5. Given that data is from the poisson distribution with Parameter λ. $+a$ $-b$

for estimating λ using bayesian inference, we are assuming an exponential prior:

$$f_{prior}(x) = \lambda \cdot e^{-\lambda x} \quad \forall\ x \geq 0.$$

$$P_{dat}(x) = \frac{\lambda^{x} \cdot e^{-\lambda}}{x!}$$

but we know that exponential distribution is Special case of the Gamama distribution & Gamma distribution is Conjugate prior of poisson distribution. (by conjugate prior property ~~are~~ even after k numbee of iterations, the ~~graph~~ ~~will~~ posterior distribution of λ will be in gamma distribution.

let $f_{prior} = \lambda \cdot e^{-\lambda x}$ ~~Exp(~~ and given intial mean as β. so $\lambda = 1/\beta$.

$$f_{prior} = 1/\beta \cdot e^{-x/\beta} = \text{Exp}(1/\beta) \quad \text{— (1)}$$

gamma distribution is $f(a, b) = \dfrac{b^{a} \cdot x^{a-1} \cdot e^{-bx}}{\Gamma(a)}$

So the equation (1) would be in gamma.

$$\text{Gamma}(1, 1/\beta) = \frac{1/\beta \cdot e^{-x/\beta}}{1}.$$

so intial a & b values are 1, $1/\beta$.

using bayesian inference.

$f_{Posterior}(\lambda/D) \propto$ Likelyhood $\times f_{prior}(\lambda)$

$$f_{post}(\lambda) \propto \left(\prod_{i=1}^{n} \frac{\lambda^{x_i} \cdot e^{-\lambda}}{x_i!}\right) \times \frac{b^a \cdot \lambda^{a-1} \cdot e^{-b\lambda}}{\Gamma(a)}$$

removing terms which are not dependant on λ (as we have proportionality.

$$f_{post}(\lambda) \propto \lambda^{\sum x_i} \cdot e^{-n\lambda} \cdot \lambda^{a-1} \cdot e^{-b\lambda}$$

$$f_{post}(\lambda) \propto \lambda^{\sum x_i + a - 1} \cdot e^{-\lambda(n+b)}$$

this is ~~proportional~~ a ~~to~~ Gamma distribution

with parameters $\sum x_i + a$ $n+b$.

$$\Rightarrow \text{Gamma}(\sum x_i + a, n+b)$$

f_{prior} = Gamma (a, b)

$f_{posterior}$ = Gamma $(\sum x_i + a, n+b)$

// Ans

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finding constant here.

$$l = \Sigma x_i + a$$
$$m = n + b$$

$$f_{post}(\lambda) = c \cdot \lambda^{\cancel{\Sigma x_i + a}\, l-1} \cdot e^{-\lambda(m)}$$

$$\int_0^\infty f(\lambda) \cdot d\lambda = 1$$

$$\int_0^\infty c \cdot \lambda^{l-1} \cdot e^{-\lambda m} = 1$$

$\downarrow$

this value will be equal to.

$$\cancel{}\; c \cdot m^{-l} \cdot \Gamma(l) = 1$$

$$c = \frac{m^l}{\Gamma(l)}$$

$$f_{post} = \frac{m^l}{\Gamma(l)} \cdot \lambda^{l-1} \cdot e^{-\lambda m}.$$

$$= \text{Gamma}(l, m)$$