5a) The outer loop "for i in range (0, n)" iterates n times from 0 to "n-1" $\longrightarrow O(n)$
The inner loop runs once when $i = 0$ and twice when $i = 1$ once $i = 2$, the loop stops running. This is regardless of the size of n.
Hence the combined time complexity is $O(n)$

5b) There is only one loop, during which n is operated upon until the resulting n is n-5 this constant subtraction ensures that the single loop is is completed in linear time regardless of
time complexity $\rightarrow O(n)$

5c) There is one loop however in this loop i is scaled $2(i)$ ~~
thus the iterations are completed in $O(\log n)$ times.
as the value of n is squared, the number of iterations doubles.
time complexity $\rightarrow O(\log n)$

5d) Here there are two loops, the outer loop runs n^2 times, while the in the inner loop, $j = j/2 \rightarrow \log(n)$ time complexity. operation.
Combined, this forms $O(n^2 \times \log n) \rightarrow O(n^2 \log n)$ time complexity

5e) With each iteration $i = i^2$. i grows exponentially rather than linearly so the iterations grow to become 2^{2^k} where k is the number of iterations.
so for the loop,

the loop stops when $2^{2^k} \geq n$
$\log 2^{2^k} \geq \log n$
$\log 2^k \geq \log(\log n)$

$k \geq \log(\log n)$
$\therefore$ time complexity $\rightarrow O(\log(\log n))$

4a) $f(n) = n\log n^3$ & $g(n) = n\log n$

if $f(n) \in \Theta(g(n))$ then there exists constants c_1 & c_2 such that

$$c_1\, g(n) \le f(n) \le c_2\, g(n)$$

let $c_1 = 1$ & $c_2 = 4$

$$n\log n \le 3n\log n \le 4n\log n \rightarrow \text{True}$$

$\therefore$ $f(n) \in \Theta(g(n))$.

4b) $f(n) = \frac{1}{3}n^2 + 10n - 2$ & $g(n) = n^2$

if $f(n) \in \Omega(g(n))$, then there exists constant "c" where $f(n) \geq c\,g(n)$ for all $n \geq K$

let $c = \frac{1}{3}n$

$\therefore$ $\frac{1}{3}n^2 + 10n - 2 \geq \frac{1}{3}n^2$

$$10n - 2 \geq 0 \rightarrow \text{True for } n \geq \tfrac{2}{5}$$

$\therefore$ $f(n) \in \Omega(g(n))$.

4c) $f(n) = n^3$ & $g(n) = 100n^2 + 10^6 n$

if $f(n) \in O(g(n))$ then there exists a constant, "c" for which $f(n) \le c\,g(n)$ for all $n \geq K$

$\therefore$ $\frac{f(n)}{g(n)} \le c$ as $n \rightarrow \infty$

$$\lim_{n\to\infty} \frac{n^3}{100n^2 + 10^6 n} = \infty$$

$\therefore$ $f(n)$ is not an element of $O(g(n))$.

4d) $f(n) = \sqrt{n^2+1}$ & $g(n) = n^2 + 3$

if $f(n) \in o(g(n))$ then; $f(n) < g(n)$

$$\frac{f(n)}{g(n)} < 1$$

as n tends to infinity; $\frac{f(n)}{g(n)}$ should tend to zero.

Test: $$\lim_{n \to \infty} \left[\frac{(n^2+1)^{1/2}}{n^2+3} \right] = \frac{(\infty)}{(\infty^2)} \simeq 0 \longrightarrow \underline{\underline{\text{TRUE}}}$$

$\therefore$ $\underline{\underline{f(n) \in o(g(n))}}$

4e) $f(n) = n^2 \sin(1/n) + 2n$ & $g(n) = n^2$

if $f(n) \in \omega(g(n))$ then. $f(n) > g(n)$

$$\therefore \lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$$

Test: $$\lim_{n \to \infty} \left[\frac{n^2(\sin 1/n) + 2n}{n^2} \right] = \infty \longrightarrow \text{FALSE}$$

$\therefore$

Sine is upperbound by 1

As $n \to \infty$ $\frac{2n}{n^2} \to 0$ & $\frac{n^2 \sin 1/n}{n^2}$ does not increase

$\therefore$ $f(n)$ is not $\in \omega(g(n))$