E2-243
Programming Exercise - 1

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Instructions:

- Do not submit your code, output files, etc.
- There will one lab exam towards the end of the semester that will test your understanding of the concepts taught in class. The questions in the lab exam will be somewhat similar to these questions in both content and implementation complexity. If you do not program these exercises, handling the lab exam will not be easy! In a way, programming these assignments yourself will be your preparation for the lab exam.
- You may use any discrete plot like the 'stem' function in MATLAB.
- Whenever $x \in$ interval, you may have to take appropriate discrete points in the interval to realize the functions in matlab. Please use appropriate commands for continuous plots when x is continuous.

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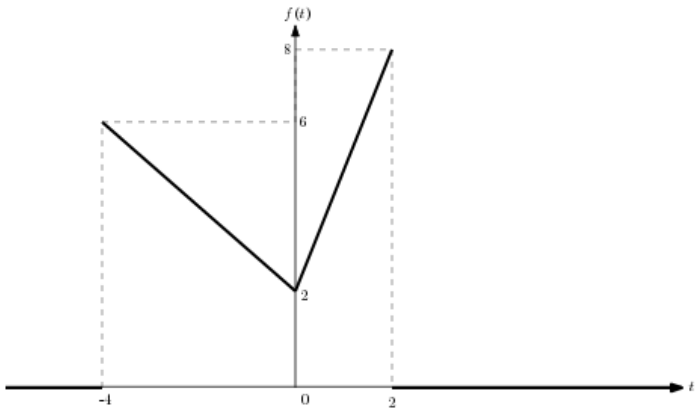
1. Let $f : \mathbb{R} \to \mathbb{R}$ be the function whose graph is sketched below:



Plot the following functions:

(a) $f(t)$
(b) $g(t) = f(t + 2)$
(c) $g(t) = f(\frac{5}{4}t)$
(d) $g(t) = f(2t - 4)$
(e) $g(t) = f(2 - t)$

2. Let T be a positve real number. The "ramp" function $R_T : \mathbb{R} \to \mathbb{R}$ is defined as follows:

$$R_T(t) = \begin{cases} \frac{t+T}{2T}, & \text{if } -T \leq t \leq T \\ 1, & \text{if } t \geq T \\ 0, & \text{if } t \leq T \end{cases}$$

Plot the following functions:

(a) $R_T(t)$

(b) $g(t) = R_T(t+T) - R_T(t-T)$

(Use $T = 1, 5, 10$).

3. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined as $f(t) = \exp(-|t|)$. Then answer the following by plotting the function.

(a) Range of f.

(b) From (a) comment if the function is onto or not.

(c) Is f one-one.

4. Let $f : A \to B$ where $A, B \subseteq \mathbb{R}$ are finite. Consider the following problems.

(a) Find $f(X)$ for some $X \subseteq A$.

(b) Find $f^{-1}(Y)$ for some $Y \subseteq B$.

(c) Is f one-to-one?

(d) Is f onto?

Can you write a program that takes A, B, f, X, Y as user-defined input and answers the above questions.

5. Consider a sequence of sets E_n as follows:

$$E_0 = [0, 1]$$
$$E_1 = \left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$$
$$E_2 = \left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{6}{9}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right]$$
$$\dots$$
$$\dots$$

E_{n+1} is obtained by removing middle third from each of the intervals that constitute E_n, and taking the union of the so obtained intervals.

Write a program that demonstrates E_n, given input n.

6. Plot the following regions :

(a) $\{(x_1, x_2) \in \mathbb{R}^2 : (|x_1|^r + |x_2|^r)^{\frac{1}{r}} \leq 1\}$ for $r = 1, 2, 3$. Observe that as r increases the region is expanding.

(b) Write a program that plot the region $\{(x_1, x_2) \in \mathbb{R}^2 : (|x_1|^c + |x_2|^c)^{\frac{1}{c}} \leq 1\}$, given input c.

(c) $\{(x_1, x_2) \in \mathbb{R}^2 : \max(|x_1|, |x_2|) \leq 1\}$.