Finite Word Length Effects in DSP Systems (9 Marks)

1. Introduction (2 Marks)
Digital Signal Processing (DSP) systems use digital hardware such as microprocessors or DSP processors to represent and manipulate signals using binary numbers.
However, digital hardware has finite word length, meaning the number of bits available to represent numbers is limited (e.g., 16-bit, 32-bit). Hence, numerical representations, arithmetic operations, and coefficients are approximate.
These limitations introduce various errors, collectively called finite word length effects.
These effects impact accuracy of signals, stability of filters, and output noise and distortion.

2. Fixed-Point and Floating-Point DSP Arithmetic (2 Marks)
(a) Fixed-Point Representation
Numbers are represented with a fixed number of bits for integer and fractional parts. Example: 1 sign bit + 15 magnitude bits in a 16-bit DSP.
Range is limited, and precision is fixed. Common in low-cost DSPs due to speed and simplicity.
Issues: Quantization errors in representation and overflow or underflow during arithmetic operations.

(b) Floating-Point Representation
Number represented as $x = m \times 2^e$, where m = mantissa, e = exponent.
Offers larger dynamic range and better precision. Used in high-performance DSPs like TMS320C67xx.
Advantages: Reduced quantization error and no overflow for wide range of values.
Disadvantages: More complex hardware, higher cost and power consumption.

3. ADC Quantization Noise (1.5 Marks)
When an ADC converts an analog signal into digital form, the continuous amplitude values are quantized into discrete levels, introducing quantization error or quantization noise.
If step size is Δ, then quantization error e(n) lies between $-\Delta/2 \le e(n) \le \Delta/2$.
Quantization Noise Power: $P_q = \Delta^2/12$
SQNR: SQNR = 6.02N + 1.76 dB, where N = number of bits in ADC.
Higher bit ADC $\Rightarrow$ smaller $\Delta \Rightarrow$ lower quantization noise.

4. Finite Word Length Effects in IIR Digital Filters (2 Marks)
IIR filters are sensitive to coefficient accuracy because their poles determine stability.
(a) Coefficient Quantization Errors:
In practical systems, filter coefficients cannot be represented with infinite precision, they are quantized to a finite number of bits.
Effect: Quantization changes pole and zero locations slightly. Small changes can cause stability problems or distorted frequency response.
Example: If ideal pole is at 0.95, quantized pole may shift to 0.97.
Thus, coefficient quantization affects: Stability and filter magnitude/phase characteristics.

5. Finite Word Length Effects in FFT Algorithms (1.5 Marks)
Each arithmetic operation in FFT (multiplication/addition) is performed with finite precision.
Intermediate results are rounded or truncated, introducing round-off errors.
Round-off Error Characteristics: Random in nature, acts like additive white noise, increases with number of FFT stages (log■N).
Mean square value of round-off noise $\propto 1/(12 \times 2^{2B})$, where B = number of bits used.
Impact: Degraded amplitude accuracy and small spurious spectral components.

Additional 7-Mark Questions

Q1. Illustrate the quantization noise in ADC. (7 Marks)
Quantization noise arises during analog-to-digital conversion when continuous input values are mapped to discrete levels. The difference between actual and quantized values is quantization error.
For a step size Δ, error lies in range ($-\Delta/2$, $+\Delta/2$). The power of quantization noise is $\Delta^2/12$.
Increasing number of bits reduces step size, hence reduces noise. The SQNR formula, SQNR = 6.02N + 1.76 dB, shows that each additional bit improves signal quality by about 6 dB.

Q2. List out the advantages and disadvantages of floating-point DSP processors. (7 Marks)
Advantages:
1. Large dynamic range
2. High precision calculations
3. Reduced risk of overflow/underflow
4. Suitable for complex algorithms

Disadvantages:
1. Higher hardware complexity
2. More expensive
3. Higher power consumption
4. Slower than fixed-point for simple operations

Q3. Explain the usage of a DSP Processor for any two day-to-day applications. (7 Marks)
(a) Audio Processing: DSPs are used in music players and mobile phones for noise cancellation, equalization, and sound enhancement.
(b) Communication Systems: Used in mobile base stations and modems for filtering, modulation, and error correction.

Q4. Explain the effects of coefficient quantization in IIR and FIR filters. (7 Marks)
In IIR filters, coefficient quantization shifts pole locations, potentially causing instability or distortion.
In FIR filters, quantization mainly affects amplitude response, since FIRs are inherently stable.
Therefore, quantization leads to minor ripple in passband/stopband but no instability.

Q5. Write a short note on truncation and rounding with examples. (7 Marks)
Truncation: Removing least significant bits (LSBs) without changing remaining bits. Example: 0.101110 truncated to 0.1011.
Rounding: Adjusting number to nearest representable value. Example: 0.101110 rounded to 0.1100.
Effect: Rounding gives smaller error than truncation; both cause quantization noise.