Q 4

	$\nabla f(\bar{x})$	Eigen values of $\nabla^2 f(\bar{x})$	at $\bar{x}$
Q. 4(i)	$\begin{pmatrix} 20x_1+10x_2+4 \\ 10x_1+2x_2-10 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	24.4536 & −2.4536	Saddle point.
Q. 4 (ii)	$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	36 & 16	Local minima

Q 5

	FS subroutine $\hat{x}$	tolerance	NSC	GS subroutine $\hat{x}$	tolerance	NSC
(i) N = 5	0.92308	0.23077	8	0.9929	0.200225	14
(i) N = 10	0.8958	0.02083	18	0.89278	0.020045	30
(i) N = 20	0.88894	0.00017	38	0.88898	0.00015	64
(ii) N = 5	1.1538	0.23077	8	1.2126	0.2002258	14
(ii) N = 10	1.20833	0.020833	18	1.2054	0.020045	30
(ii) N = 20	1.2045	0.0001644	38	1.2045	0.00015068	

mary

	Closed	Bounded	M	/ $\hat{x}$
(i)	✓	✓	178885	(1.6, -0.8)
(ii)	✓	✓	1	$\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

Q3 (i)

(b) ~~Elon~~ Siri is correct and Cortona's claim is false.

Q2

$\beta = \cancel{1.623 \times 10^{-4}}$ 623

$\epsilon = 1.623 \times 10^{-4}$

	ϵ	0.1ϵ	0.01ϵ	0.0001ϵ
δ =	9490804	949080323	9.49×10^{10}	9.49×10^{14}
Q. 2(i)	3081	30808	308072	30807148
Q. 2(ii)	56	176	556	5551
Q. 2(iii)	319	410	501	683
Q. 2(iv)	9	9	9	10
Q. 2(v)				

$$k_\epsilon = \left\lceil \frac{\ln\left(\frac{1.623\times10^{-4}}{0.5128}\right)}{\ln(0.975)} \right\rceil$$

$$= 319$$

$k_{0.1\epsilon} = 410$

$k_{0.01\epsilon} = 501$

$k_{0.0001\epsilon} = 683$

(v) $e_1 = 0.5$, $e_k = 1.9 e_{k-1}^2$, $k \geq 2$,

using ~~py~~ Python program,

$k_\epsilon = 9$

$k_{0.1\epsilon} = 9$

$k_{0.01\epsilon} = 9$

$k_{0.0001\epsilon} = 10$

Is there a pen & paper method available?

$e_k = 0.5/k^2$

$k = \left\lceil \sqrt{\dfrac{0.5}{e_k}} \right\rceil$

$k_\epsilon = \left\lceil \sqrt{\dfrac{0.5}{1.623\times10^{-4}}} \right\rceil$

$= 56$

$k_{0.1\epsilon} = 176$

$k_{0.01\epsilon} = 556$

$k_{0.001\epsilon} = 5551$

(iv) $e_1 = 0.5$

$e_k = 0.975 e_{k-1}$

$\Rightarrow (E - 0.975) e_k = 0$

$\Rightarrow e_k = c_1 (0.975)^k$

$e_1 = c_1 \times 0.975 = 0.5$

$\Rightarrow c_1 = \dfrac{0.5}{0.975} = 0.5128$

$\Rightarrow \ln e_k = \ln c_1 + k \ln (0.975)$

$k = \left\lceil \dfrac{\ln\left(\frac{e_k}{c_1}\right)}{\ln (0.975)} \right\rceil$

$$K_{0.01\epsilon} = \left\lceil \left(\frac{0.5}{0.01 \times 1.623 \times 10^{-4}} \right)^2 \right\rceil$$

$$\approx 9.49 \times 10^{10}$$

$$K_{0.0001\epsilon} \approx 9.49 \times 10^{14}$$

(ii) $e_k = \frac{0.5}{k}$

$$k = \left\lceil \frac{0.5}{e_k} \right\rceil$$

$$K_\epsilon = \left\lceil \frac{0.5}{1.623 \times 10^{-4}} \right\rceil = 3081$$

$$K_{0.1\epsilon} = 30808$$

$$K_{0.01\epsilon} = 308072$$

$$K_{0.0001\epsilon} = 30807148$$

2]

$\beta = 623$

$\epsilon = (0.001\beta+1)10^{-4}$

$= (0.001 \times 623+1)10^{-4}$

$= 1.623 \times 10^{-4}$

(i) $e_k = 0.5/\sqrt{k}$

$\sqrt{k} = 0.5/e_k$

$k = \left\lceil \left(\frac{0.5}{e_k}\right)^2 \right\rceil$

$e_k < \epsilon,$

$k_\epsilon = \left\lceil \left[\frac{0.5}{\epsilon}\right]^2 \right\rceil$

$= \left\lceil \left(\frac{0.5}{1.623 \times 10^{-4}}\right)^2 \right\rceil$

$= \lceil 9490803.222 \rceil$

$= 9490804$

$k_{0.1\epsilon} = \left\lceil \left(\frac{0.5}{1.623 \times 10^{-5}}\right)^2 \right\rceil$

$= 949080323$

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MI DUAL CAMERA

Scanned by CamScanner

To find eigenvalues,

$$\begin{vmatrix} 32-\lambda & 8 \\ 8 & 20-\lambda \end{vmatrix} = (32-\lambda)(20-\lambda) - 64$$

$$= 640 - 52\lambda + \lambda^2 - 64$$

$$= \lambda^2 - 52\lambda + 576 = 0$$

$$\lambda = \frac{52 \pm \sqrt{52^2 - 4\times 576}}{2}$$

$$= 36 \;\&\; 16$$

Point is on local minima

4 (ii)

$$f(\bar{x}) = 16x_1^2 + 8x_1x_2 + 10x_2^2 + 12x_1 - 6x_2 + 2$$

$$\nabla f(\bar{x}) = \begin{bmatrix} 32x_1 + 8x_2 + 12 \\ 8x_1 + 20x_2 - 6 \end{bmatrix}$$

$$\nabla f(\bar{x} = [-0.5, 0.5])$$

$$= \begin{bmatrix} -16 + 4 + 12 \\ -4 + 10 - 6 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}}$$

$$\nabla^2 f = \begin{bmatrix} \frac{\partial}{\partial x_1}(32x_1 + 8x_2 + 12) & \frac{\partial}{\partial x_1}(\cancel{32}\ 8x_1 + 20x_2 - 6) \\ \frac{\partial}{\partial x_2}(32x_1 + 8x_2 + 12) & \frac{\partial}{\partial x_2}(8x_1 + 20x_2 - 6) \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 8 \\ 8 & 20 \end{bmatrix}$$

To find eigen values.

$$\begin{vmatrix} 20-\lambda & 10 \\ 10 & 2-\lambda \end{vmatrix} = (20-\lambda)(2-\lambda) - 100 = 0$$

$$40 - 22\lambda + \lambda^2 - 100 = 0$$

$$\lambda^2 - 22\lambda - 60 = 0$$

$$\lambda = \frac{22 \pm \sqrt{22^2 + 240}}{2} = 24.4536,$$

$\&$

-2.4536

Point on ~~local minima~~ is saddle points.

Calculations

(i) $f(\bar{x}) = 10x_1^2 + 10x_1x_2 + x_2^2 + 4x_1 - 10x_2 + 2$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 20x_1 + 10x_2 + 4 \\ 10x_1 + 2x_2 - 10 \end{bmatrix}$$

$$\nabla f(\bar{x} = (1.8, -4))$$

$$= \begin{bmatrix} 20 \times 1.8 - 40 + 4 \\ 18 - 8 - 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\nabla^2 f = \cancel{\nabla(\nabla f^T)}$$

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1}(20x_1 + 10x_2 + 4) & \frac{\partial}{\partial x_1}(10x_1 + 2x_2 - 10) \\ \frac{\partial}{\partial x_2}(20x_1 + 10x_2 + 4) & \frac{\partial}{\partial x_2}(10x_1 + 2x_2 - 10) \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 10 \\ 10 & 2 \end{bmatrix}$$

Checking Cortona's claim

~~Co~~ f is coercive,

Coercive

$\Rightarrow$

$$\lim_{\|x\| \to \infty} f(x) \to \infty$$

Let $p^T = [1 \ -1 \ 0]$ $\quad q^T = [1 \ 0 \ -1]$

& $x^T = [x_1 \ x_2 \ x_3] \mid x_1 \to x_2 \to x_3 \to \infty$

Now ~~f is not~~ we can see f is not coercive

Have doubt about the definition & the way I did

Seems Cortona is also right.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(ii) Given,

$$f(\bar{x}) = (p^T x)^2 + (q^T x)^2, \quad p, q, x \in \mathbb{R}^3$$

$$p \neq 0, \; q \neq 0, \; p + q \neq 0$$

Sid: $\exists \, \bar{x} \in \mathbb{R}^3$ ~~with~~ $\wedge \; \|\bar{x}\|_2 = 1 \; \wedge f(\bar{x}) = 0$

Cortona: f is coercive.

Checking claim by Sid,

~~$\exists$~~ Take any $\bar{x}$, divide each component ~~you will get ||~~ with $\|\bar{x}\|$, you will get a vector with unit norm,

take $p = [p_1 \; p_2 \; p_3]^T$ & $q = [q_1 \; q_2 \; q_3]$

& $\bar{x}^T = [x_1 \; x_2 \; x_3]$

Now assume $\exists$

$$\begin{bmatrix} p^T \\ q^T \end{bmatrix} [\bar{x}] = 0$$

$$= \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

We need 3 equations to find unique solution.

We can find multiple $\bar{x}$s satisfying $f(\bar{x}) = 0$ & divide each component by $\|\bar{x}\|$ to get a vector x with unit norm $\Rightarrow$ claim by Sid is correct

$\|A\|_F = \sqrt{a^2 + c^2 + 2b^2}$

$= \sqrt{a^2 + (1-a)^2 + 2b^2}$

$\|A\|_F^2 = 1 \cancel{-2a + 2b^2}$ $\quad 2a^2 + 1 - 2a + 2b^2$

To maximize $\|A\|_F^2$, $\quad a(1-a) - b^2 \geq 0$

Make $a(1-a) = b^2$

$a = 0.5 \quad b = 0.5$

$\|A\|_F^2 = 1 - 2a + 2a(1-a)$. $\quad \frac{d}{da}(2a^2 + 1 - 2a + 2a(1-a))$

$= 1 - 2a \cancel{+2a} - 2a^2 \quad 4a - 2 + 2 - 4a$

$= 1 - 2a^2$

$2a^2 + 1 - 2a + 2(a(1-a))$

$4a - 2 + 2 - 4a$

Differentiating

$-4a = 0 \Rightarrow a = 0$

$c = 1$

$\|A\|_F = 1$

$\Rightarrow$

~~One Sn~~ Bounded

2

$\hat{X} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$X = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

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(ii) Given,

$S = \{ X \in \mathbb{R}^{2\times 2} ; X$ is symmetric & +ve semi definite, trace $(X) = 1 \}$

Let A be an element in S.

$$\begin{vmatrix} a-\lambda & b \\ b & c-\lambda \end{vmatrix}$$

$(a-\lambda)(c-\lambda) - b^2 = 0$

$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ $\because$ symmetric

$a + c = 1 \Rightarrow \cancel{a + (1-a) = 1}$

$1 - a = c$

$ac - b^2 \geq 0$ $\because$ +ve semi definite

Existance of limiting points

$\Rightarrow$ S is closed

$\cancel{ac = b^2} \geq 0$

Assumption: Frobenius norm is $\leq M$ if the sets of matrices are bounded

This is the equation of a tilted ellipse

$\Rightarrow$ Q is bounded

$M = 1.78885$ → length of semi major axis (calculated using Wolfram Alpha)

&

$\hat{x} = (1.6, -0.8)$ → ~~Ve~~ One vertex of the ellipse.

Q1

(i) Given,

$Q \in R^2$

$Q = \{ \bar{x} = (x_1, x_2) \in \mathbb{R}^2 : 9x_1^2 + 16x_1x_2 + 21x_2^2 \le 16 \}$.

Q is closed since the limiting points ~~is~~ are inside the sets.

Testing if Q is bounded

Let $f(\bar{x}) = 9x_1^2 + 16x_1x_2 + 21x_2^2$

At minima/maxima

$$\nabla f(\bar{x}) = \begin{bmatrix} 18x_1 + 16x_2 \\ 16x_1 + 42x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \bar{x}^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

at (0,0), the function value is zero ~~⇒ it's mi~~ ~~Q~~

(0,0) is minima since there exist $\bar{x}$ where we get bigger values.

For