

How many array accesses does the following code fragment make as a function of  $N$ ?  
(Assume the compiler does not optimize away any array accesses in the innermost loop.)

☐  $\sim 3N^2$

☒  $\sim \frac{3}{2} N^2 \lg N$

☐  $\sim \frac{3}{2} N^3$

☐  $\sim 3N^3$

```
int sum = 0;
for (int i = 0; i < N; i++)
    for (int j = i+1; j < N; j++)
        for (int k = 1; k < N; k = k*2)
            if (a[i] + a[j] >= a[k]) sum++;
```

Loop 1

$i \Rightarrow 0, 1, 2, 3, \dots, N-1$  {  $N$  times }

Loop 2

$j \Rightarrow \{1, 2, \dots, N-1\}, \{2, 3, \dots, N-1\}, \{3, 4, \dots\}, \dots$

count of the loop 2

$$= (N-1) + (N-2) + (N-3) + \dots + 0$$

$$= \frac{(N)(N-1)}{2} \quad (2^{\text{nd}} \text{ loop runs } \frac{(N)(N-1)}{2} \text{ times})$$

Loop 3

$k \Rightarrow 1, 2, 4, 8, \dots$

if  $N=10$ ,  $k \Rightarrow 1, 2, 4, 8$ .

$N=33$   $k \Rightarrow 1, 2, 4, 8, 16, 32$  {  $\log_2(33-1)$  } + 1

$N=N$   $k \Rightarrow 1, 2, \dots$

count of loop 3 =  $\log_2(N-1) + 1 \approx \log_2(N)$

for large  $N$ .

Total order of growth =  $\frac{N^2 \log N}{2}$ , Array access =  $(3N^2 \log N)/2$