

Give the order of growth (as a function of N) of the running times of each of the following code fragment:

```
int sum = 0;
for (int i = 1; i < N; i *= 2)
    for (int j = 0; j < N; j++)
        sum++;
```

1st loop $i \Rightarrow 1, 2, 4, 8, \dots$

If $N = 3$, $i \Rightarrow 1, 2$; $\text{count}(i) = 2$

$N = 5$, $i \Rightarrow 1, 2, 4$; $\text{count}(i) = 3$

$N = 33$, $i \Rightarrow 1, 2, 4, 8, 16, 32$; $\text{count}(i) = 6$

If $N = 2^x + 1$, $i \Rightarrow 1, 2, 4, \dots, (N-1)$
 $\Rightarrow 1, 2, 4, \dots, 2^x$

The ~~add~~ count of $i \Rightarrow x + 1$

$\text{count}(i) =$

$x + 1$

\Rightarrow

$$2^x = (N-1)$$

$$x = \log(N-1)$$

$$x + 1 = 1 + \log(N-1)$$

for large (N)

$$(x+1) \approx \log N$$

$$\begin{cases} \text{for large } N \\ N-1 \approx N \\ 1 + \log N \\ \approx \log N \end{cases}$$

2nd loop executes N times.

$$\text{Total execution} = N * \log N \Rightarrow \underline{\underline{N \log N}}$$