Unit I

Introduction to Statistics

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Statistics

Statistics

Statistics is a mathematical body of science that pertains to the collection, analysis, interpretation or explanation, and presentation of data.

Statistics is concerned with the use of data in the context of uncertainty and decision making in the face of uncertainty.

Statistics

Statistics deals with variability. You're different from everybody else. Today differs from both yesterday and tomorrow. In an experiment designed to detect whether psychotherapy improves self-esteem, self-esteem scores will differ among subjects in the experiment, whether or not psychotherapy improves self-esteem.

1. **Statistics** is the study of how to collect, organize, analyze, and interpret numerical information from data.

2. Statistics is both the science of uncertainty and the technology of extracting information from data.

Types of Statistics

Descriptive statistics consists of methods for organizing and summarizing information.

Inferential statistics consists of methods for drawing and measuring the reliability of conclusions about a population based on information obtained from a sample of the population.

Descriptive Statistics

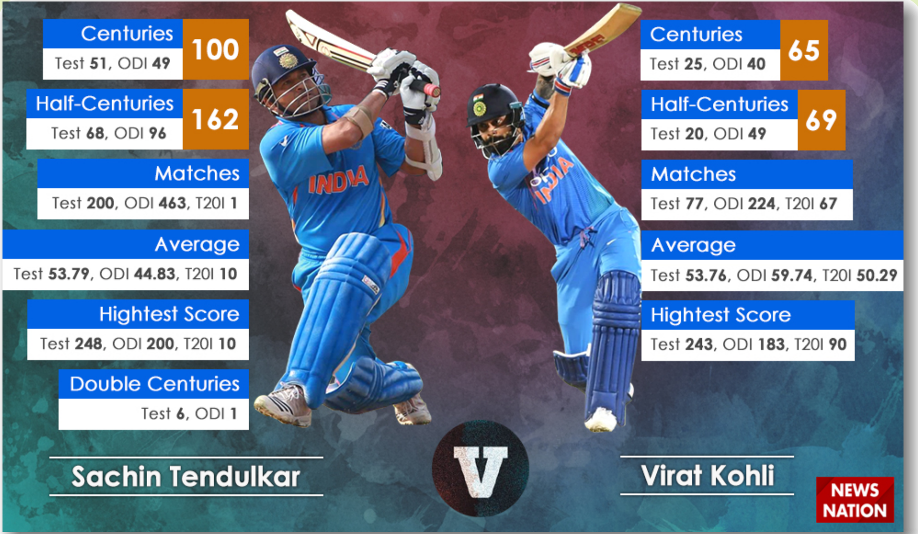
Statistics exists because of the prevalence of variability in the real world. In its simplest form, known as **descriptive statistics**, statistics provides us with tools [*tables, graphs, averages, ranges, correlations*] for organizing and summarizing the inevitable variability in collections of actual observations or scores. Examples are:

1. A tabular listing, ranked from most to least, of the total number of romantic affairs during college reported anonymously by each member of your stat class

2. A graph showing the annual change in global temperature during the last 30 years

3. A report that describes the average difference in grade point average (GPA) between college students who regularly drink alcoholic beverages and those who don't

Descriptive Statistics

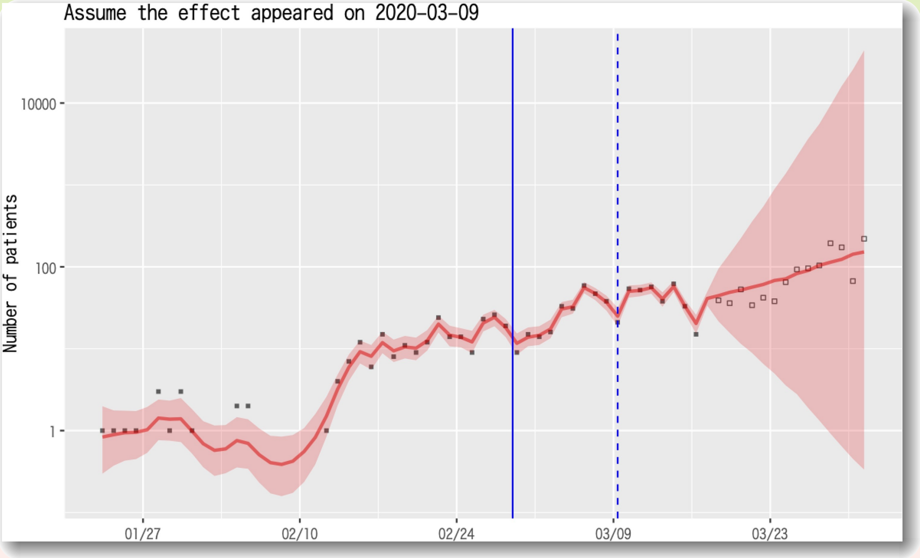


Inferential Statistics

Statistics also provides tools [*a variety of tests and estimates*] for generalizing beyond collections of actual observations. This more advanced area is known as inferential statistics. Tools from inferential statistics permit us to use a relatively small collection of actual observations to evaluate, for example:

1. A pollster's claim that a majority of all U.S. voters favor stronger gun control laws
2. A researcher's hypothesis that, on average, meditators report fewer headaches than do nonmeditators
3. An assertion about the relationship between job satisfaction and overall happiness

Inferential Statistics



Statistics

Question

Indicate whether each of the following statements typifies descrip- tive statistics (because it describes sets of actual observations) or inferential statistics (because it generalizes beyond sets of actual observations).

1. Students in my statistics class are, on average, 23 years old.
2. The population of the world exceeds 7 billion (that is, 7,000,000,000 or 1 million multiplied by 7000).
3. Either four or eight years have been the most frequent terms of office actually served by U.S. presidents.
4. Sixty-four percent of all college students favor citizenship amendment law.

Data

Data

Data are characteristics or information, usually numerical, that are collected through observation. In a more technical sense, data are a set of values of qualitative or quantitative variables about one or more persons or objects

Data

"facts and statistics collected together for reference or analysis."

Qualitative

approximates and characterizes.

Quantitative

association of unique numerical value

Levels of Measurement

Levels of Measurement I

What is?

Looming behind any data, the level of measurement specifies the extent to which a number (word,letter) actually represents some attribute and, therefore, has implications for the appropriateness of various arithmetic operations and statistical procedures.

1. The **nominal level** of measurement applies to data that consist of names, labels, or categories. There are no implied criteria by which the data can be ordered from smallest to largest.
2. The **ordinal level** of measurement applies to data that can be arranged in order. However, differences between data values either cannot be determined or are meaningless.
3. The **interval level** of measurement applies to data that can be arranged in order. In addition, differences between data values are meaningful.

Levels of Measurement II

4. The **ratio level** of measurement applies to data that can be arranged in order. In addition, both differences between data values and ratios of data values are meaningful. Data at the ratio level have a true zero.

Question

Identify the type of data.

1. Taos, Acoma, Zuni, and Cochiti are the names of four Native American pueblos from the population of names of all Native American pueblos in Arizona and New Mexico.
2. In a high school graduating class of 319 students, Jim ranked 25th, June ranked 19th, Walter ranked 10th, and Julia ranked 4th, where 1 is the highest rank.
3. Body temperatures (in degrees Celsius) of trout in the Yellowstone River.
4. Length of trout swimming in the Yellowstone River.

Basic Terminology I

Individuals

Individuals are the people or objects included in the study.

Variable

A variable is a characteristic of the individual to be measured or observed. The variables in a study may be quantitative or qualitative in nature.

- A quantitative variable has a value or numerical measurement for which operations such as addition or averaging make sense.
- A qualitative variable describes an individual by placing the individual into a category or group, such as male or female.

Basic Terminology II

Population & Sample

In statistics, a population refers to any complete collection of observations or potential observations, whereas a sample refers to any smaller collection of actual observations drawn from a population.

Population & Sample Data

In population data, the data are from every individual of interest. In sample data, the data are from only some of the individuals of interest.

Population & Sample Data

In population data, the data are from every individual of interest. In sample data, the data are from only some of the individuals of interest.

Basic Terminology III

Population parameter | A population parameter is a numerical measure that describes an aspect of a population.

Sample statistic | A sample statistic is a numerical measure that describes an aspect of a sample.

Basic Terminology IV

Question

How important is music education in school (K–12)? The Harris Poll did an online survey of 2286 adults (aged 18 and older) within the United States. Among the many questions, the survey asked if the respondents agreed or disagreed with the state- ment, “Learning and habits from music education equip people to be better team players in their careers.” In the most recent survey, 71% of the study participants agreed with the statement.

1. Identify the individuals of the study and the variable
2. Do the data comprise a sample? If so, what is the underlying population?
3. Is the variable qualitative or quantitative?
4. Identify a quantitative variable that might be of interest.

Univariate, Bivariate and Multivaiate

Univariate, Bivariate and Multivaiate

Data in statistics is sometimes classified according to how many variables are in a particular study. For example, “height” might be one variable and “weight” might be another variable. Depending on the number of variables being looked at, the data might be univariate, or it might be bivariate.

“How Many variables?”

Univariate Analysis

What is?

Univariate analysis is the simplest form of analyzing data. “Uni” means “one”, so in other words your data has only one variable. It doesn’t deal with causes or relationships (unlike regression) and it’s major purpose is to describe; It takes data, summarizes that data and finds patterns in the data.

Univariate Descriptive Statistics

Some ways you can describe patterns found in univariate data include central tendency (mean, mode and median) and dispersion: range, variance, maximum, minimum, quartiles (including the interquartile range), and standard deviation.

You have several options for describing data with univariate data.

- Frequency Distribution Tables.
- Bar Charts & Histograms.
- Frequency Polygons & Pie Charts.

Bivariate Analysis

What is?

Bivariate analysis means the analysis of bivariate data. It is one of the simplest forms of statistical analysis, used to find out if there is a relationship between two sets of values. It usually involves the variables X and Y.

Common types of bivariate analysis include:

- Scatter plots
- Regression Analysis
- Correlation Coefficients

Multivariate Analysis

What is?

Multivariate analysis is used to study more complex sets of data than what univariate analysis methods can handle. This type of analysis is almost always performed with software, as working with even the smallest of data sets can be overwhelming by hand.

There are more than 20 different ways to perform multivariate analysis. Which one you choose depends upon the type of data you have and what your goals are. For example, if you have a single data set you have several choices:

- Additive trees, multidimensional scaling, cluster analysis are appropriate for when the rows and columns in your data table represent the same units and the measure is either a similarity or a distance.
- Principal component analysis (PCA) decomposes a data table with correlated measures into a new set of uncorrelated measures.
- Correspondence analysis is similar to PCA. However, it applies to contingency tables.

Organizing Data

Organizing Data

Some situations generate an overwhelming amount of data. We can often make a large or complicated set of data more compact and easier to understand by organizing it in a table, chart, or graph.

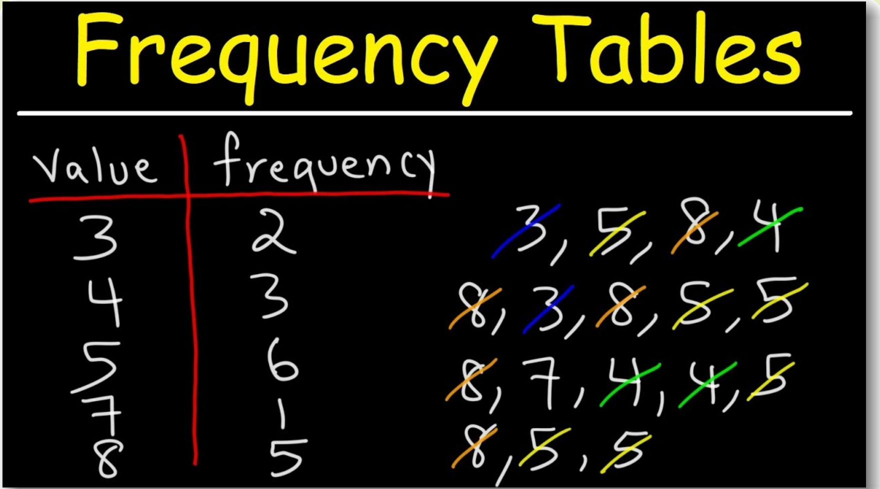
Frequency distributions

Frequency distributions

> A frequency distribution helps us to detect any pattern in the data (assuming a pattern exists) by superimposing some order on the inevitable variability among observations. For example, the appearance of a familiar bell-shaped pattern in the frequency distribution of reaction times of airline pilots to a cockpit alarm suggests the presence of many small chance factors whose collective effect must be considered in pilot retraining or cockpit redesign.

Frequency Distributions

A frequency distribution provides a table of the values of the observations and how often they occur.



Frequency Distribution of Qualitative Data I

A frequency distribution of qualitative data is a listing of the distinct values and their frequencies.

To Construct a Frequency Distribution of Qualitative Data :

1. List the distinct values of the observations in the data set in the first column of a table.
2. For each observation, place a tally mark in the second column of the table in the row of the appropriate distinct value.
3. Count the tallies for each distinct value and record the totals in the third column of the table.

Frequency Distribution of Qualitative Data II

TABLE 2.1

Political party affiliations of the students in introductory statistics

Democratic	Other	Democratic	Other	Democratic
Republican	Republican	Other	Other	Republican
Republican	Republican	Republican	Democratic	Republican
Republican	Democratic	Democratic	Other	Republican
Democratic	Democratic	Republican	Democratic	Democratic
Republican	Republican	Other	Other	Democratic
Republican	Democratic	Republican	Other	Other
Republican	Republican	Republican	Democratic	Republican

Frequency Distribution of Qualitative Data III

TABLE 2.2

Table for constructing a frequency distribution for the political party affiliation data in Table 2.1

Party	Tally	Frequency
Democratic	~~IIII~~ ~~IIII~~ III	13
Republican	~~IIII~~ ~~IIII~~ ~~IIII~~ III	18
Other	~~IIII~~ IIII	9
		40

Relative-Frequency Distribution of Qualitative Data I

What is?

A relative-frequency distribution provides a table of the values of the observations and (relatively) how often they occur.

$$\text{Relative frequency} = \frac{\text{Frequency}}{\text{Number of observations}}$$

A relative-frequency distribution of qualitative data is a listing of the distinct values and their relative frequencies.**To Construct a Relative-Frequency Distribution of Qualitative Data :**

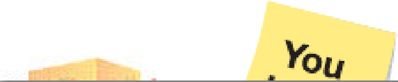
1. Obtain a frequency distribution of the data.
2. Divide each frequency by the total number of observations.

Relative-Frequency Distribution of Qualitative Data II

TABLE 2.3

Relative-frequency distribution for the political party affiliation data in Table 2.1

Party	Relative frequency	
Democratic	0.325	← *13/40*
Republican	0.450	← *18/40*
Other	0.225	← *9/40*
	1.000	



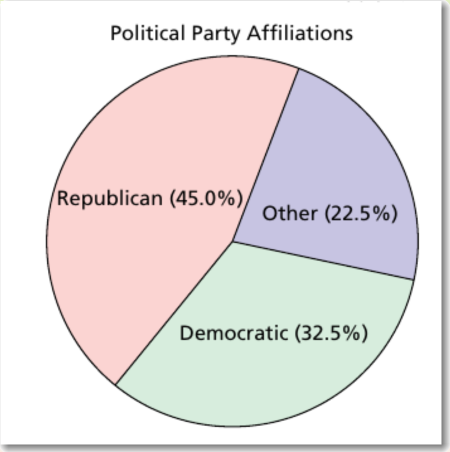
Pie-Chart for Organizing Qualitative Data I

What is? A pie chart is a disk divided into wedge-shaped pieces proportional to the relative frequencies of the qualitative data.

To Construct a Pie-Chart of Qualitative Data :

1. Obtain a relative-frequency distribution of the data
2. Divide a disk into wedge-shaped pieces proportional to the relative frequencies.
3. Label the slices with the distinct values and their relative frequencies.

Pie-Chart for Organizing Qualitative Data II



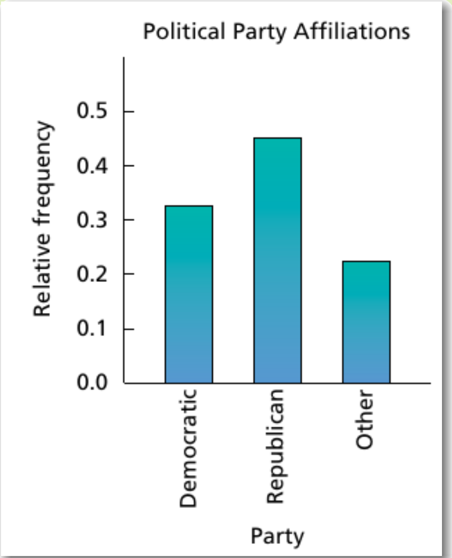
Bar Chart for Organizing Qualitative Data I

A bar chart displays the distinct values of the qualitative data on a horizontal axis and the relative frequencies (or frequencies or percents) of those values on a vertical axis. The relative frequency of each distinct value is represented by a vertical bar whose height is equal to the relative frequency of that value. The bars should be positioned so that they do not touch each other.

To Construct a Bar Chart of Qualitative Data :

- Obtain a relative-frequency distribution of the data
- Draw a horizontal axis on which to place the bars and a vertical axis on which to display the relative frequencies.
- For each distinct value, construct a vertical bar whose height equals the relative frequency of that value.
- Label the bars with the distinct values, the horizontal axis with the name of the variable, and the vertical axis with “Relative frequency.”

Bar Chart for Organizing Qualitative Data II



Organizing Quantitative Data I

classes

To organize quantitative data, we first group the observations into classes (also known as categories or bins) and then treat the classes as the distinct values of qualitative data. Consequently, once we group the quantitative data into classes, we can construct frequency and relative-frequency distributions of the data in exactly the same way as we did for qualitative data.

Why classes?

The reason for grouping is to organize the data into a sensible number of classes in order to make the data more accessible and understandable.

Three commonsense and important guidelines for grouping quantitative data into classes are:

Organizing Quantitative Data II

1. The number of classes should be small enough to provide an effective summary but large enough to display the relevant characteristics of the data. A rule of thumb is that the number of classes should be between 5 and 20.

2. Each observation must belong to one, and only one, class. That is, each observation should belong to some class and no observation should belong to more than one class.

3. Whenever feasible, all classes should have the same width. Roughly speaking, this guideline means that, if possible, all classes should cover the same number of pos- sible values.

Organizing Quantitative Data III

Several methods can be used to group quantitative data into classes. Here we discuss three of the most common methods:

- single-value grouping,
- limit grouping, and
- cutpoint grouping.

Organizing Quantitative Data IV

Single-Value Grouping

In some cases, the most appropriate way to group quantitative data is to use classes in which each class represents a single possible value. Such classes are called single-value classes, and this method of grouping quantitative data is called single-value grouping.

Thus, in single-value grouping, we use the distinct values of the observations as the classes, a method completely analogous to that used for qualitative data. Single-value grouping is particularly suitable for discrete data in which there are only a small number of distinct values.

Organizing Quantitative Data V

Number of TV sets in each of 50 randomly selected households

1	1	1	2	6	3	3	4	2	4
3	2	1	5	2	1	3	6	2	2
3	1	1	4	3	2	2	2	2	3
0	3	1	2	1	2	3	1	1	3
3	2	1	2	1	1	3	1	5	1

The (single-value) classes are the distinct values of the data in Table, which are the numbers 0, 1, 2, 3, 4, 5, and 6.

Number of TVs	Frequency	Relative frequency
0	1	0.02
1	16	0.32
2	14	0.28
3	12	0.24
4	3	0.06
5	2	0.04
6	2	0.04
	50	1.00

Organizing Quantitative Data VI

Limit Grouping

A second way to group quantitative data is to use class limits. With this method, each class consists of a range of values. The smallest value that could go in a class is called the lower limit of the class, and the largest value that could go in the class is called the upper limit of the class.

This method of grouping quantitative data is called limit grouping. It is particularly useful when the data are expressed as whole numbers and there are too many distinct values to employ single-value grouping.

Organizing Quantitative Data VII

Days to maturity for 40 short-term investments

70	64	99	55	64	89	87	65
62	38	67	70	60	69	78	39
75	56	71	51	99	68	95	86
57	53	47	50	55	81	80	98
51	36	63	66	85	79	83	70

Days to maturity	**Tally**	**Frequency**	**Relative frequency**
30–39	|||	3	0.075
40–49	|	1	0.025
50–59	~~||||~~ |||	8	0.200
60–69	~~||||~~ ~~||||~~	10	0.250
70–79	~~||||~~ ||	7	0.175
80–89	~~||||~~ ||	7	0.175
90–99	||||	4	0.100
		40	1.000

Organizing Quantitative Data VIII

Terms Used in Limit Grouping

- **Lower class limit:** The smallest value that could go in a class.
- **Upper class limit:** The largest value that could go in a class.
- **Class width:** The difference between the lower limit of a class and the lower limit of the next-higher class.
- **Class mark:** The average of the two class limits of a class.

Organizing Quantitative Data IX

Cutpoint Grouping

A third way to group quantitative data is to use class cutpoints. As with limit grouping, each class consists of a range of values. The smallest value that could go in a class is called the lower cutpoint of the class, and the smallest value that could go in the next-higher class is called the upper cutpoint of the class. Note that the lower cutpoint of a class is the same as its lower limit and that the upper cutpoint of a class is the same as the lower limit of the next higher class.

The method of grouping quantitative data by using cutpoints is called cutpoint grouping. This method is particularly useful when the data are continuous and are expressed with decimals.

Organizing Quantitative Data X

Weights, in pounds, of 37 males aged 18–24 years

129.2	185.3	218.1	182.5	142.8
155.2	170.0	151.3	187.5	145.6
167.3	161.0	178.7	165.0	172.5
191.1	150.7	187.0	173.7	178.2
161.7	170.1	165.8	214.6	136.7
278.8	175.6	188.7	132.1	158.5
146.4	209.1	175.4	182.0	173.6
149.9	158.6			

Weight (lb)	Frequency	Relative frequency
120–under 140	3	0.081
140–under 160	9	0.243
160–under 180	14	0.378
180–under 200	7	0.189
200–under 220	3	0.081
220–under 240	0	0.000
240–under 260	0	0.000
260–under 280	1	0.027
	37	0.999

Organizing Quantitative Data XI

Terms Used in Cutpoint Grouping

- **Lower class cutpoint:** The smallest value that could go in a class.
- **Upper class cutpoint:** The smallest value that could go in the next-higher class (equivalent to the lower cutpoint of the next-higher class).
- **Class width:** The difference between the cutpoints of a class.
- **Class midpoint:** The average of the two cutpoints of a class.

Organizing Quantitative Data XII

Choosing the Grouping Method

Grouping method	When to use
Single-value grouping	Use with discrete data in which there are only a small number of distinct values.
Limit grouping	Use when the data are expressed as whole numbers and there are too many distinct values to employ single-value grouping.
Cutpoint grouping	Use when the data are continuous and are expressed with decimals.

Saying It With Picture : Quantitative Data

A Convinient method for organizing and summarizing data is to draw a picture of some kind. Some common methods for graphically displaying quantitative data are

1. Histograms,
2. Dotplots,
3. Stem-and-leaf diagrams, and
4. Frequency Polygon

Histograms of Quantitative Data I

what is? | A histogram provides a graph of the values of the observations and how often they occur.

- A histogram of quantitative data is the direct analogue of a bar chart of qualitative data, where we use the classes of the quantitative data in place of the distinct values of the qualitative data.
- However, to help distinguish a histogram from a bar chart, we position the bars in a histogram so that they touch each other. Frequencies, relative frequencies, or percents can be used to label a histogram.
- For single-value grouping, we use the distinct values of the observations to label the bars, with each such value centered under its bar.
- For limit grouping or cutpoint grouping, we use the lower class limits (or, equivalently, lower class cutpoints) to label the bars. **Note:** *Some statisticians and technologies use class marks or class midpoints centered under the bars.*

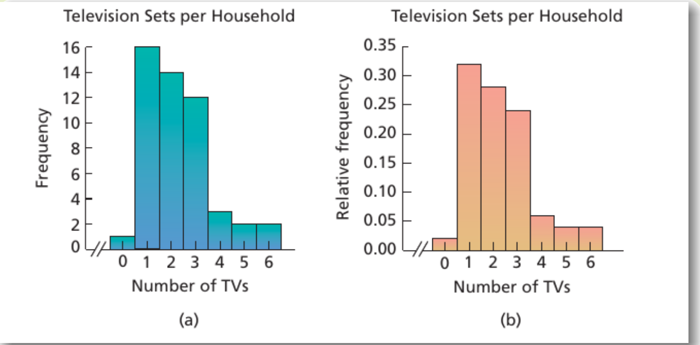
Histograms of Quantitative Data II

How to Construct?

1. Obtain a frequency (relative-frequency, percent) distribution of the data.
2. Draw a horizontal axis on which to place the bars and a vertical axis on which to display the frequencies (relative frequencies, percents).
3. For each class, construct a vertical bar whose height equals the frequency (relative frequency, percent) of that class.
4. Label the bars with the classes, as explained in Definition 2.9, the horizontal axis with the name of the variable, and the vertical axis with ‘Frequency’ (‘Relative frequency,’ ‘Percent’).

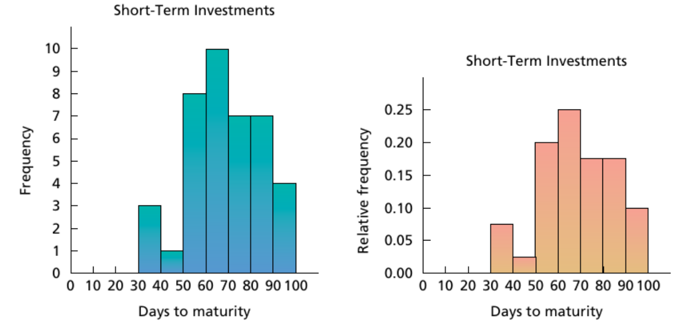
Histograms of Quantitative Data III

Number of TVs	Frequency	Relative frequency
0	1	0.02
1	16	0.32
2	14	0.28
3	12	0.24
4	3	0.06
5	2	0.04
6	2	0.04
	50	1.00



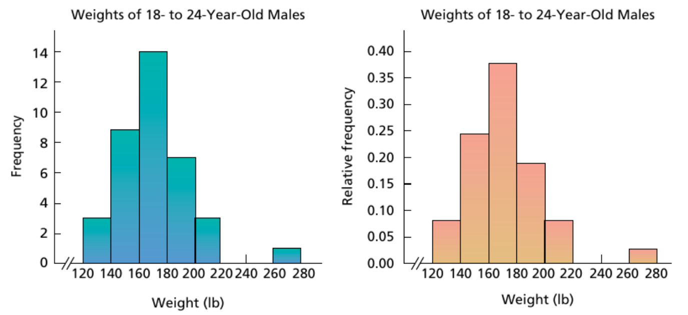
Histograms of Quantitative Data IV

Days to maturity	Tally	Frequency	Relative frequency
30–39	|||	3	0.075
40–49	|	1	0.025
50–59	~~||||~~ |||	8	0.200
60–69	~~||||~~ ~~||||~~	10	0.250
70–79	~~||||~~ ||	7	0.175
80–89	~~||||~~ ||	7	0.175
90–99	||||	4	0.100
		40	1.000



Histograms of Quantitative Data V

Weight (lb)	Frequency	Relative frequency
120–under 140	3	0.081
140–under 160	9	0.243
160–under 180	14	0.378
180–under 200	7	0.189
200–under 220	3	0.081
220–under 240	0	0.000
240–under 260	0	0.000
260–under 280	1	0.027
	37	0.999



DotPlot of Quantitative Data

what is it ?

A dotplot is a graph in which each observation is plotted as a dot at an appropriate place above a horizontal axis. Observations having equal values are stacked vertically.

1. Draw a horizontal axis that displays the possible values of the quantitative data.
2. Record each observation by placing a dot over the appropri-ate value on the horizontal axis.
3. Label the horizontal axis with the name of the variable.

Prices, in dollars, of 16 DVD players

210	219	214	197
224	219	199	199
208	209	215	199
212	212	219	210

Stem-and-Leaf Diagram of Quantitative Data I

what is it ?

In a stem-and-leaf diagram (or stemplot), each observation is separated into two parts, namely, a stem–consisting of all but the rightmost digit and a leaf, the rightmost digit.

1. Think of each observation as a stem—consisting of all but the rightmost digit—and a leaf, the rightmost digit.
2. Write the stems from smallest to largest in a vertical column to the left of a vertical rule.
3. Write each leaf to the right of the vertical rule in the row that contains the appropriate stem.
4. Arrange the leaves in each row in ascending order.

Stem-and-Leaf Diagram of Quantitative Data II

CONSTRUCTING STEM AND LEAF DISPLAY FROM WEIGHTS OF MALE STATISTICS STUDENTS

RAW SCORES					STEM AND LEAF DISPLAY
160	165	135	175		
193	168	245	165	13	3 5 5
226	169	170	185	14	5
152	160	156	154	15	2 7 1 7 8 0 2 0 2 6 9 8 2 6 4 7 6
180	170	160	179	16	0 3 5 8 9 0 0 0 6 5 5 5
205	150	225	165	17	2 0 0 0 2 5 9
163	152	190	206	18	0 0 5
157	160	159	165	19	3 0 0 0
151	190	172	157	20	5 6
157	150	190	156	21	
220	133	166	135	22	6 0 5
145	180	158		23	
158	152	152		24	5
172	170	156			

Stem-and-Leaf Diagram of Quantitative Data

Days to maturity for 40 short-term investments

70	64	99	55	64	89	87	65
62	38	67	70	60	69	78	39
75	56	71	51	99	68	95	86
57	53	47	50	55	81	80	98
51	36	63	66	85	79	83	70

Stem-and-Leaf Diagram of Quantitative Data

Days to maturity for 40 short-term investments

70	64	99	55	64	89	87	65
62	38	67	70	60	69	78	39
75	56	71	51	99	68	95	86
57	53	47	50	55	81	80	98
51	36	63	66	85	79	83	70

(a)

3	8 6 9
4	7
5	7 1 6 3 5 1 0 5
6	2 4 7 3 6 4 0 9 8 5
7	0 5 1 0 9 8 0
8	5 9 1 7 0 3 6
9	9 9 5 8

(b)

Stems	Leaves
3	6 8 9
4	7
5	0 1 1 3 5 5 6 7
6	0 2 3 4 4 5 6 7 8 9
7	0 0 0 1 5 8 9
8	0 1 3 5 6 7 9
9	5 8 9 9

Frequency Polygon of Quantitative Data I

An important variation on a histogram is the frequency polygon, or line graph. Frequency polygons may be constructed directly from frequency distributions.

what is it ? | A line graph for quantitative data that also emphasizes the continuity of continuous variables.

Construction of Frequency Polygon from Histogram :

1. Construct the Histogram.
2. Place dots at the midpoints of each bar top or, in the absence of bar tops, at mid–points for classes on the horizontal axis, and connect them with straight lines.

Frequency Polygon of Quantitative Data II

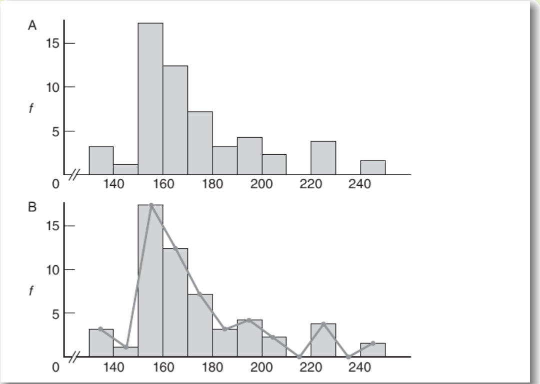
3

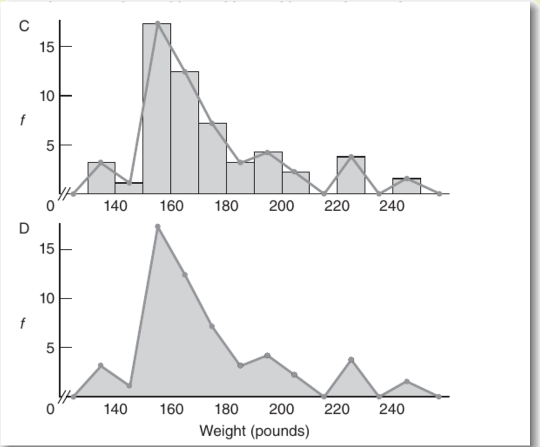
Anchor the frequency polygon to the horizontal axis. First, extend the upper tail to the midpoint of the first unoccupied class on the upper flank of the histogram. Then extend the lower tail to the midpoint of the first unoccupied class on the lower flank of the histogram. Now all of the area under the frequency polygon is enclosed completely.

4

Finally, erase all of the histogram bars, leaving only the frequency polygon. Frequency polygons are particularly useful when two or more frequency distributions or relative frequency distributions are to be included in the same graph.

Frequency Polygon of Quantitative Data III



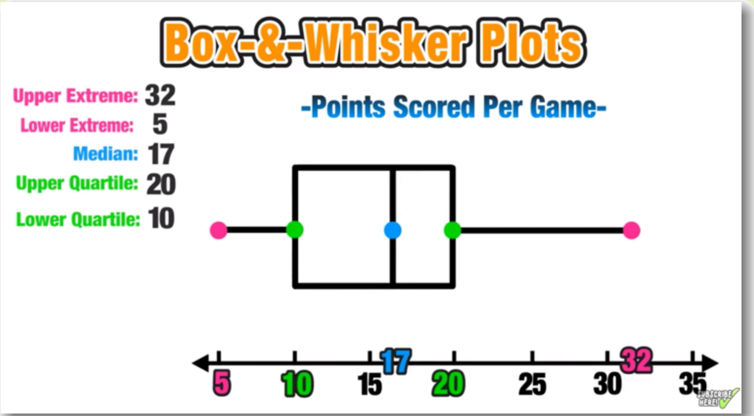


Box and Whisker plot of Quantitative Data I

what is it ?

A box and whisker plot is a type of chart often used in explanatory data analysis to visually show the distribution of numerical data and skewness through displaying the data quartiles (or percentiles) and averages.
Boxand whisker plots show the five-number summary of a set of data: including the minimum score, first (lower) quartile, median, third (upper) quartile, and maximum score.

Box and Whisker plot of Quantitative Data II



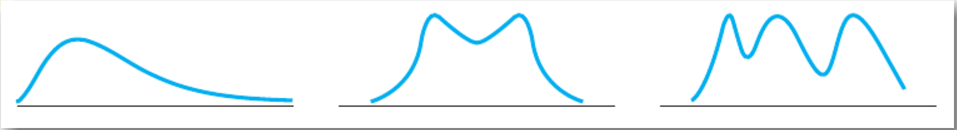
Distribution Shapes I

Distribution of a Data Set

The distribution of a data set is a table, graph, or formula that provides the values of the observations and how often they occur.

Modality

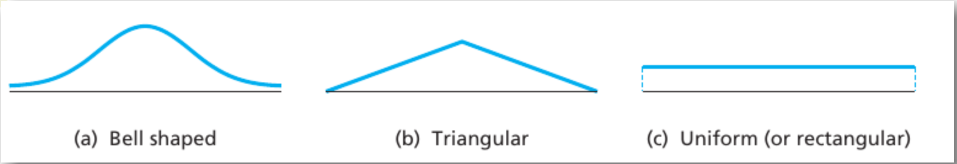
When considering the shape of a distribution, you should observe its number of peaks (highest points). A distribution is unimodal if it has one peak, bimodal if it has two peaks, and multimodal if it has three or more peaks.



Distribution Shapes II

Symmetry

Another important consideration when examining the shape of a distribution is sym- metry. A distribution that can be divided into two pieces that are mirror images of one another is called sym- metric.



Distribution Shapes III

Skewness

A unimodal distribution that is not symmetric is either right skewed or left skewed.

- A right-skewed distribution rises to its peak rapidly and comes back toward the horizontal axis more slowly. *its 'right tail' is longer than its 'left tail.'*
- A left-skewed distribution rises to its peak slowly and comes back toward the horizontal axis more rapidly. *its 'left tail' is longer than its 'right tail.'*

Distribution Shapes IV