# Assignment 1
## DSECLZC416 - Mathematical Foundations for Data Science

**Instructions**

1. Assignments have to be handwritten and uploaded as a single pdf file with name BITSID.pdf

2. Assignments sent via email would not be accepted

3. Submissions beyond 1st of June, 17.00 hrs would not be graded

Answer all the questions (5 x 2 = 10 marks)

**Q1)** Let $B$ be a skew symmetric matrix with real entries.

1. Prove that $I - B$ is non singular

2. If $A = (I + B)(I - B)^{-1}$, then prove that $A^{-1} = A^T$

**Q2)** Let $\mathcal{M} = \{m_1, m_2, .....m_r\}$ and $\mathcal{N} = \{m_1, m_2, ...., m_r, v\}$ be two sets of vectors from the same vector space $V$ over a field $F$. Prove that span$\{\mathcal{M}\}$ = span$\{\mathcal{N}\}$ if and only if $v \in$ span$\{\mathcal{M}\}$.

**Q3)** If $T : V \rightarrow W$ is a linear transformation from a vector space $V$ to a vector space $W$ then prove that Rank $T$ + Nullity $T = \dim V$.

**Q4)** Find the eigenvalues and eigenvectors for the matrix $A_{n \times n}$ whose elements are given by $a_{ij} = \begin{cases} \alpha & \text{if } i = j \\ 1 & \text{if } |i - j| = 1 \\ 0 & \text{otherwise} \end{cases}$ , where $\alpha$ is a constant.

**Q5)** Find the singular value decomposition of $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \end{pmatrix}$ and determine the angle of rotation induced by $U$ and $V$. Also, write the rank 1 decomposition of $A$ in terms of the columns of $U$ and rows of $V$. Can we do dimensionality reduction in this case?