E2-243

Programming Exercise - 4

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Instructions:

- Do not submit your code, output files, etc.
- There will one lab exam towards the end of the semester that will test your understanding of the concepts taught in class. The questions in the lab exam will be somewhat similar to these questions in both content and implementation complexity. If you do not program these exercises, handling the lab exam will not be easy! In a way, programming these assignments yourself will be your preparation for the lab exam.
- You may use any discrete plot like the 'stem' function in MATLAB.
- Whenever $x \in$ interval, you may have to take appropriate discrete points in the interval to realize the functions in matlab. Please use appropriate commands for continuous plots when x is continuous.

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1. Suppose $f : [a, b] \to [a, b]$ is continuous and differentiable on (a, b). Suppose $|f'(x)| < \alpha$ for all $x \in (a, b)$ where $\alpha < 1$. Then f has unique fixed point in $[a, b]$. Also, given $p \in [a, b]$, the sequence

$$x_1 = p$$
$$x_{n+1} = f(x_n), n \geq 1$$

converges to the unique fixed point.
Consider a function

$$f(x) = \frac{1}{3}(2 - e^x + x^2), x \in [0, 1]$$

Write a program that takes input $p \in [0, 1]$, $N \in \mathbb{Z}_{++}$, and $\epsilon > 0$, computes $X_n, n \geq 1$ iteratively and terminates if

(a) $|x_n - x_{n-1}| < \epsilon$, or

(b) $n = N$, or

(c) $x_n \notin (0, 1)$

Also the program outputs x_n in cases (a) and (b) and an error message in case (c).

2. Suppose $f : [a, b] \to \mathbb{R}$ is continuous, and $f(a)$ and $f(b)$ have opposite *signs*. Then f has a root in (a,b). Consider two sequences $\{a_n\}$ and $\{b_n\}$ obtained as follows:

- $a_1 = a, b_1 = b$
- for $n \geq 1$, $c = \frac{a_n + b_n}{2}$,

if $f(c) = 0$ **then**
  Stop
else if $sign(f(c)) = sign(f(a_n))$ **then**
  $a_{n+1} = c$
  $b_{n+1} = b_n$
else
  $a_{n+1} = a_n$
  $b_{n+1} = c$
end if

It can be checked that both $\{a_n\}$ and $\{b_n\}$ converge to a root of f.

Consider a function $f(x) = x^3 - x - 2$

Write a program that takes input $a, b \in \mathbb{R}$, $N \in \mathbb{Z}_{++}$, and $\epsilon > 0$, terminates with an error message if $sign(f(a)) = sign(f(b))$ otherwise computes a_n, b_n, $n \geq 1$ iteratively and terminates if

(a) $|b_n - a_n| < \epsilon$, or

(b) $n = N$

Also the program outputs a_n and b_n.