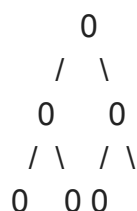


Binary Trees:

- Binary trees are data structures used for storing data. They are hierarchical data structures.
- Binary tree nodes at most can hold 2 children nodes.
- A real life application is used such as a file system within a computer.
- The height of a tree starts at the root and works its way down.
- A **full binary tree** (sometimes proper **binary tree** or **2-tree**) is a **tree** in which every node other than the leaves has two children.
- A binary tree T with n levels is **complete** if all levels except possibly the **last are completely full**, and the last level has all its nodes to the left side.
- A **traversal** is a process that visits all the nodes in the tree. Since a tree is a nonlinear data structure, there is no unique traversal.
- There are three different types of depth-first traversals:
 - PreOrder traversal - visit the **parent** first and then **left** and **right children**.
 - InOrder traversal - visit the **left** child, then the **parent** and the **right child**.
 - PostOrder traversal - visit **left child**, then the **right child** and then the **parent**.
 - All the above operations can easily be done by recursion.
- The height h of a complete binary tree with N nodes is at most $O(\log N)$. We can easily prove this by counting nodes on each level, starting with the root, assuming that each level has the maximum number of nodes:
$$n = 1 + 2 + 4 + \dots + 2^{h-1} + 2^h = 2^{h+1} - 1, \text{ h being the height of the tree}$$
- Some of the operations for a binary tree include: inserting, removing, searching, height of the tree and the 3 depth first search traversals which are preorder, inorder, and postorder.

Complete Binary Tree:



Full Binary Tree:



- This is a complete binary tree because it has its children of the nodes at the leftmost level
- The full binary tree is because all the nodes have 2 children and even though the second level right node has no children, this is still considered a full tree. So its 2 children or none at all.

References:

<http://www.cs.cmu.edu/~adamchik/15-121/lectures/Trees/trees.html>

<http://courses.cs.vt.edu/~cs3114/Fall09/wmcquain/Notes/T03a.BinaryTreeTheorems.pdf>