

E2-243

Final Lab Exam

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Date: 04 February 2021

Time: 10:00am-12:30pm

Instructions:

- Please read all the questions carefully. All the questions are self explanatory. All questions are compulsory.
 - You are free to use any programming language for coding. But we prefer you to use Matlab.
 - Error Handling: In your code, you should make sure that the input parameters to your function satisfy any assumptions that you make about them. For example, if we have asked you to write a function that takes a $m \times n$ matrix A , then you should check in your code that the matrix passed as input to your function is indeed of size $m \times n$. You should check for all possible pathological inputs.
 - Credit will be given for code hygiene. Please use appropriate comments and indentation. Please use proper variable names.
 - You should save all your files for **a particular question in a separate folder**. For example, all your MATLAB files for Question 1 should be in the folder 'Q1'. Also, include **a copy of the input and corresponding output which you have used to run the program** in the Matlab command window in a **text/word file**. At the end of the exam, you are required to **ZIP/compress** them and create a single zip file. The naming format for the ZIP file is **<YourName>-<SRNumber>.zip**.
 - Submission: The zip file should be emailed to both of us, ***prernaarote@iisc.ac.in*, and *zithas@iisc.ac.in***. There is no other paper submission for lab exam.
 - The timestamp of the email would be considered as submission time. Please ensure that you **submit the assignment by 12:30 pm. Late submissions will not be evaluated**. Also, please ensure that you **submit only one final version**.
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1. Real Analysis: (6 Marks)

- (a) Write a program that, given $0 < r < \sqrt{5}$, produces a rational p such that $p < \sqrt{5}$ and $\frac{r}{2} < |\sqrt{5} - p| < r$. **(2 marks)**
- (b) Consider a function,

$$f(x) = 1 - \left(1 - \frac{1 + x + x^2}{1 + 2x + 3x^2}\right)^2, x \in [0, 1]$$

and the iteration,

$$x_{n+1} = \frac{x_n}{3} + 2\frac{f(x_n)}{3}$$

Write a program that takes input $x_1 \in [0, 1]$, $N \in \mathbb{Z}_{++}$, $\epsilon > 0$ and

- i. produces first N iterates of the sequence $\{x_n\}$
- ii. gives the first N_ϵ iterates where $N_\epsilon = \min\{n : |x_n - x_{n+1}| < \epsilon\}$

(2 marks)

- (c) Write a program that outputs first 10 iterations (use an iterative method) and root of the function $f(x) = x^3 - 6x^2 + 11x - 6$ in $[2.5, 4]$. **(2 marks)**

2. Linear Algebra - Diagonalization of a matrix: **(7 Marks)**

- (a) Write a MATLAB function **findAM(A)** that takes a 3×3 matrix as the input and returns its eigenvalues and their algebraic multiplicities. **(1 mark)**
- (b) Write a MATLAB function **findGM(A)** that outputs the eigenvalues and their geometric multiplicities). **(2 marks)**
- (c) Write a MATLAB function **isDiagonalizable(A)** that uses the above functions and outputs 1 or 0 depending on whether A is or is not diagonalizable. **(2 marks)**
- (d) Write a MATLAB function $[P, D, P^{-1}] = \mathbf{diagonalize(A)}$ that takes a square matrix A as input and returns P, D and P^{-1} if A is diagonalizable. Verify that $A = PDP^{-1}$. **(2 marks)**

Note : You can use the standard MATLAB function **eig()** to find the eigenvalues of a matrix.

3. Probability - Random variables **(7 Marks)**

- (a) X_1, X_2 are exponential random variables with parameter 1 and 2 respectively. $Y = X_1 + X_2$
Write a program that, given a , outputs $Pr(Y \leq a)$ **(1 mark)**
- (b) Let Y_1, Y_2, \dots, Y_{10} be iid random variables with same distribution as Y in part (a). Let Z_j is the j^{th} minimum of Y_1 to Y_{10} .
Write a program that, given j and b , outputs $Pr(Z_j < b)$ **(3 marks)**
- (c) Define $D_k = Z_{k+1} - Z_k$, $k = 1$ to 9.
Write a program, that given k and c , outputs $Pr(D_k < c)$ **(3 marks)**