CS5691 - ASSIGNMENT 1

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Motivation: The task of this assignment is to decompose a 256x256 gray scale image using SVD and EVD. The main agenda behind this assignment is to get us familiar with matrix decompositions which can aid us hugely in dimensionality reduction of extremely large data sets. Using this we would know which features are contributing majorly towards the final result or change. These techniques can also be used to use less storage which would enable faster data transfer(image transfers etc) without much deviation from the original data.

Through this assignment we are required to compare both techniques and analyse the pros and cons of both techniques and to come to a conclusion on which technique can be used in a generalised way on a wide range of datasets.

Challenges: Some of the challenges faced during solving the assignment was the datatype of values in image matrix needed to be type casted to np.int64, or else it gave a large number of negative values while computing Σ^2 in SVD. But even after converting to int64 one value was negative in Σ^2, but it was too negligible value andd was the smallest in the array so either taking it 0 or positive, value didn't really effect the result.

Optimisations to consider: As the value AA^t in svd calculation was symmetric, usage of eigh(special function for hermitian matrices) instead of regular eig optimises the performance. Even though it is negligible here, it can be kept in my while dealing with larger datasets.

Description of My program: My code file "CS19B028.py" on running takes an input of image "39.jpg" and creates 2 folders *evd_images* and *svd_images* which contains the reconstructed imaged of evd and svd respectively for all values of k from 0 to 256. This way we can gradually observe changes for all images carefully.

It also creates 2 more folders $ErrorImages\_EVD$ and $ErrorImages\_SVD$ which contains compaision between reconstructed image and error image for values of k which equals powers of 2.

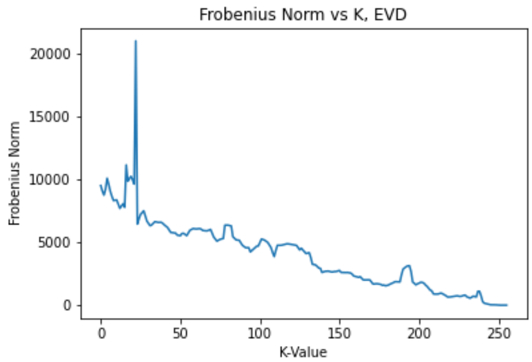
It also creates 2 images name "*evd_graph*" and "*svd_graph*" which contains the graphs of Frobenius norm vs K for EVD and SVD respectively.

EVD: Any Square Matrix A can be written in the form of

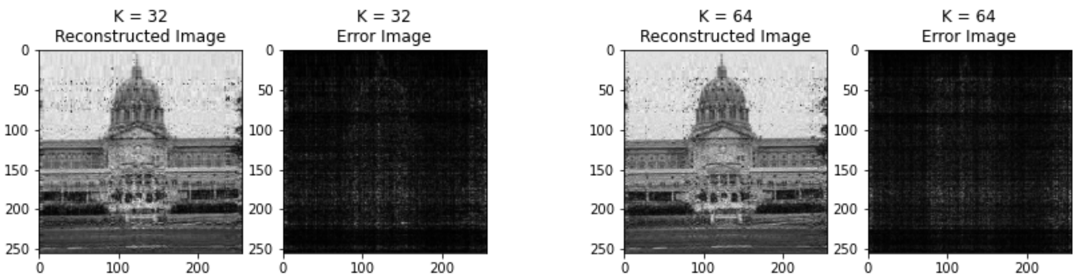
$$A = X\lambda X^{-1}$$

Where X is a square matrix with each of its is columns being the Eigen Vectors of Matrix A and corresponding diagonal elements of λ being the Eigen value of the respective Eigen vectors.

Experimental Results (EVD): In most of the cases on increasing the K-value (rank approximation of the matrix), the image tends to become more detailed and contrast increases. It loses blurriness and becomes clearer. However this trend is not always followed but is just the overall scenario. There appears to be some values of K at which an increase in value of K causes the image to become more blurry and deviated from the original image. From the graph(for image 39) below of Frobenius norm vs K of the EVD images it is pretty clear that the trend is not monotonic but a more common one.



Inferences: In most of the cases the changed brought by increasing/decreasing the value of K are sharp and sudden but do not follow a generalised trend, So accurately predicting/generalising the changes brought by EVD for a wide range of images is not always possible. So this may not be suggestible as a general technique for dimensional reduction of images. However it is known fact that when the k value tends to maximum rank/order of the matrix then the reconstructed image tends to original image and Frobenius norm value tends to 0.



The changes between image of k with 32 and image of k with 64 is negligible and both images are still blurry, So from this it is clear that in EVD to observe considerable changes in image after some point the change in the value of K must be massive, which defeats the puspose of dimensional reduction. In EVD even required contrast is attained early sometimes, the details are missing and are changing right until reaching the maximum value of K. EVD can only be performed for square matrices which makes it uselss if the dataset is not comprised of square matrices.

SVD : For any Matrix A (need not be square) we can write it as

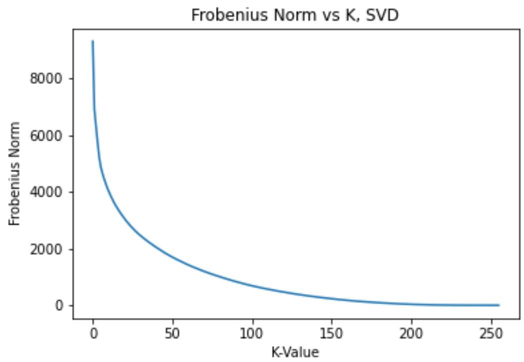
$$A = U\Sigma V^t$$

Where Σ is a diagonal matrix and U and V are unitary matrices.

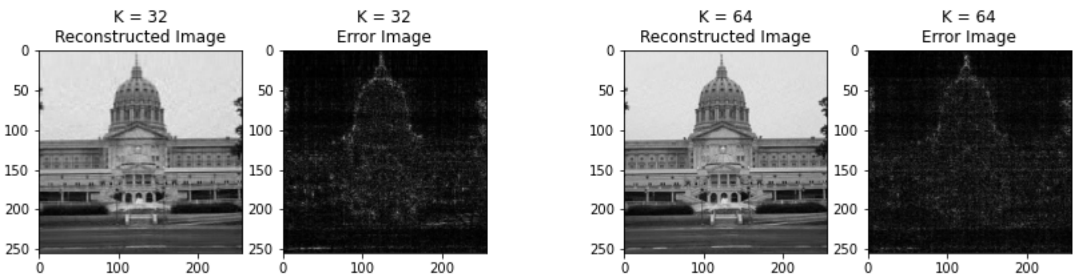
$$A^t A = V\Sigma^2 V^t$$

For any matrix A, AA^t becomes a symmetric matrix which implies that it is also a square matrix.

Experimental Results (SVD): In SVD the changes are rather smooth than EVD and are monotonic. Increase in value of K ensure the decrease in Frobenius norm value, and the changes are notable seen only up to a certain point and after that the Frobenius norm value tends to zero like shown in the graph below.



Inferences: The changes in slight change of K in SVD are always continuous unlike those of EVD. On increase in value of K image always becomes clearer but after some certain point the changes become negligible and image almost resembles the original image, So with SVD we can generalise a limit for K and SVD can be used as a general method of dimensionality reduction for almost every image. For many image K of range 40 to 60 was an ideal value and image almost resembled the original image with out much change.



Even though the change from $k = 32$ to $k = 64$ is only small, but the image is already clear resembles the original image for $k = 32$ so considerable change will not be observed. In SVD the only thing changing after certain point is the contrast but all the details can be clearly observed right from a lower value of K itself. SVD is also advantageous as it can clearly be used for matrices of all orders, and the matrix doesnt need to be a square matrix.