# Home Work 2

Due Date: 8am, 29th October 2019

(Attempt all questions. Total 50 marks)

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You need to show detailed calculations / explainations for your answers.

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**Q. 1]** Consider the function $f : \mathbb{R}^n \to \mathbb{R}$ given by

$$f(\mathbf{x}) \;=\; \frac{1}{2}\mathbf{x}^\top Q\mathbf{x} \;-\; \mathbf{b}^\top \mathbf{x},$$

where $\mathbf{b} \in \mathbb{R}^n$ and $Q \in \mathbb{R}^{n\times n}$ is symmetric & positive definite. Let $\mathbf{x}^*$ be the unknown point at which $f$ achieves minima. Assume that algorithms are initialized with an $\mathbf{x}_0$ such that $\mathbf{x}_0 - \mathbf{x}^*$ is parallel to an eigen vector of $Q$. How many iterations will be needed to find a minimizer of $f$ if the following methods were used

a. Gradient descent with exact line search

b. Conjugate gradient

c. Newton's method [5+4+1 marks

**Q. 2]** Consider $f(x_1, x_2) \;=\; 100(x_2 - x_1^2)^2 + (1 - x_1)^2$. Report the function value at the end of each iteration of the following:

(a) Apply two iterations of Newton's method initialized with $(0, 0)$. [4 marks

(b) Apply two iterations of Gradient Descent with fixed stepsize 0.05 and initialized with $(0, 0)$. [2 marks

**Q. 2]** (c)

Consider $f(x_1, x_2) \;=\; \frac{5}{2}x_1^2 + \frac{1}{2}x_2^2 + 2x_1x_2 - 3x_1 - x_2$. Find the minimizer of $f$ using the conjugate gradient algorithm starting from $(0, 0)$. [4 marks

**Q. 3]** Implement (in Matlab/Python) the conjugate gradient algorithm to solve the system of linear equations $\mathbf{Ax} = \mathbf{b}$, where $(i, j)$-th entry in $\mathbf{A} \in \mathbb{R}^{n\times n}$ is $\frac{1}{i+j-1}$ and all the coordinates in $\mathbf{b} \in \mathbb{R}^n$ are 1. Start the algorithm from zero vector. For $n = 5, 8, 12, 20$ report the number of iterations required to reduce the error $\|\mathbf{Ax}-\mathbf{b}\|_2$ below $10^{-6}$. For $n = 20$ plot the error (log scale) vs iteration. No need to submit code. [8+2 marks

**Q. 4]** Consider steepest descent algorithm (with exact line search) applied to the function

$$f(x_1, x_2) \;=\; 5x_1^2 + 5x_2^2 - x_1x_2 - 11x_1 + 11x_2 + 11.$$

(a) What would be the rate of convergence of the algorithm? [5 marks

(b) Starting with $(0, 0)$, how many iterations would it take (at most) to reduce the function value to $10^{-11}$? [5 marks

**Q. 5]** Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x_1, x_2) \;=\; \frac{3}{2}(x_1^2 + x_2^2) + (1+a)x_1x_2 - (x_1 + x_2) + b,$$

where $a, b \in \mathbb{R}$ are some unknown parameters.
(a) Find the largest range of values of $a$ and $b$ such that the unique global minimizer of $f$ exists, and write down the minimizer in terms of $a, b$. [4+2 marks
(b) Consider gradient descent algorithm (with fixed stepsize 0.4) applied to $f$. Find the largest range of values of $a$ and $b$ for which the algorithm converges to a global minimizer of $f$ from any arbitrary initial point. [4 marks