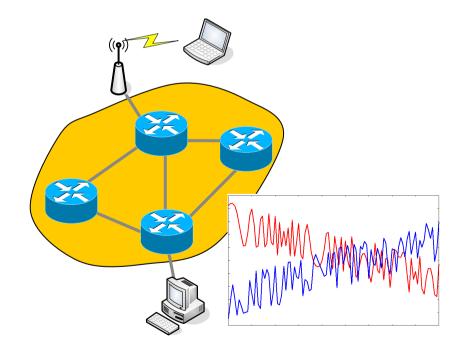


Chapter 2

Simulation Examples



2.1

Contents

- Simulation using Tables
- Simulation of Queueing Systems
- Examples
 - A Grocery
 - Call Center
 - Inventory System
- Appendix: Random Digits

Simulation using Tables

Simulation using a Table

- Introducing simulation by manually simulating on a table
 - Can be done via pen-and-paper or by using a spreadsheet

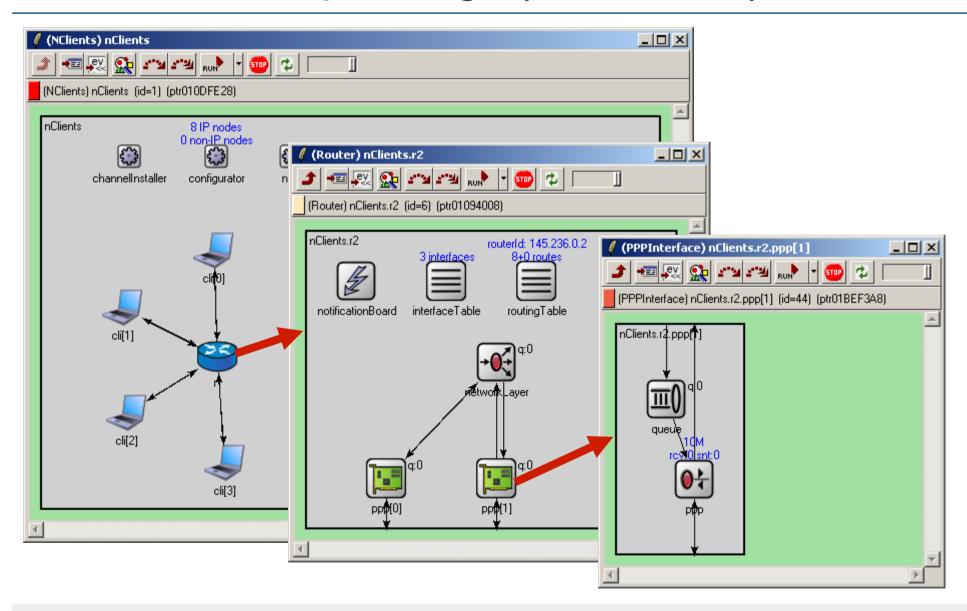
Repetition		Inputs						
i	x_{i1}	x_{i2}	•••	X _{ij}	•••	x_{ip}	y_i	Static meta data
1]]
2								Dynamic data
								during
•								simulation
•] run
n								

Simulation using a Table

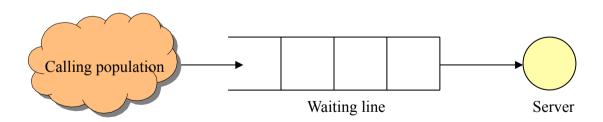
- Three steps
 - 1. Determine the characteristics of each input to the simulation.
 - 2. Construct a simulation table consisting of
 - $p \text{ inputs } x_{ij}$, j=1,2,...,p
 - one response y_i , i=1,2,...,n
 - 3. For each repetition i, generate a value for each of the p inputs x_{ij} and calculate the response y_i .

Repetition		Inputs						
i	x_{i1}	x_{i2}	•••	x _{ij}	•••	x_{ip}	y_i	
1								
2								
•								
n								



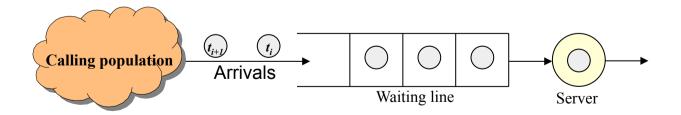


- A queueing system is described by
 - Calling population
 - Arrival rate
 - Service mechanism
 - System capacity
 - Queueing discipline

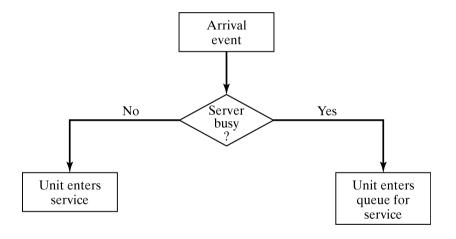


- Single server queue
 - Calling population is infinite
 - → Arrival rate does not change
 - Units are served according FIFO
 - Arrivals are defined by the distribution of the time between arrivals ⇒ inter-arrival time
 - Service times are according to a distribution
 - Arrival rate must be less than service rate ⇒ stable system
 - Otherwise waiting line will grow unbounded → unstable system

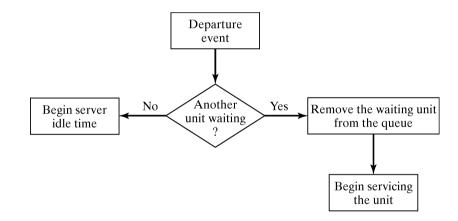
- Queueing system state
 - System
 - Server
 - Units (in queue or being served)
 - Clock
 - State of the system
 - Number of units in the system
 - Status of server (idle, busy)
 - Events
 - Arrival of a unit
 - Departure of a unit



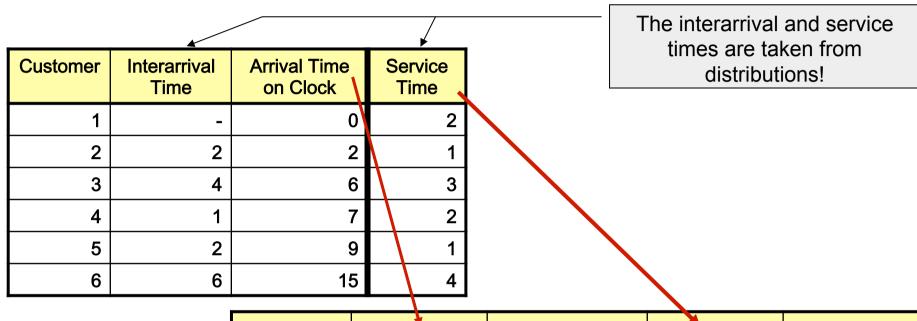
- Arrival Event
 - If server idle unit gets service, otherwise unit enters queue.



- Departure Event
 - If queue is not empty begin servicing next unit, otherwise server will be idle.



- How do events occur?
 - Events occur randomly
 - Interarrival times $\in \{1,...,6\}$
 - Service times $\in \{1,...,4\}$



The simulation run is build by meshing clock, arrival, and service times!

			Time ock]	Time Service Begins [Clock]	Service Time [Duration]	Time Service Ends [Clock]
-	$\overline{}$	1	0	0	2	2
	•	2	2	2	1	3
		3	6	6	3	9
		4	7	9	2	11
		5	9	11	1	12
		6	15	15	4	19

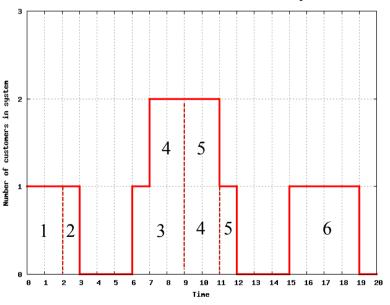
Chronological ordering of events

Clock Time	Customer Number	Event Type	Number of customers
0	1	Arrival	1
2	1	Departure	0
2	2	Arrival	1
3	2	Departure	0
6	3	Arrival	1
7	4	Arrival	2
9	3	Departure	1
9	5	Arrival	2
11	4	Departure	1
12	5	Departure	0
15	6	Arrival	1
19	6	Departure	0

Interesting observations

- Customer 1 is in the system at time 0
- Sometimes, there are no customers
- Sometimes, there are two customers
- Several events may occur at the same time

Number of customers in the system



Examples

Example 1: A Grocery



Example 1: A Grocery

Analysis of a small grocery store

One checkout counter

 Customers arrive at random times from {1,2,...,8} minutes

Service times vary from {1,2,...,6}
 minutes

Consider the system for 100 customers

- Problems/Simplifications
 - Sample size is too small to be able to draw reliable conclusions
 - Initial condition is not considered

Interarrival Time [minute]	Probability	Cumulative Probability
1	0.125	0.125
2	0.125	0.250
3	0.125	0.375
4	0.125	0.500
5	0.125	0.625
6	0.125	0.750
7	0.125	0.875
8	0.125	1.000

Service Time [minute]	Probability	Cumulative Probability
1	0.10	0.10
2	0.20	0.30
3	0.30	0.60
4	0.25	0.85
5	0.10	0.95
6	0.05	1.00

Example 1: A Grocery

Simulation run for 100 customers

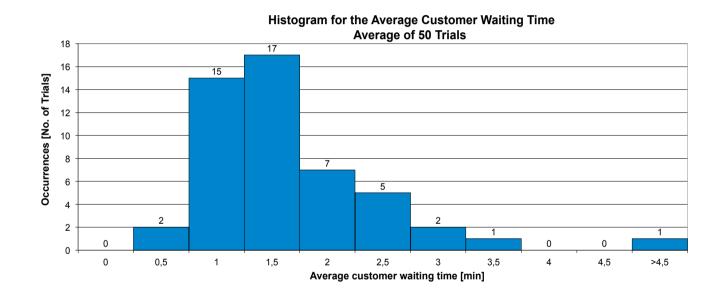
		Perfor	mance M	easure				
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	-	0	4	0	4	0	4	0
2	1	1	2	4	6	3	5	0
3	1	2	5	6	11	4	9	0
4	6	8	4	11	15	3	7	0
5	3	11	1	15	16	4	5	0
6	7	18	5	18	23	0	5	2
				•••				
100	5	415	2	416	418	1	3	0
Total	415		317			174	491	101

Example 1: A Grocery, Some statistics

Average waiting time	$\overline{w} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers}} = \frac{174}{100} = 1.74 \text{min}$
Probability that a customer has to wait	$p(\text{wait}) = \frac{\text{Number of customer who wait}}{\text{Number of customers}} = \frac{46}{100} = 0.46$
Proportion of server idle time	$p(\text{idle server}) = \frac{\sum \text{Idle time of server}}{\text{Simulation run time}} = \frac{101}{418} = 0.24$
Average service time	$\bar{s} = \frac{\sum \text{Service time}}{\text{Number of customers}} = \frac{317}{100} = 3.17 \text{ min}$
	$E(s) = \sum_{s=0}^{\infty} s \cdot p(s) = 0.1 \cdot 10 + 0.2 \cdot 20 + \dots + 0.05 \cdot 6 = 3.2 \text{min}$
Average time between arrivals	$\overline{\lambda} = \frac{\sum \text{Times between arrivals}}{\text{Number of arrivals - 1}} = \frac{415}{99} = 4.19 \text{min}$
	$E(\lambda) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{min}$
Average waiting time of those who wait	$\overline{w}_{waited} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers that wait}} = \frac{174}{54} = 3.22 \text{min}$
Average time a customer spends in system	$\bar{t} = \frac{\sum \text{Time customers spend in system}}{\text{Number of customers}} = \frac{491}{100} = 4.91 \text{min}$
	$\bar{t} = \bar{w} + \bar{s} = 1.74 + 3.17 = 4.91 \text{min}$

Example 1: A Grocery, Some statistics

- Interesting results for a manager, but
 - longer simulation run would increase the accuracy



- Some interpretations
 - Average waiting time is not high
 - Server has not undue amount of idle time, it is well loaded ;-)
 - Nearly half of the customers have to wait (46%)

Examples

Example 2: Call center





- Consider a Call Center where technical personnel take calls and provide service
- Two technical support people (2 server) exists
 - Able more experienced, provides service faster
 - Baker newbie, provides service slower
- Rule
 - Able gets call if both people are idle
 - Try other rules
 - Baker gets call if both are idle
 - Call is assigned randomly to Able and Baker
- Goal of study: Find out how well the current rule works

• Interarrival distribution of calls for technical support

Time between Arrivals [Minute]	Probability	Cumulative Probability	Random-Digit Assignment
1	0.25	0.25	01 - 25
2	0.40	0.65	26 - 65
3	0.20	0.85	66 - 85
4	0.15	1.00	86 - 00

Service time distribution of Baker

Service Time [Minute]	J		Random-Digit Assignment
3	0.35	0.35	01 - 35
4	0.25	0.60	36 - 60
5	0.20	0.80	61 - 80
6	0.20	1.00	81 - 00

Service time distribution of Able

Service Time [Minute]	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01 - 30
3	0.28	0.58	31 - 58
4	0.25	0.83	59 - 83
5	0.17	1.00	84 - 00

Simulation proceeds as follows

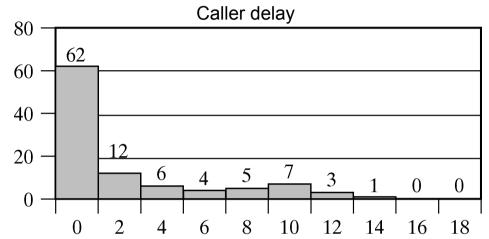
- Step 1:
 - For Caller k, generate an interarrival time A_k . Add it to the previous arrival time T_{k-1} to get arrival time of Caller k as $T_k = T_{k-1} + A_k$
- Step 2:
 - If Able is idle, Caller k begins service with Able at the current time T_{now}
 - Able's service completion time $T_{fin,A}$ is given by $T_{fin,A} = T_{now} + T_{svc,A}$ where $T_{svc,A}$ is the service time generated from Able's service time distribution. Caller k's waiting time is $T_{wait} = 0$.
 - Caller k's time in system, T_{sys} , is given by $T_{sys} = T_{fin,A} T_k$
 - If Able is busy and Baker is idle, Caller begins with Baker. The remainder is in analogous.
- Step 3:
 - If Able and Baker are both busy, then calculate the time at which the first one becomes available, as follows: $T_{beg} = \min(T_{fin,A}, T_{fin,B})$
 - Caller k begins service at T_{beg} . When service for Caller k begins, set $T_{now} = T_{beg}$.
 - Compute $T_{fin,A}$ or $T_{fin,B}$ as in Step 2.
 - Caller k's time in system is $T_{sys} = T_{fin,A} T_k$ or $T_{sys} = T_{fin,B} T_k$

Simulation run for 100 calls

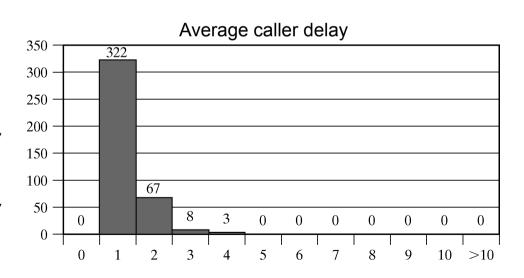
Caller Nr.	Interarrival Time	Arrival Time	When Able Avail.	When Baker Avail.	Server Chosen	Service Time	Time Service Begins	Able's Service Compl. Time	Baker's Service Compl. Time	Caller Delay	Time in System
1	-	0	0	0	Able	2	0	2		0	2
2	2	2	2	0	Able	2	2	4		0	2
3	4	6	4	0	Able	2	6	8		0	2
4	2	8	8	0	Able	4	8	12		0	4
5	1	9	12	0	Baker	3	9		12	0	3
	•••										
100	1	219	221	219	Baker	4	219			0	4
Total										211	564

Example 2: Call Center Problem, Some statistics

- One simulation trial of 100 caller
 - 62% of callers had no delay
 - 12% of callers had a delay of 1-2 minutes



- 400 simulation trials of 100 caller
 - 80.5% of callers had delay up to 1 minute
 - 19.5% of callers had delay more than 1 minute

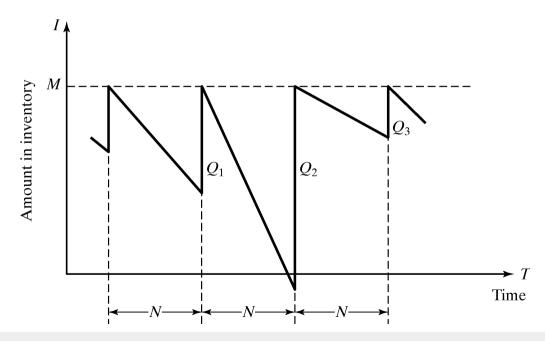


Examples

Example 3: Inventory System

Example 3: Inventory System

- Important class of simulation problems: inventory systems
- Performance measure: Total cost (or total profit)
- Parameters
 - N Review period length
 - M Standard inventory level
 - Q_i Quantity of order i to fill up to M



Example 3: Inventory System

- Events in an (M, N) inventory system are
 - Demand for items
 - Review of the inventory
 - Receipt of an order at the end of each review
- To avoid shortages, a buffer stock is needed
- Cost of stock
 - Storage space
 - Guards

Appendix

Random digits

Appendix: Random Digits

- Producing random numbers from random digits
 - Select randomly a number,
 e.g.

• One digit: 0.9

• Two digits: 0.19

• Three digits: 0.219

- Proceed in a systematic direction, e.g.
 - first down then right
 - first up then left

04	737	08225	35614	24826	88319	05595	58701	57365	74759
	259	85982	13296	89326	74863	99986	68558	06391	50248
	856	14016	18527	11634	96908	52146	53496	51730	03500
	612	54714	46783	61934	30258	61674	07471	67566	31635
30	712	58582	05704	23172	86689	94834	99057	55832	21012
696	607	24145	43886	86477	05317	30445	33456	34029	09603
37	792	27282	94107	41967	21425	04743	42822	28111	09757
014	488	56680	73847	64930	11108	44834	45390	86043	23973
663	248	97697	38244	50918	55441	51217	54786	04940	50807
514	453	03462	61157	65366	61130	26204	15016	85665	97714
92	168	82530	19271	86999	96499	12765	20926	25282	39119
	463	07331	54590	00546	03337	41583	46439	40173	46455
	097	78780	04210	87084	44484	75377	57753	41415	09890
	400	45972	44111	99708	45935	03694	81421	60170	58457
	554	13863	88239	91624	00022	40471	78462	96265	55360
74.))4	13003	00239	91024	00022	40471	10402	90203	33300
31:	567	53597	08490	73544	72573	30961	12282	97033	13676
078	321	24759	47266	21747	72496	77755	50391	59554	31177
090	056	10709	69314	11449	40531	02917	95878	74587	60906
100	922	37025	80731	26179	16039	01518	82697	73227	13160
299	923	02570	80164	36108	73689	26342	35712	49137	13482
296	502	29464	99219	20308	82109	03898	82072	85199	13103
	135	94661	87724	88187	62191	70607	63099	40494	49069
	926	34092	34334	55064	43152	01610	03126	47312	59578
	039	19212	59160	83537	54414	19856	90527	21756	64783
660		38480	74636	45095	86576	79337	39578	40851	53503
000	,,,,	30400	74050	T3073	00370	77331	37310	40051	33303
781	166	82521	79261	12570	10930	47564	77869	16480	43972
946	572	07912	26153	10531	12715	63142	88937	94466	31388
564	106	70023	27734	22254	27685	67518	63966	33203	70803
677	726	57805	94264	77009	08682	18784	47554	59869	66320
075	516	45979	76735	46509	17696	67177	92600	55572	17245
430	070	22671	00152	81326	89428	16368	57659	79424	57604
369	917	60370	80812	87225	02850	47118	23790	55043	75117
039	919	82922	02312	31106	44335	05573	17470	25900	91080
467		22558	64303	78804	05762	70650	56117	06707	90035
161		61281	86823	20286	14025	24909	38391	12183	89393
744	= 4 1	75000	90660	97690	77750	∠00 ₹ 1	55202	05662	99226
745		75808	89669	87680	72758 42986	60851	55292	95663	88326
829		31285	01850	72550		57518	01159	01786	98145
313		26809	77258	99360	92362	21979	41319	75739	98082
171		75522	15687	07161	99745	48767	03121	20046	28013
004	604	88068	68631	98745	97810	35886	14497	90230	69264

Summary

- This chapter introduced simulation concepts by means of examples
- Example simulations were performed on a table manually
 - Use a spreadsheet for large experiments (Excel, OpenOffice)
 - Input data is important
 - Random variables can be used
 - Output analysis important and difficult
 - The used tables were of ad hoc, a more methodic approach is needed