E2-243
Programming Exercise - 3

TA: Zitha Sasindran, Prerna Arote

Instructions:

- Do not submit your code, output files, etc.
- There will one lab exam towards the end of the semester that will test your understanding of the concepts taught in class. The questions in the lab exam will be somewhat similar to these questions in both content and implementation complexity. If you do not program these exercises, handling the lab exam will not be easy! In a way, programming these assignments yourself will be your preparation for the lab exam.
- You may use any discrete plot like the 'stem' function in MATLAB.
- Whenever $x \in$ interval, you may have to take appropriate discrete points in the interval to realize the functions in matlab. Please use appropriate commands for continuous plots when x is continuous.

---

1. Given a complex number z, consider the sequence

$$S_n = \sum_{k=1}^{n} z^k \ , \ n \geq 1$$

The series $\sum z^n$ converges if and only if $|z| < 1$. Let $|z| < 1$ and $S_n \to S$. Then

$$|S_n - S| = \left| \sum_{k=n+1}^{\infty} z^k \right| \leq \sum_{k=n+1}^{\infty} \left| z^k \right| = \frac{|z|^{n+1}}{1 - |z|}$$

. Write a program that takes z, N and ϵ as input and

(a) If $|z| \geq 1$, returns a message that the series does not converge.

(b) If $|z| < 1$, plots $[S_1, S_2, \cdots S_N]$.

(c) If $|z| < 1$, plots $[S_1, S_2, \cdots S_{N_\epsilon}]$ where $N_\epsilon = \min\{n : \frac{|z|^{n+1}}{1-|z|} \leq \epsilon\}$.

2. For any matrix $A \in \mathbb{R}^{m \times n}$

$$\|A\|_2 = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{\frac{1}{2}}$$

is a norm, referred to as the Frobenius norm. Let $A \in \mathbb{R}^{m \times m}$ be a square matrix with eigenvalues $\lambda_1, \lambda_2 \cdots \lambda_m$ then

- A^{-1} exists if and only if $|\lambda_i| > 0$
- if $0 < |\lambda_i| < 1, \ \forall i$ then

$$A^{-1} = \sum_{n=0}^{\infty} A^n \quad , \text{Note} : \ A^0 = I.$$

Define $S_n = \sum_{k=0}^{n} A^k$. Write a program that takes A and ϵ as input and

(a) If any eigenvalue is 0, returns a message that A is not invertible.

(b) If none of the eigenvalue is 0 but any eigenvalue has absolute value 1 or larger, returns a message that the series does not converge.

(c) If all the eigenvalues have absolute values between 0 and 1, output $N_\epsilon = \min\{n : \|AS_n - I\|_2 \leq \epsilon\}$ and $\|AS_{N_\epsilon} - I\|_2$.