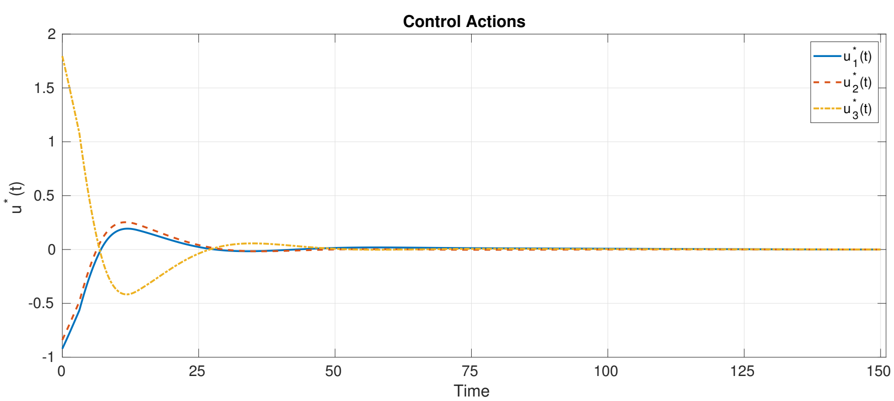

Figure 8: Control actions

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