Assignment 4 Report: Perceptron Learning Algorithms

MATH/CSCI 485

Assignment: Implementation of Heuristic and Gradient Descent Perceptron
Dataset Used: data.csv (2D linearly separable binary classification)

Introduction

This report details the development and evaluation of two perceptron-based learning algorithms: the heuristic approach of perceptron and a perceptron with gradient descent approach through classification thresholds. Both models were trained on a 2D binary classification dataset with three different learning rates (0.01, 0.1, and 1.0). The objective was to understand how learning rate affects convergence, model accuracy, and decision boundary behavior. Each algorithm's performance was assessed through accuracy, weight evolution, and log loss curves.

Data Loading

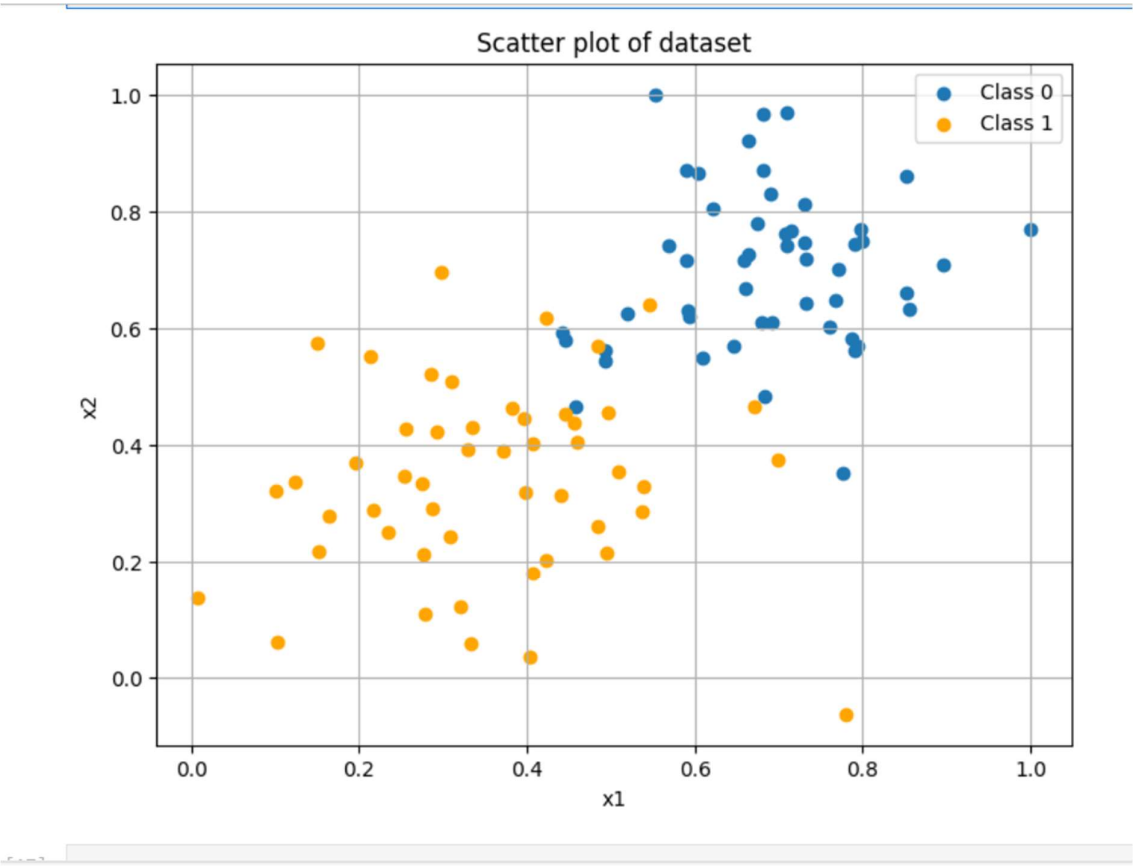
The dataset was successfully loaded using pandas. It contains two input features (x1, x2) and a binary label y (0 or 1). Visualization showed clear linear separability:

- Class 0: blue points
- Class 1: orange points

This setup is ideal for experimenting with simple linear classifiers such as perceptrons.

```
[1]: import numpy as np
     import pandas as pd
     import matplotlib.pyplot as plt

     df = pd.read_csv("data.csv", header=None)
     df.columns = ["x1", "x2", "y"]
     X = df[["x1", "x2"]].values
     y = df["y"].values

     plt.figure(figsize=(8, 6))
     plt.scatter(X[y==0][:, 0], X[y==0][:, 1], color='tab:blue', label='Class 0')
     plt.scatter(X[y==1][:, 0], X[y==1][:, 1], color='orange', label='Class 1')
     plt.xlabel("x1")
     plt.ylabel("x2")
     plt.title("Scatter plot of dataset")
     plt.legend()
     plt.grid(True)
     plt.show()
```

Output:



Part 1: Heuristic Perceptron

Algorithm Details

This implementation uses the classic perceptron update rule based on gradient descent,

Prediction: $\hat{y} = \text{sigmoid}(w \cdot x + b)$

- Error: $e = y - \hat{y}$
- Weight update: $w = w + lr * e * x$
- Bias update: $b = b + lr * e$

This approach allows for smooth, continuous updates and can be viewed as logistic regression with online training.

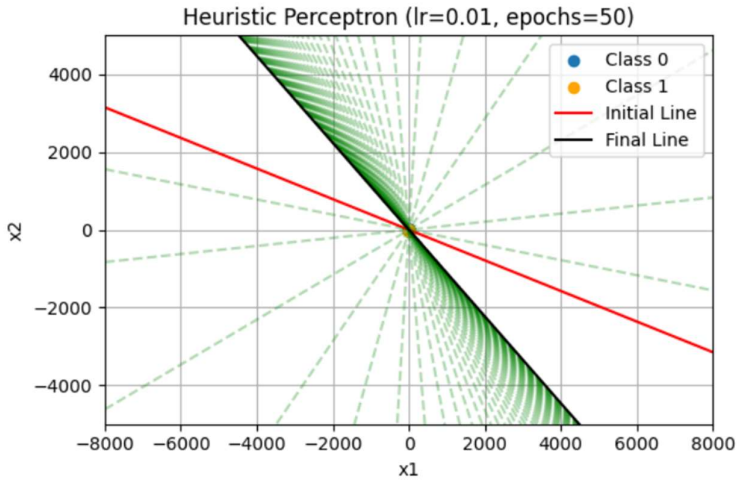
```
[17]: def heuristic_with_output(X, y, lr=1.0, epochs=50):
          np.random.seed(42)
          w = np.random.rand(2)
          b = np.random.rand(1)[0]

          plt.figure(figsize=(6, 4))
          plt.scatter(X[y == 0][:, 0], X[y == 0][:, 1], color='tab:blue', label='Class 0')
          plt.scatter(X[y == 1][:, 0], X[y == 1][:, 1], color='orange', label='Class 1')

          x_vals = np.linspace(-8000, 8000, 100)
          y_vals = -(w[0] * x_vals + b) / w[1]
          plt.plot(x_vals, y_vals, 'r-', label='Initial Line')

          for epoch in range(epochs):
              for xi, yi in zip(X, y):
                  z = np.dot(w, xi) + b
                  y_hat = sigmoid(z)
                  error = yi - y_hat
                  b += lr * error
                  w += lr * error * xi

              if epoch < epochs - 1:
                  y_vals = -(w[0] * x_vals + b) / w[1]
                  plt.plot(x_vals, y_vals, 'g--', alpha=0.3)

          y_vals = -(w[0] * x_vals + b) / w[1]
          plt.plot(x_vals, y_vals, 'k-', label='Final Line')

          plt.title(f"Heuristic Perceptron (lr={lr}, epochs={epochs})")
          plt.xlabel("x1")
          plt.ylabel("x2")
          plt.xlim(-8000, 8000)
          plt.ylim(-5000, 5000)
          plt.legend()
          plt.grid(True)
          plt.tight_layout()
          plt.show()

          z_all = np.dot(X, w) + b
          predictions = (sigmoid(z_all) >= 0.5).astype(int)
          accuracy = (predictions == y).mean() * 100

          print(f"Training with learning rate: {lr}")
          print(f"Iterations to converge: {epochs}")
```

```
    print(f"Training with learning rate: {lr}")
    print(f"Iterations to converge: {epochs}")
    print(f"Final weights: {w}")
    print(f"Final bias: {b}")
    print(f"Accuracy: {accuracy:.2f}%\n")

# With multiple learning rates
for learning_rate in [0.01, 0.1, 1.0]:
    heuristic_with_output(X, y, lr=learning_rate, epochs=50)
```

Learning Rate Experiments

LR = 0.01

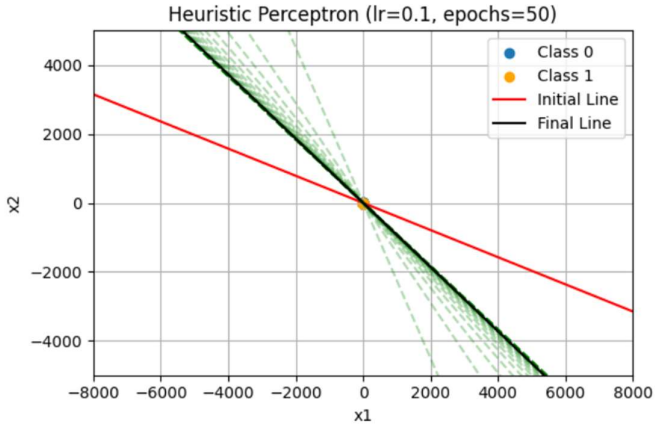
- **Accuracy:** 93.00%
- **Weights & Bias:** Small in magnitude
- **Observation:**
    - The model converged gradually
    - Decision boundary evolved slowly
    - Small updates led to stable, smooth loss reduction




```
Training with learning rate: 0.01
Iterations to converge: 50
Final weights: [-2.17297082 -1.94522975]
Final bias: 1.8630456324635594
Accuracy: 93.00%
```

LR = 0.1

- **Accuracy:** 93.00%
- **Observation:**
  - Balanced speed and stability
  - The boundary shifted significantly in early epochs and settled smoothly
  - Produced the cleanest convergence of all settings

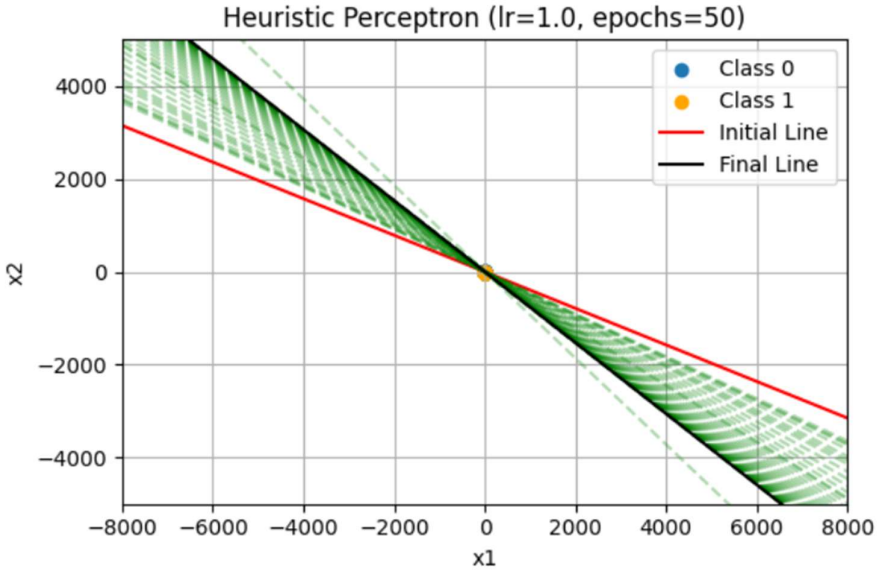



```
Training with learning rate: 0.1
Iterations to converge: 50
Final weights: [-7.18306184 -7.70666596]
Final bias: 7.2470943416691505
Accuracy: 93.00%
```

LR = 1.0

- **Accuracy:** 91.00%

- **Observation:**
    - Updates were very large
    - Boundary changes were abrupt
    - The model converged, but overshooting may have reduced final accuracy




```
Training with learning rate: 1.0
Iterations to converge: 50
Final weights: [-13.63492232 -17.82248411]
Final bias: 13.914486261894538
Accuracy: 91.00%
```

Part 2: Gradient Descent Perceptron

Algorithm Details

This approach uses discrete updates based on classification:

- Compute $\hat{y} = \text{sigmoid}(w \cdot x + b)$
- Convert to class: $\hat{y}\_class = 1$ if $\hat{y} \geq 0.5$ else 0
- If misclassified:
    - If $\hat{y}\_class = 0$ and true label is 1:
        - $w = w + lr * x$, $b = b + lr$
    - If $\hat{y}\_class = 1$ and true label is 0:
        - $w = w - lr * x$, $b = b - lr$

[23]:

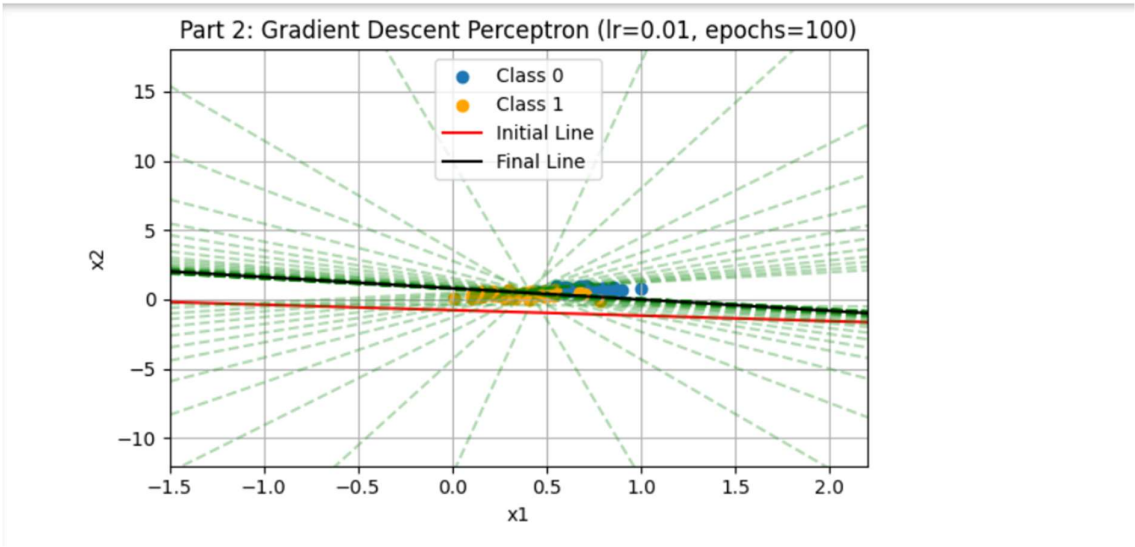
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Sigmoid function
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

# Log loss function
def log_loss(y_true, y_pred):
    eps = 1e-15
    y_pred = np.clip(y_pred, eps, 1 - eps)
    return -np.mean(y_true * np.log(y_pred) + (1 - y_true) * np.log(1 - y_pred))

def gradient_descent(X, y, lr=0.1, epochs=100):
    np.random.seed(42)
    w = np.random.rand(2)
    b = np.random.rand(1)[0]
    log_losses = []

    plt.figure(figsize=(6, 4))
    plt.scatter(X[y == 0][:, 0], X[y == 0][:, 1], color='tab:blue', label='Class 0')
    plt.scatter(X[y == 1][:, 0], X[y == 1][:, 1], color='orange', label='Class 1')
    x_vals = np.linspace(-1.5, 2.2, 100)
    y_vals = -(w[0] * x_vals + b) / w[1]
    plt.plot(x_vals, y_vals, 'r-', label='Initial Line')

    for epoch in range(epochs):
        for xi, yi in zip(X, y):
            z = np.dot(w, xi) + b
            y_hat = sigmoid(z)
            pred_class = 1 if y_hat >= 0.5 else 0

            if pred_class != yi:
                if pred_class == 0:
                    b += lr
                    w += lr * xi
                else:
                    b -= lr
                    w -= lr * xi

        if epoch < epochs - 1:
            y_vals = -(w[0] * x_vals + b) / w[1]
            plt.plot(x_vals, y_vals, 'g--', alpha=0.3)
```

```
        if epoch % 10 == 0:
            preds = sigmoid(np.dot(X, w) + b)
            loss = log_loss(y, preds)
            log_losses.append((epoch, loss))

    y_vals = -(w[0] * x_vals + b) / w[1]
    plt.plot(x_vals, y_vals, 'k-', label='Final Line')

    plt.title(f" Gradient Descent Perceptron (lr={lr}, epochs={epochs})")
    plt.xlabel("x1")
    plt.ylabel("x2")
    plt.xlim(-1.5, 2.2)
    plt.ylim(-12, 18)
    plt.legend()
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    plt.figure(figsize=(6, 4))
    epochs_, losses = zip(*log_losses)
    plt.plot(epochs_, losses, marker='o', color='orange')
    plt.xlabel("Epoch")
    plt.ylabel("Log Loss")
    plt.title("Log Loss (Every 10 Epochs)")
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    predictions = (sigmoid(np.dot(X, w) + b) >= 0.5).astype(int)
    accuracy = np.mean(predictions == y) * 100

    print(f"Training with learning rate: {lr}")
    print(f"Iterations to converge: {epochs}")
    print(f"Final weights: {w}")
    print(f"Final bias: {b}")
    print(f"Accuracy: {accuracy:.2f}%\n")

for lr in [0.01, 0.1, 1.0]:
    part2_gradient_descent(X, y, lr=lr, epochs=100)
```

Learning Rate Experiments

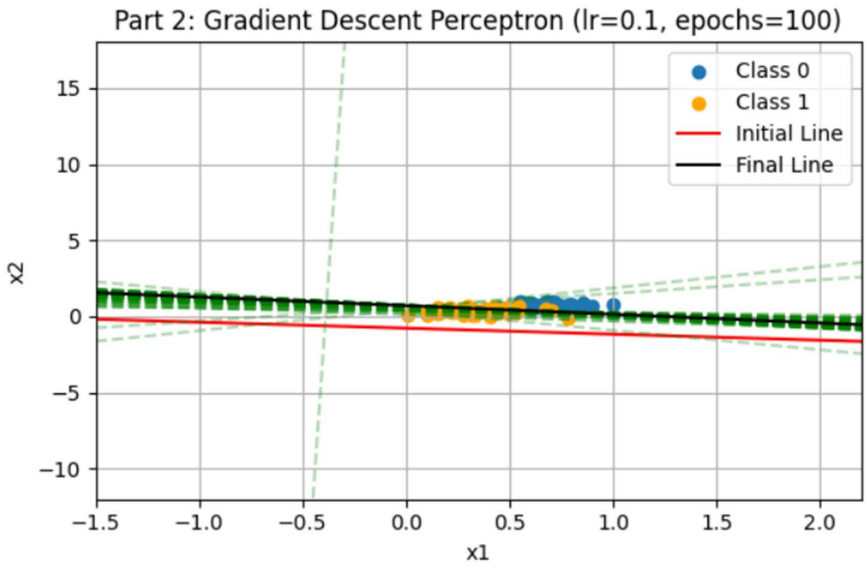
LR = 0.01

- **Accuracy:** 91.00%
- **Observation:**
    - Stable and very gradual improvements
    - Loss decreased smoothly with minimal oscillation
    - Decision boundaries shifted gently



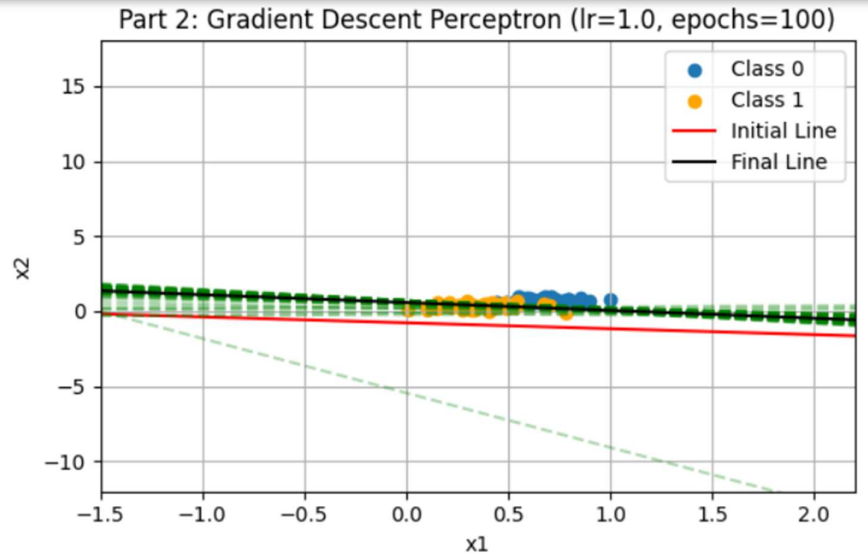
LR = 0.1

- **Accuracy:** 93.00%
- **Observation:**
    - Quick convergence with consistent updates
    - Produced the most accurate and balanced result
    - Loss dropped quickly and stabilized early



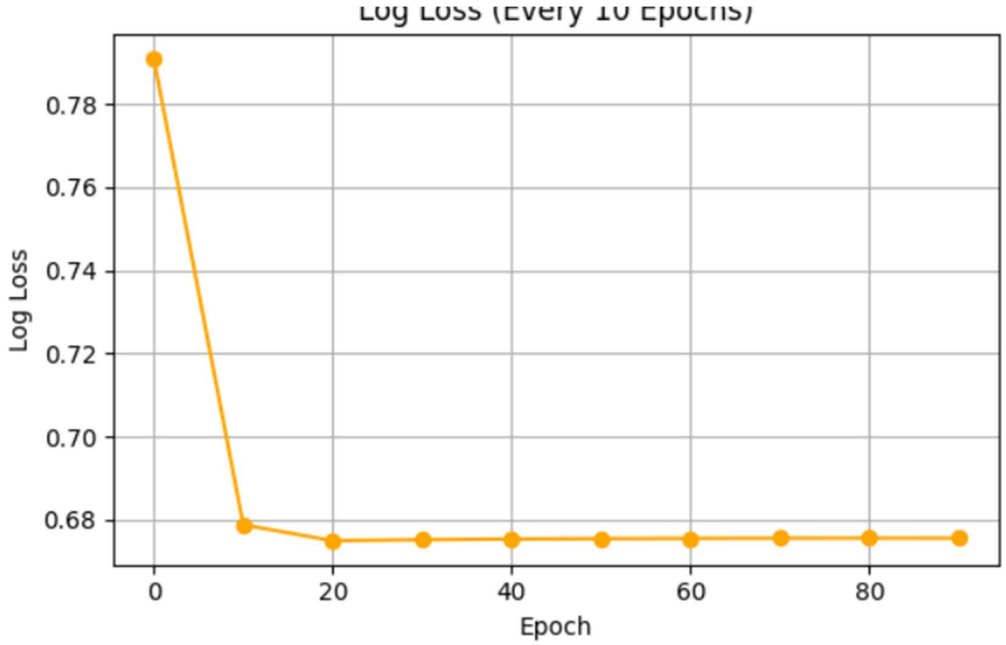
LR = 1.0

- **Accuracy:** 79.00%
- **Observation:**
  - Initial training was unstable with large swings
  - Eventually converged, but to a suboptimal boundary
  - Loss showed early spikes before settling



Log Loss Curve Analysis(0.01):

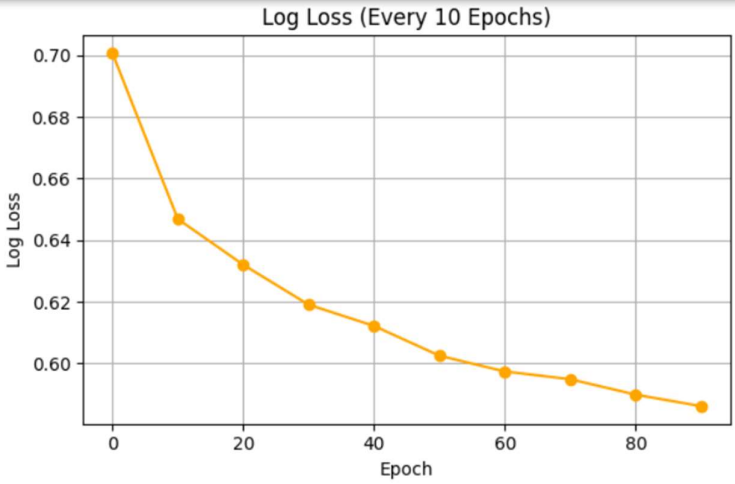
Smooth, steady decline with no instability




```
Training with learning rate: 0.01
Iterations to converge: 100
Final weights: [-0.09272239 -0.11407515]
Final bias: 0.09199394181140458
Accuracy: 91.00%
```

Log Loss Curve Analysis(0.1):

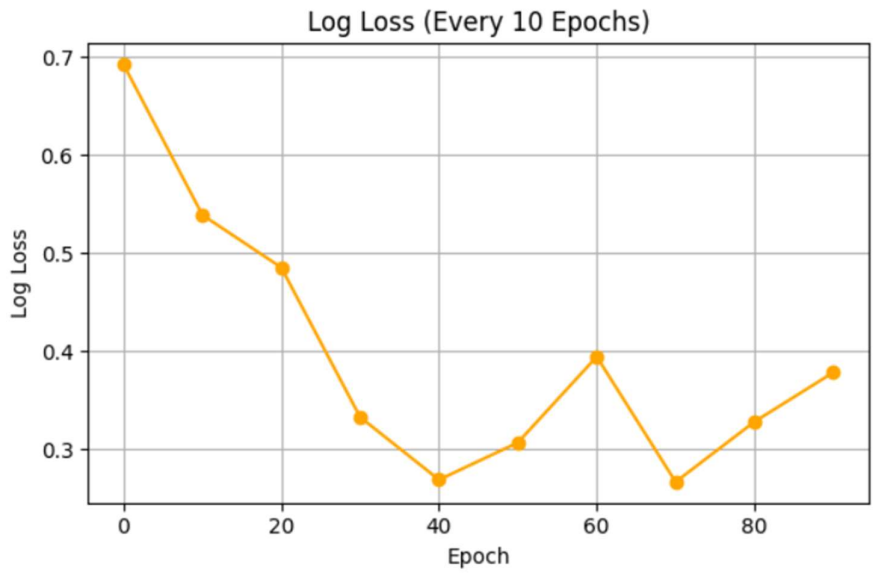
Fastest loss drop; clean convergence to the lowest final loss




```
Training with learning rate: 0.1
Iterations to converge: 100
Final weights: [-0.50753188 -0.89464919]
Final bias: 0.6319939418114051
Accuracy: 93.00%
```

Log Loss Curve Analysis(1.0):

Initial spikes due to overshooting; eventual recovery but higher final loss




```
Training with learning rate: 1.0
Iterations to converge: 100
Final weights: [-5.16954988 -9.89549269]
Final bias: 5.731993941811405
Accuracy: 79.00%
```

Overall Comparison

Metric	Part 1 (Heuristic)	Part 2 (Gradient Descent)
Update Rule	Based on sigmoid error (y - ŷ)	Based on misclassification only
Learning Style	Smooth, continuous updates	Discrete step adjustments
Accuracy Range	91% – 93%	79% – 93%
Log Loss Curve	Smooth decline	Depends on learning rate
Best LR	0.1	0.1
Final Outcome	Highly stable and interpretable	Works well at correct LR

CONCLUSION

- The heuristic perceptron (Part 1) behaved like a soft-margin classifier with smooth decision boundary convergence.
- The step-based perceptron (Part 2), while effective at low/moderate learning rates, was prone to instability at high learning rates.
- Learning rate was the most influential factor across both approaches.
- LR = 0.1 proved to be the most balanced, delivering high accuracy and fast convergence in both models.
- The experiments highlighted the strengths of gradient-based learning for fine-tuning and the trade-offs of step-based logic for fast binary classification.