

Assignment-based Subjective Questions:

1. From your analysis of the categorical variables from the dataset, what could you infer about their effect on the dependent variable?

- Clear weather has more positive impact towards cnt
- Day of the week has very minimal variations across the 7 days
- cnt gradually increases as calendar month increases and after mid-year seems to be reduction in cnt
- year on year increase between 2018 and 2019 is increasing trend
- Spring has the lowest demand for cnt and Fall has the highest demand
- Working day and holiday has very minimal variations

2. Why is it important to use `drop_first=True` during dummy variable creation?

- Drop first ensures that unnecessary additional dummy column is not created and n-1 columns are created where n is number of levels.

3. Looking at the pair-plot among the numerical variables, which one has the highest correlation with the target variable?

- temp variable is highest correlated with target variable

4. How did you validate the assumptions of Linear Regression after building the model on the training set?

- Major assumption of Linear Regression is error terms are normally distributed which is validated in the python notebook by plotting distplot of residuals which shows normal distribution of error terms.

5. Based on the final model, which are the top 3 features contributing significantly towards explaining the demand of the shared bikes?

- temp impacts positively
- Weather Light Rain and Snow impacts negatively
- year impacts positively

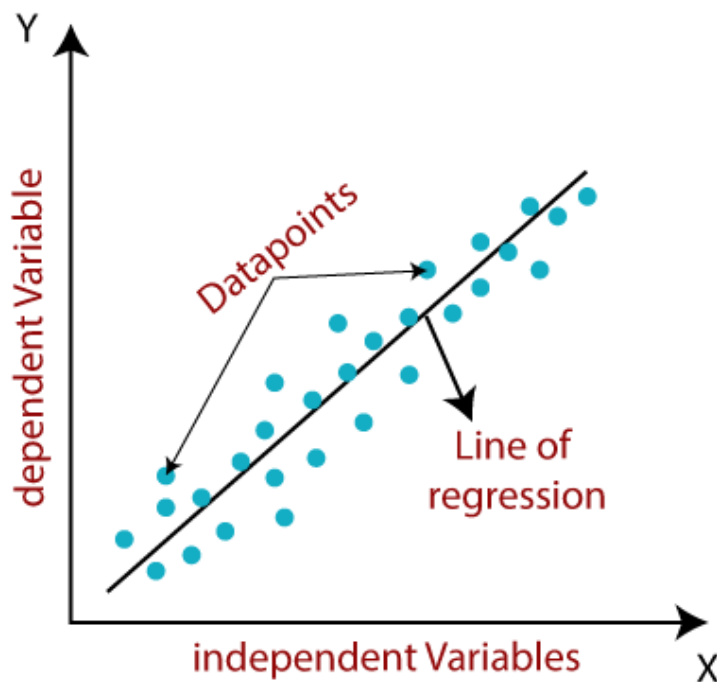
General Subjective Questions:

1. Explain the linear regression algorithm in detail.

Linear regression is one of the easiest and most popular Machine Learning algorithms. It is a statistical method that is used for predictive analysis. Linear regression makes predictions for continuous/real or numeric variables such as **sales, salary, age, product price**, etc.

Linear regression algorithm shows a linear relationship between a dependent (y) and one or more independent (x) variables, hence called as linear regression. Since linear regression shows the linear relationship, which means it finds how the value of the dependent variable is changing according to the value of the independent variable.

The linear regression model provides a sloped straight line representing the relationship between the variables. Consider the below image:



Mathematically, we can represent a linear regression as:

$$y = a_0 + a_1x + \epsilon$$

Here,

Y= Dependent Variable (Target Variable)

X= Independent Variable (predictor Variable)

a_0 = intercept of the line (Gives an additional degree of freedom)

a_1 = Linear regression coefficient (scale factor to each input value).

ϵ = random error

The values for x and y variables are training datasets for Linear Regression model representation.

Types of Linear Regression

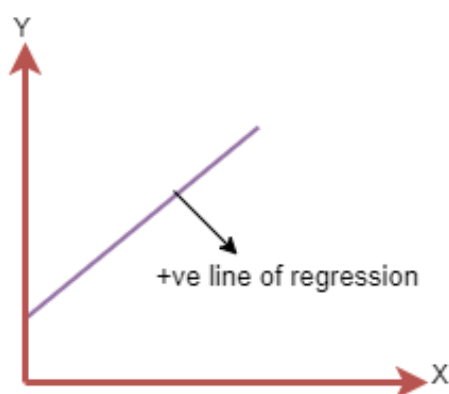
Linear regression can be further divided into two types of the algorithm:

- **Simple Linear Regression:**
If a single independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Simple Linear Regression.
- **Multiple Linear regression:**
If more than one independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Multiple Linear Regression.

Linear Regression Line

A linear line showing the relationship between the dependent and independent variables is called a **regression line**. A regression line can show two types of relationship:

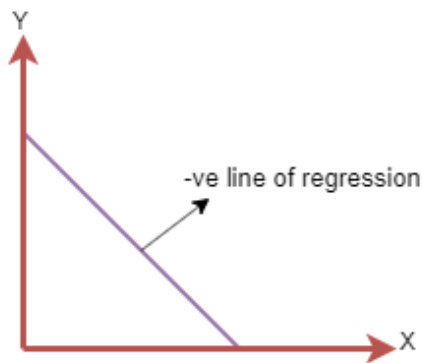
- **Positive Linear Relationship:**
If the dependent variable increases on the Y-axis and independent variable increases on X-axis, then such a relationship is termed as a Positive linear relationship.



The line equation will be: $Y = a_0 + a_1X$

- **Negative Linear Relationship:**
If the dependent variable decreases on the Y-axis and independent variable

increases on the X-axis, then such a relationship is called a negative linear relationship.



The line of equation will be: $Y = -a_0 + a_1X$

Finding the best fit line:

When working with linear regression, our main goal is to find the best fit line that means the error between predicted values and actual values should be minimized. The best fit line will have the least error.

The different values for weights or the coefficient of lines (a_0 , a_1) gives a different line of regression, so we need to calculate the best values for a_0 and a_1 to find the best fit line, so to calculate this we use cost function.

Cost function:

- The different values for weights or coefficient of lines (a_0 , a_1) gives the different line of regression, and the cost function is used to estimate the values of the coefficient for the best fit line.
- Cost function optimizes the regression coefficients or weights. It measures how a linear regression model is performing.
- We can use the cost function to find the accuracy of the **mapping function**, which maps the input variable to the output variable. This mapping function is also known as **Hypothesis function**.

For Linear Regression, we use the **Mean Squared Error (MSE)** cost function, which is the average of squared error occurred between the predicted values and actual values. It can be written as:

For the above linear equation, MSE can be calculated as:

$$MSE = \frac{1}{N} \sum_{i=1}^n (y_i - (a_1 x_i + a_0))^2$$

Where,

N=Total number of observation

Y_i = Actual value

(a₁x_i+a₀)= Predicted value.

Residuals: The distance between the actual value and predicted values is called residual. If the observed points are far from the regression line, then the residual will be high, and so cost function will high. If the scatter points are close to the regression line, then the residual will be small and hence the cost function.

Gradient Descent:

- Gradient descent is used to minimize the MSE by calculating the gradient of the cost function.
- A regression model uses gradient descent to update the coefficients of the line by reducing the cost function.
- It is done by a random selection of values of coefficient and then iteratively update the values to reach the minimum cost function.

Model Performance:

The Goodness of fit determines how the line of regression fits the set of observations. The process of finding the best model out of various models is called **optimization**. It can be achieved by below method:

1. R-squared method:

- R-squared is a statistical method that determines the goodness of fit.
- It measures the strength of the relationship between the dependent and independent variables on a scale of 0-100%.
- The high value of R-square determines the less difference between the predicted values and actual values and hence represents a good model.
- It is also called a **coefficient of determination**, or **coefficient of multiple determination** for multiple regression.
- It can be calculated from the below formula:

$$R\text{-squared} = \frac{\text{Explained variation}}{\text{Total Variation}}$$

Assumptions of Linear Regression

Below are some important assumptions of Linear Regression. These are some formal checks while building a Linear Regression model, which ensures to get the best possible result from the given dataset.

- **Linear relationship between the features and target:**
Linear regression assumes the linear relationship between the dependent and independent variables.
- **Small or no multicollinearity between the features:**
Multicollinearity means high-correlation between the independent variables. Due to multicollinearity, it may difficult to find the true relationship between the predictors and target variables. Or we can say, it is difficult to determine which predictor variable is affecting the target variable and which is not. So, the model assumes either little or no multicollinearity between the features or independent variables.
- **Homoscedasticity Assumption:**
Homoscedasticity is a situation when the error term is the same for all the values of independent variables. With homoscedasticity, there should be no clear pattern distribution of data in the scatter plot.
- **Normal distribution of error terms:**
Linear regression assumes that the error term should follow the normal distribution pattern. If error terms are not normally distributed, then confidence intervals will become either too wide or too narrow, which may cause difficulties in finding coefficients.
It can be checked using the **q-q plot**. If the plot shows a straight line without any deviation, which means the error is normally distributed.
- **No autocorrelations:**
The linear regression model assumes no autocorrelation in error terms. If there will be any correlation in the error term, then it will drastically reduce the accuracy of the model. Autocorrelation usually occurs if there is a dependency between residual errors.

2. Explain the Anscombe's quartet in detail.

Anscombe's quartet comprises four datasets that have nearly identical simple statistical properties, yet appear very different when graphed. Each dataset consists of eleven (x,y) points. They were constructed in 1973 by the statistician Francis Anscombe to demonstrate both the importance of graphing data before analysing it and the effect of outliers on statistical properties.

Simple understanding:

Once Francis John "Frank" Anscombe who was a statistician of great repute found 4 sets of 11 data-points in his dream and requested the council as his last wish to plot those points. Those 4 sets of 11 data-points are given below.

I		II		III		IV	
x	y	x	y	x	y	x	y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

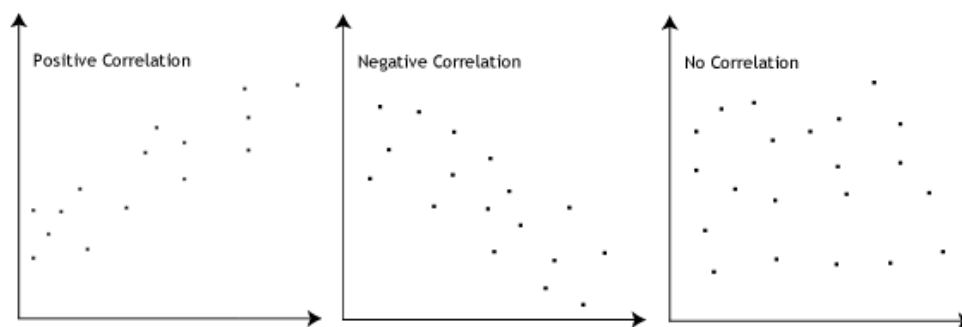
After that, the council analysed them using only descriptive statistics and found the mean, standard deviation, and correlation between x and y.

3. What is Pearson's R?

In statistics, the Pearson correlation coefficient (PCC), also referred to as Pearson's r, the Pearson product-moment correlation coefficient (PPMCC), or the bivariate correlation, is a measure of linear correlation between two sets of data. It is the covariance of two variables, divided by the product of their standard deviations; thus it is essentially a normalised measurement of the covariance, such that the result always has a value between -1 and 1.

The Pearson's correlation coefficient varies between -1 and +1 where:

- $r = 1$ means the data is perfectly linear with a positive slope (i.e., both variables tend to change in the same direction)
- $r = -1$ means the data is perfectly linear with a negative slope (i.e., both variables tend to change in different directions)
- $r = 0$ means there is no linear association
- $r > 0 < 5$ means there is a weak association
- $r > 5 < 8$ means there is a moderate association
- $r > 8$ means there is a strong association



Pearson r Formula

$$r = \frac{\sum (x_i - \bar{x}) (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

Here,

- r = correlation coefficient
- x_i = values of the x-variable in a sample
- \bar{x} = mean of the values of the x-variable
- y_i = values of the y-variable in a sample
- \bar{y} = mean of the values of the y-variable

4. What is scaling? Why is scaling performed? What is the difference between normalized scaling and standardized scaling?

What is Scaling?

It is a step of data Pre-Processing which is applied to independent variables to normalize the data within a particular range. It also helps in speeding up the calculations in an algorithm.

Why is it performed?

Most of the times, collected data set contains features highly varying in magnitudes, units and range. If scaling is not done then algorithm only takes magnitude in account and not units hence incorrect modelling. To solve this issue, we have to do scaling to bring all the variables to the same level of magnitude.

It is important to note that scaling just affects the coefficients and none of the other parameters like t-statistic, F-statistic, p-values, R-squared, etc.

Normalization/Min-Max Scaling:

It brings all of the data in the range of 0 and 1.

`sklearn.preprocessing.MinMaxScaler` helps to implement normalization in python.

$$\text{MinMax Scaling: } x = \frac{x - \min(x)}{\max(x) - \min(x)}$$

Standardization Scaling:

Standardization replaces the values by their Z scores. It brings all of the data into a standard normal distribution which has mean (μ) zero and standard deviation one (σ).

$$\text{Standardisation: } x = \frac{x - \text{mean}(x)}{\text{sd}(x)}$$

`sklearn.preprocessing.scale` helps to implement standardization in python.

One disadvantage of normalization over standardization is that it loses some information in the data, especially about outliers.

5. You might have observed that sometimes the value of VIF is infinite. Why does this happen?

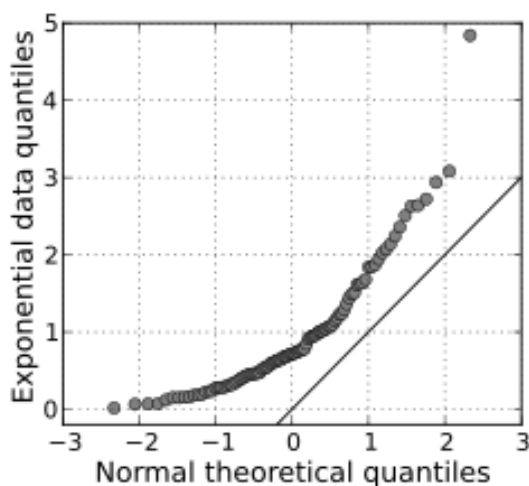
If there is perfect correlation, then $VIF = \infty$. This shows a perfect correlation between two independent variables. In the case of perfect correlation, we get $R^2 = 1$, which leads to $1/(1-R^2)$ infinity. To solve this problem we need to drop one of the variables from the dataset which is causing this perfect multicollinearity.

An infinite VIF value indicates that the corresponding variable may be expressed exactly by a linear combination of other variables (which show an infinite VIF as well).

6. What is a Q-Q plot? Explain the use and importance of a Q-Q plot in linear regression.

Q-Q Plots (Quantile-Quantile plots) are plots of two quantiles against each other. A quantile is a fraction where certain values fall below that quantile. For example, the median is a quantile where 50% of the data fall below that point and 50% lie above it. The purpose of Q-Q plots is to find out if two sets of data come from the same distribution. A 45 degree angle is plotted on the Q-Q plot; if the two data sets come from a common distribution, the points will fall on that reference line.

Q-Q plot showing the 45 degree reference line:



If the two distributions being compared are similar, the points in the Q-Q plot will approximately lie on the line $y = x$. If the distributions are linearly related, the points in the Q-Q plot will approximately lie on a line, but not necessarily on the line $y = x$. Q-Q plots can also be used as a graphical means of estimating parameters in a location-scale family of distributions.

A Q-Q plot is used to compare the shapes of distributions, providing a graphical view of how properties such as location, scale, and skewness are similar or different in the two distributions.