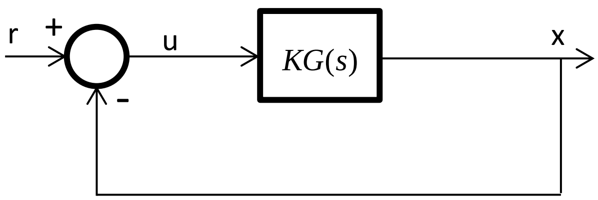
IN 227 CONTROL SYSTEMS DESIGN 2020

HOMEWORK#2

Problem#1 (15)

A system has the following open loop transfer function $G(s)=1/[(s/6+1)(s/11+1)(s/14+1)]$. It is part of a feedback control system of the kind shown below:



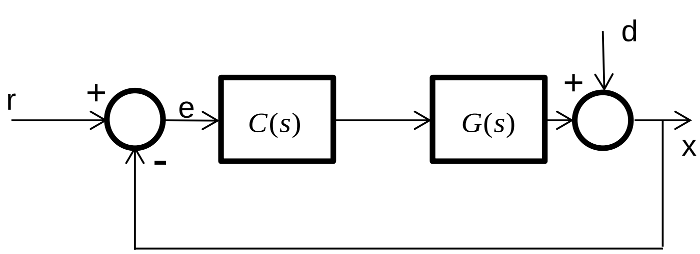
(a) Sketch the Nyquist plot of G(jω) by hand. Identify the frequencies at which G(jω) cross the imaginary axis and the real axis in the s-plane. Find the range of the gain K(>0) for which the closed-loop system is stable.

(b) In addition to being stable, it is desired that the closed-loop system have at least one pair of poles of the form $s=\sigma\pm j\omega$, wherein $-4.5\le\sigma\le-1.1$ and $0.4\le\omega\le 25$. Use the principles of Nyquist stability theory to find the range of the gain K (>0) for which the closed-loop system satisfies these requirements.

Problem #2 (15)

The theoretical model of an open loop system has the following transfer function: $G(s)=K/\{[5s/4\overline{p}+1][s/(6\overline{p})+1][5s/\overline{p}^2+1]\}$, where, $0.20\overline{p}\le K\le 0.30\overline{p}$. Here $\overline{p}=(p_1+p_2)/2$ where p_1, p_2 represent the sum of the digits (all 18 digits) of your S.R. number and that of your team member respectively.

It is part of a feedback loop of the kind shown below:



(a) Sketch the Bode plot of your plant by hand. Also, plot the Bode plot using MATLAB and use it to identify the gain margin, phase margin, gain and phase cross-over frequencies.

(b) Assume d=0. Design the controller C such that the closed-loop control system tracks steady (DC) reference perfectly and ramp reference with an error of at most 0.020. Make any other assumption necessary to execute your design. Plot the response of your closed-loop system to step and ramp inputs and validate your design.

(c) Redesign your controller if, in addition to the specifications above, your plant experiences disturbances in the range 10-15 rad/s at the location shown above and you want your control system to reject them by 97%. Assume a sinusoidal disturbance with frequency in this range and show, by simulation, that your controller satisfies the specifications.

Problem#3 (10)

For the plant given in problem#2, use root-locus method to design a compensator such that the dominant closed-loop poles are located at $-3\overline{p} \pm j6\overline{p}$ and the closed-loop system rejects DC output disturbance perfectly.

Practice questions (These will not be graded)

Common hints for all practice questions:

(1) Use asymptotic Bode plots wherever possible (for eg. to determine ω_{gc} etc...). Refrain for solving complicated quadratic/quartic equations.

(2) Approximate |1+L| as |L| for |L|>>1

(3) Use the notion of dominant poles wherever possible

Bode plot based design

(1) (1) A plant has the following transfer function: $P(s)=\dfrac{K}{s+p}$, where the gain K is nominally equal to 40 but can change by ±25% of its nominal value. Likewise, p is nominally 1 but can change by ±20% of its nominal value.

Design a feedback control system that satisfies the following specifications:

(a) The closed-loop system shows steady state error to ramp inputs of at most 0.01

(b) The rise time is at most 0.005s.

(c) The phase margin is between 35˚and 45°.

(2) Fig. (a) Shows the waveform of an output disturbance that affects a plant with the transfer function $P(s)=\dfrac{5}{(s/250)+1}$. In the figure, A is an unknown constant. Fig. (b) sketches the expected response of the closed-loop system to the applied disturbance for the case $\mathrm{A}=0.5$. Design a feedback system that achieves, approximately, the expected response.

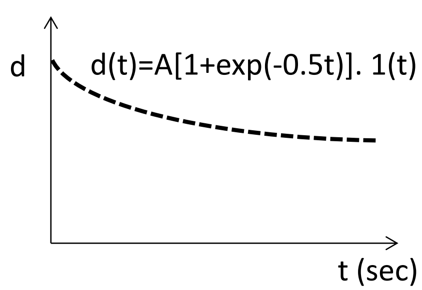


Fig. (a)

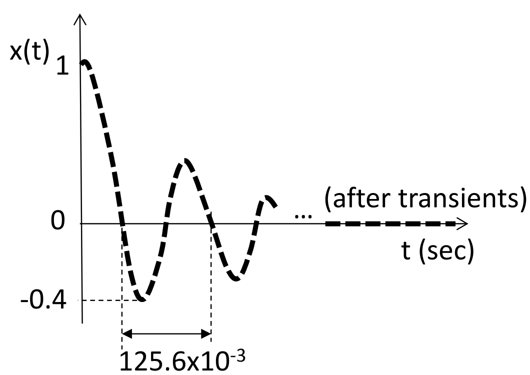
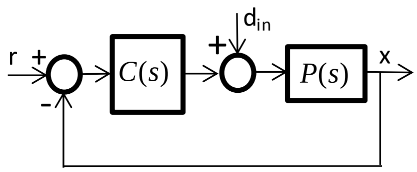


Fig. (b)

(3) The plant $P(s)=10/(s+10)$ is part of a feedback system affected by measurement noise $n(t)$. The power spectral density of $n(t)$ is 1 in the frequency range $0.1\le\omega\le 1$, and zero at other frequencies. Design a one degree of freedom control system that results in the power spectral of the output $x(t)$ to be at most 0.01 in the same frequency range, and possesses a closed-loop damping for the dominant

poles of at least 0.55. (Hint: power spectral density refers to the Fourier transform of the autocorrelation of the signal).

(4) In the feedback system shown below, the plant P(s) is given by $P(s)=\frac{K}{s+10}$. The gain K is nominally 100 but can increase by 10%. Design a 1-DOF control system that achieves the following specifications:

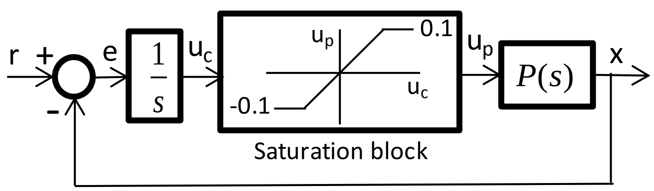


(a) The steady state change in output x(t) due to input disturbance $d_{in}(t)=t.1(t)$ is at most 0.01.

(b) The change in rise time of the closed-loop system due to change in the plant's gain is at most 0.4×10^{-3} seconds.

(c) The phase margin is at least 30°.

(5) The closed-loop system shown below controls the plant $P(s)=\frac{10(s+z)}{s^2+11s+10}$, where $9\le z\le 11$. The phase margin of the open-loop system is approximately 16° for values of z in the specified range. The saturation block saturates at ±0.1. Between these limits, the output of the block is equal to the input.



The closed-loop system is initially at rest and is provided a step input at t=0, i.e., $r(t)=1(t)$.

(a) Sketch the approximate waveform of the control error e=r-x for time $t\ge0$ when $z=9$ and when $z=11$.

(b) For the case $z=11$, estimate, approximately, the time t_1 at which the error e(t) crosses zero for the first time after application of the input.

(6) A plant P(s) is given by $P(s)=\frac{K}{(s/10+1)(s/400+1)}$. The gain K is nominally 10 but can vary by $\pm20\%$.

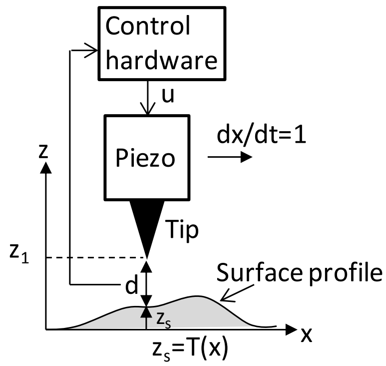
Design a 1-DOF control system that achieves the following specifications:

(a) The closed-loop system tracks sinusoidal references in the range 1-2rad/s with an accuracy of at least 95%. Further the magnitude of variation in tracking accuracy is required to be at most 0.5% when K varies.

(b) The phase margin is at least 15° and at most 40°.

[Hint: use the expression dT/T=(1/(1+L))(dP/P) for part (a)]

(7) In a scanning probe microscope, a sharp tip is positioned above a surface of unknown height profile z_s=T(x). The absolute position of the tip is z_1. The measured gap 'd' between the tip and the surface (see figure) is regulated at a set-point 'd_0' as the tip is moved along the X-axis. The regulation is performed by a piezo-actuator with transfer function P(s)=z_1/u=1/(s+1). The surface height profile is estimated as $\widehat{T}(x)=z_1(x)-d_0$. The velocity with which the tip is moved along the X-axis is dx/dt=1 nm/s.



(a) Draw the block-diagram of the control system for regulating the gap d at its set-point d_0.

(b) It is desired that the maximum steady state error in estimating topography T(x)=5x be 0.1 nm, and the damping in closed-loop response be at least 0.4. Design a control system to achieve these specifications

(8) A plant has the following transfer function: $P(s)=\dfrac{K}{s+25}$, where K is nominally equal to 250 but can vary by $\pm 25\%$.

Design a feedback control system that satisfies the following specifications:

(a) The closed-loop system rejects ramp output disturbances at least by 99% in steady state

(b) The rise time of the closed loop system to step inputs is at least 0.02s and the peak overshoot is at most 25%

(9) A plant has the following transfer function: $P(s)=\dfrac{K}{s+p}$, where K and p are nominally equal to 100 and 10 respectively. However, both can vary by $\pm 25\%$.

Design a feedback control system that satisfies the following specifications:

(a) The closed-loop system shows zero steady state error to step inputs

(b) The closed loop system rejects disturbances in the frequency range 0.1-1rad/s by atleast 98%.

(c) The phase margin is atleast 40˚.

(10) (2) A plant has the following transfer function: $P(s)=\dfrac{K}{s(s/p+1)}$, where K and p are nominally equal to 10 and 10 respectively. However, both can vary by $\pm 30\%$.

Design a feedback control system that satisfies the following specifications:

(a) The closed-loop system shows steady state error of 5% to ramp references

(b) The phase margin is atleast 40˚.

(11) A plant has the following transfer function: $G(s)=2\times 10^5/[s^2+105s+500]$.

(a) Sketch the bode plot, identifying the corner frequencies, DC gain, the gain cross-over frequency and the phase margin.

(b) A disturbance in the frequency range ω=0.01-0.1 rad/s needs to be suppressed to 0.1% of its value. The phase margin of the open-loop system is required to be at least 35˚. Design a one degree of freedom control system that achieves these design objectives.

(12) A plant has the following transfer function: $P(s) = \dfrac{K}{s+p}$, where the gain K is nominally equal to 100 but can change by ±30% of its nominal value. Likewise, p is nominally 2 but can change by ±20% of its nominal value.

Design a feedback control system that satisfies the following specifications:

(a) The closed-loop system shows zero steady state error to step inputs

(b) The smallest gain cross-over frequency is 20 rad/s

(c) The smallest phase margin is 35˚.

(13) A plant has the following transfer function: $P(s) = \dfrac{K}{s+2}$, where the gain K is nominally equal to 100 but can change by ±30% of its nominal value.

Design a feedback control system that satisfies the following specifications:

(a) The closed-loop system shows zero steady state error to step inputs

(b) The smallest gain cross-over frequency is 20 rad/s

(c) The smallest phase margin is 35˚.

(14) A plant has the following transfer function: $G(s) = 10^5 / [s^2 + 110s + 1000]$.

(a) Sketch the bode plot, identifying the corner frequencies, DC gain, the gain cross-over frequency and the phase margin.

(b) A disturbance in the frequency range ω=0.01-0.1 rad/s needs to be suppressed to 0.1% of its value. The phase margin of the open-loop system is required to be at least 40˚. Design a one degree of freedom control system that achieves these design objectives.

(15) A plant has the following transfer function: $G(s) = 10^3 / [s^2 + 10s]$. It is part of a one degree of freedom control system.

(a) Sketch the bode plot of the plant and identify the corner frequencies, the gain cross-over frequency and the phase margin.

(b) The steady state error of the overall control system to a ramp reference input (r(t)=t) is desired to be 0.1%. The phase margin of the open-loop system is required to be at least 40˚. Design a one degree of freedom control system that achieves these design objectives.

(16) A plant has the following transfer function: $G(s) = 5 \times 10^4 / [s^2 + 60s + 500]$.

(a) Sketch the bode plot, identifying the corner frequencies, DC gain, the gain cross-over frequency and the phase margin.

(b) A disturbance in the frequency range ω=0.05-0.5 rad/s needs to be suppressed to 0.1% of its value. The phase margin of the open-loop system is required to be at least 40˚. Design a one degree of freedom control system that achieves these design objectives

Root locus based design

(1) A plant has the following transfer function: $P(s) = \dfrac{10}{s+2}$. Use root locus to design a feedback control system that satisfies the following specifications:

(a) It perfectly rejects output disturbances of the kind $d(t) = A\sin^2(t+\phi)$

(b) The dominant closed-loop poles will be at $-1 \pm 1j$

(2) A plant has the following transfer function: $G(s) = 15/[(s+0.5)(s+6)]$.

(a) Sketch the root locus for this plant.

(b) Design a controller that guarantees that the dominant poles of the system are at $-1 \pm j3$ and the steady state error to DC input is zero