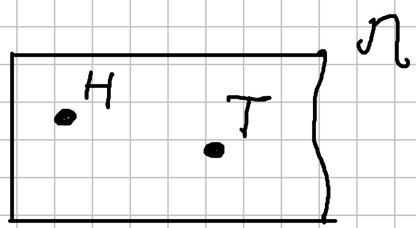
Probability :→ A mathematical framework for dealing with uncertain situation

Sample space :→ [Ω, S]

1. "List" of possible outcomes of a Random Experiment
→ "Set"

2. List must be
- mutually exclusive
- Collectively exhaustive

Ex Tossing a coin

$\Omega = \{H, T\} =$ 

$\Omega =$
- H and Tuesday
- H and any other day ($\neq$ Tuesday)
- T

$\Omega = \{ H/Tues , H/nTues , T \}$

Sample space
- Discrete
- Continuous

Discrete sample space

we have 6 faced die

Ex we are tossing the die just once

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

Ex Two rolls of a tetrahedral die

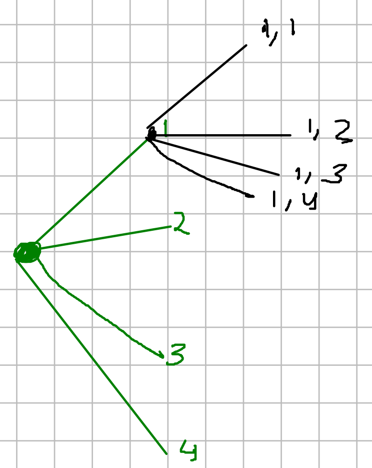
Roll 2nd ↑ Y

1, 4	2, 4	3, 4	4, 4
1, 3	2, 3	3, 3	4, 3
1, 2	2, 2	3, 2	4, 2
1, 1	2, 1	3, 1	4, 1

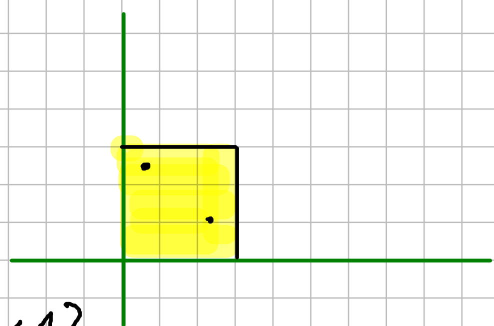
X → First Roll

$= \alpha_1, \alpha_2, \alpha_3$
α α_5

Sequential models



Continuous Sample space



$$\Omega = \{(x, y) \mid 0 \leq x \,\&\, y \leq 1\}$$

Event : A subset of Ω

$\rightarrow$ we assign probability to events and not outcomes

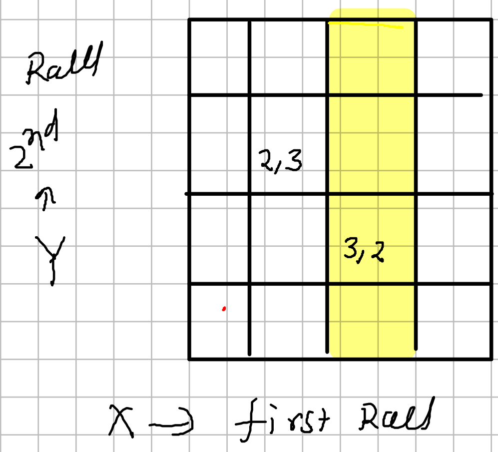
Axioms of Probability :$\Rightarrow$

1. Non-Negativity : $P(A) \geq 0$
2. Normalization : $P(\Omega) = 1$
3. Additivity :

   If $A \cap B = \phi$

   then $\underline{P(A \cup B)} = \underline{P(A) + P(B)}$

Ex Tossing of a Tetrahedral die twice



→ Let every possible outcome (x, y) be equally likely

$A = \{\underset{A_1}{(1,1)}, \underset{A_2}{(2,3)}\}$

$P(A) = p(A_1) + p(A_2)$

$= \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$

B = first Roll Results in $\underline{3}$.

$B = \{(\underline{3},1), (\underline{3},2), (\underline{3},3), (\underline{3},4)\}$

$P(B) = \underline{P((3,1))} + \underline{P((3,2))} + \underline{P((3,3))} + \underline{P((3,4))}$

$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4}$

Discrete uniform Probability Law :→

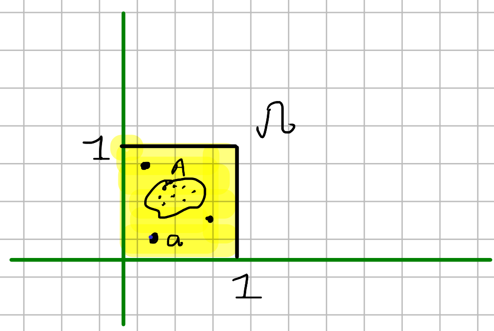
" Let all the outcomes (elements) be equally likely "

$$p(A) = \frac{\text{number of elements in } A}{\text{total number of sample points}}$$

⇒ computing probability is basically counting if we are using DPL.

⇒ key words :→ fair, unbaised, equally, likely

Continuous uniform probability low :→



probability = area

$$\boxed{P \propto \text{area}}$$

$P(A) = \text{area}(A)$

$A = \Omega / a$

$p(A) = 1$

Ex Sample space : $\{1, 2, 3, 4, \dots\}$

$$P(n) = 2^{-n}$$

$$p(\{2, 4, 6, 8, \dots\}) = p(2) + p(4) + p(6) + p(8) + \dots$$

Countable number of set

$$p(A_1 \cup A_2 \cup A_3 \cup A_4 \dots) = p(A_1) + p(A_2) + p(A_3) + \dots$$

$$A_i \cap A_j = \phi$$

$$= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} + \dots$$

$$= \frac{2^{-2}}{1 - 2^{-2}} = \frac{1/4}{1 - 1/4} = \frac{1}{4 - 1} = \frac{1}{3}$$

$$p(n/\text{odd}) = p(\{1, 3, 5, 7, 9, \dots\})$$

$$= p(1) + p(3) + p(5) + \dots = 2/3$$

axioms

1. $P(A) \geq 0$
2. $P(\Omega) = 1$
3. if $A \cap B = \phi$

   $p(A \cup B) = p(A) + p(B)$

3'. if $A_i \cap A_j = \phi$

   $p(A_1 \cup A_2 \dots) = p(A_1) + p(A_2) + \dots$

$$p(\phi) =$$

$$\Omega = \Omega \cup \phi \qquad \text{and} \qquad \Omega \cap \phi = \phi$$

$$p(\Omega) = p(\Omega) + p(\phi)$$

$$1 = 1 + p(\phi)$$

$$p(\phi) = 1 - 1 = 0$$

$$p(A^c) = 1 - p(A)$$

$$A^c \cup A = \Omega \qquad \text{and} \qquad A^c \cap A = \phi$$

$$p(\Omega) = p(A^c) + p(A)$$

$$p(A^c) = 1 - p(A)$$

$$\Rightarrow \quad P(A) \leq 1$$

we know that

$$p(A) \geq 0 \quad — \boxed{I}$$

$$p(A^c) \geq 0 \quad — \boxed{I}$$

$$A \cup A^c = \Omega$$

$$p(A \cup A^c) = p(\Omega)$$

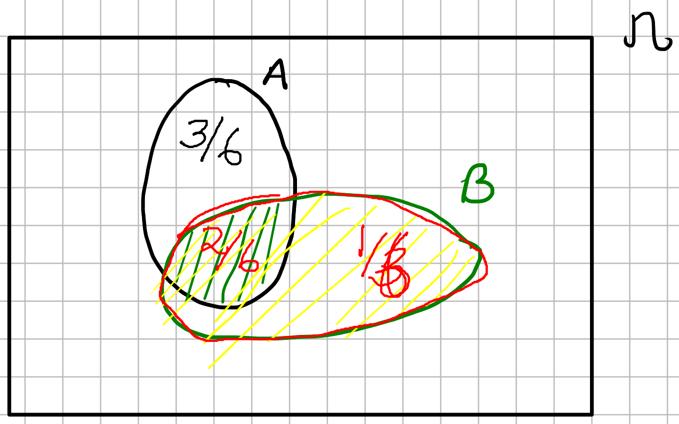
$$p(A) + p(A^c) = p(\Omega) \quad — \boxed{III}$$

$$p(A) + p(A^c) = 1 \quad — \boxed{II}$$

$$p(A) = 1 - p(A^c) \Rightarrow p(A) \leq 1$$

Conditional Probability

$P(A) = P(A/\Omega)$



$P(A/B) = \frac{2}{3}$

$P(A/B)$ = probability of A, given that B has occured

— B is our new universe

$$P(A/B) = \frac{n(A\cap B)}{n(B)} = \frac{n(A\cap B)/n(\Omega)}{n(B)/n(\Omega)}$$

$$= \frac{P(A\cap B)}{P(B)}$$

→ Two Rolls of a tetrahedral die

Y ↑				
	1,4	2,4	3,4	4,4
	1,3	2,3	3,3	4,3
	1,2	2,2	3,2	4,2
	1,1	2,1	3,1	4,1
	→ X			

$B = \min(X,Y) = 2$

$A = \max(X,Y) = 1$

$P(A/B) = 0$

$$A_1 = \{\max(X, Y) = 2\}$$

$$P(A_1/B) = \frac{1}{5} = \frac{P(A_1 \cap B)}{P(B)} = \frac{1/16}{5/16}$$

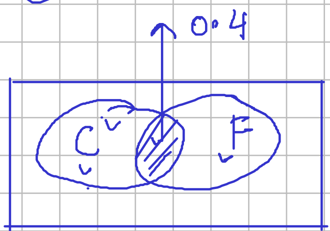
$$A_3 = \{X = 2\} \qquad P(A_3/B) = \frac{3}{5}$$

Date : 26 July, 2021

Ex: Probability that a randomly selected student from this class is a football lover is 0.6 ; probability that a randomly selected student is a cricket lover is 0.7. the probability that a randomly selected student is both cricket and football lover. is 0.4. (i) Find Probability that a randomly selected student is neither cricket nor football lover. (ii) either cricket or Football lover.

$\Rightarrow$ $P(C) = 0.7$ $\qquad P(C \cap F) = 0.4$

$P(F) = 0.6$

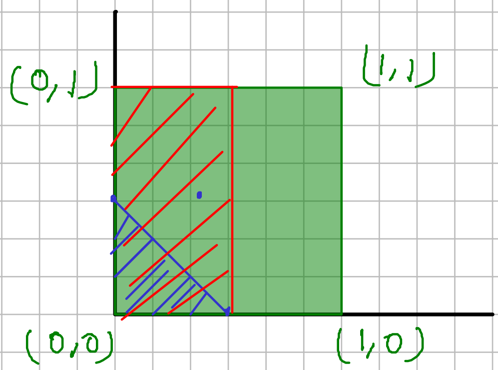


$$P(C \cup F) = P(C) + P(F) - P(C \cap F)$$

$$= 0.9$$

Inclusion - Exclusion principle

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$



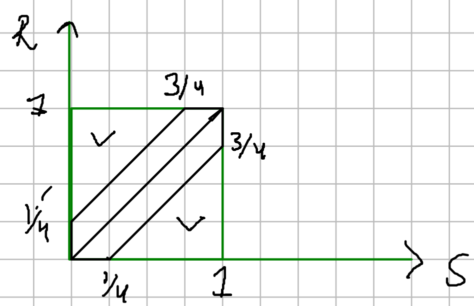
$p(A) = \text{area}(A)$

$p((x,y) = (0.3, 0.4)) = 0$

$p\left(x+y \leq \frac{1}{2}\right) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$p\left(x \leq \frac{1}{2}\right) = \frac{1}{2} \times 1 = 1/2$

<u>Question</u> : Two person Ram and shyam wants to meet up between 8:00 AM to 9:00 AM. They are only going to wait for each other for 15 minutes. what is the probability that they meet.

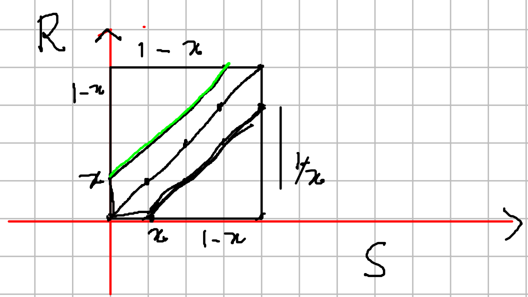


$$1 - 2 \times \frac{1}{2} \frac{3}{4} \times \frac{3}{4}$$

$$= 1 - \frac{9}{16} = \frac{16-9}{16}$$

$$= 7/16$$

$\Rightarrow$ How much Ram and shyam should wait for each other if they want 90% chance of meeting each other



$$1 - 2 \times \frac{1}{2} (1-x)(1-x)$$

$$.9 = 1 - (1-x)^2$$

$$(1-x)^2 = 1 - .9 = .1$$

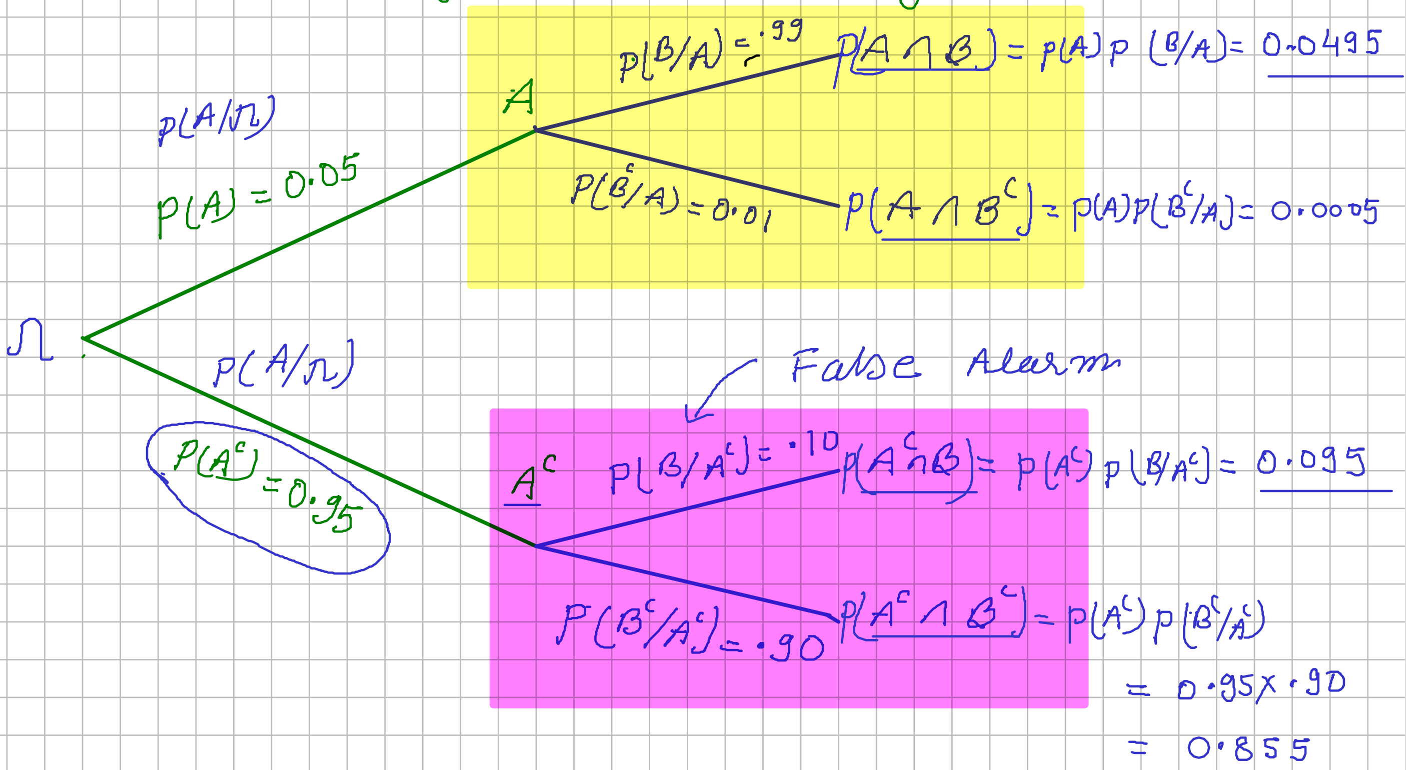
$$\boxed{A \cap B = \phi}$$

$$P(A \cup B / C) = P(A/C) + P(B/C)$$

Model based on Conditional Prob Date : 27 July, 2021

Event A : An Airplane is flying

Event B : Radar Registers something



$$P(B/A) = \frac{P(B \cap A)}{P(A)} \Rightarrow$$

$$\Rightarrow \quad P(A \cap B) = P(A)\,P(B/A)$$

$$= P(B)\,P(A/B)$$

$$P(A \cap B) = 0.0495$$

$$P(B) = P(B \cap A) + P(B \cap A^c) = .0495 + 0.0950$$

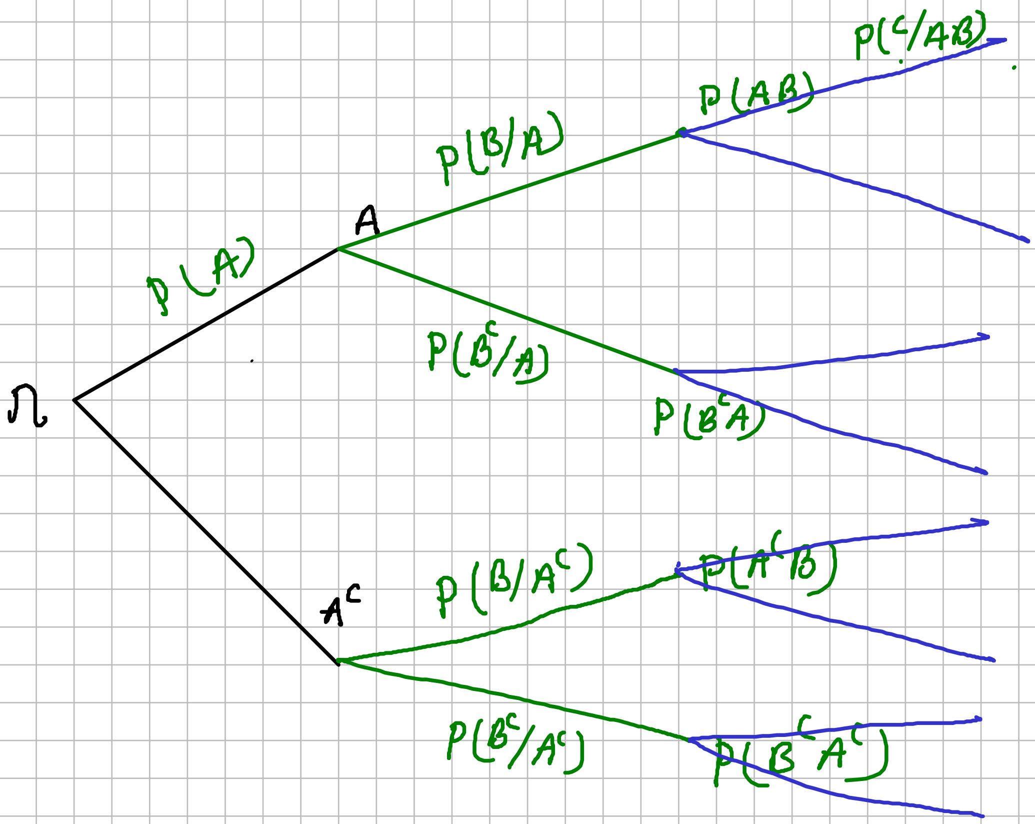
$$= .1445$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.0495}{0.1445} = 0.3425$$

Multiplication Rule

$$P(A\cap B\cap C) = P(A\cap B)\, P(C/A\cap B)$$

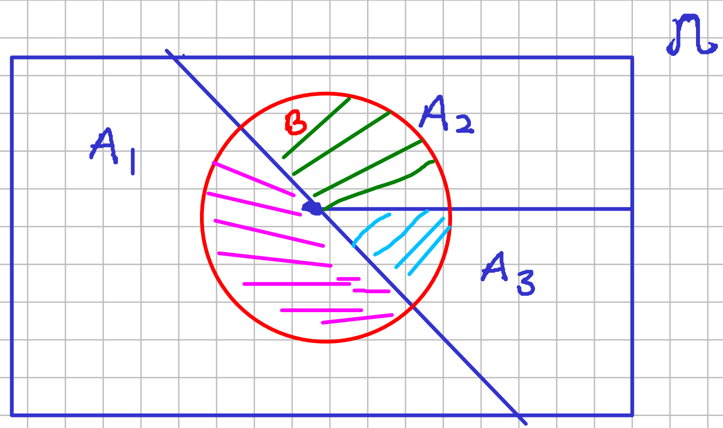
$$= P(A)\, P(B/A)\, P(C/A\cap B)$$



$P.(ABC) =$

....

Total Probability



Total probability of B

$$P(B) = P(A_1B) + P(A_2B) + P(A_3B)$$

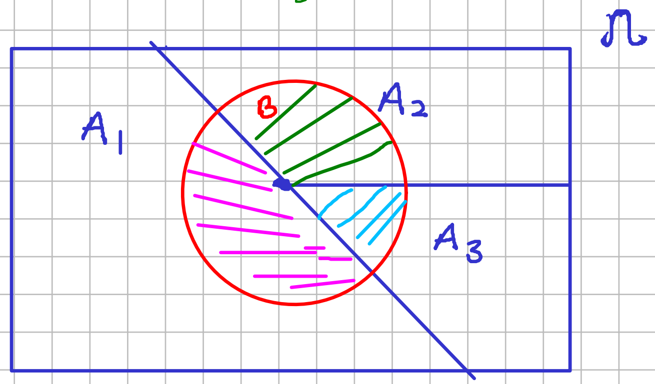
$$= P(A_1)\, P(B/A_1) + P(A_2)\, P(B/A_2) + P(A_3)\, P(B/A_3)$$

Bayes' Rule

we know

→ prior probabilities $P(A_i)$ for each A_i

→ $P(B/A_i)$ for each A_i



we want to compute $P(A_i/B)$

$$P(A_i/B) = \frac{P(A_i \cap B)}{P(B)}$$

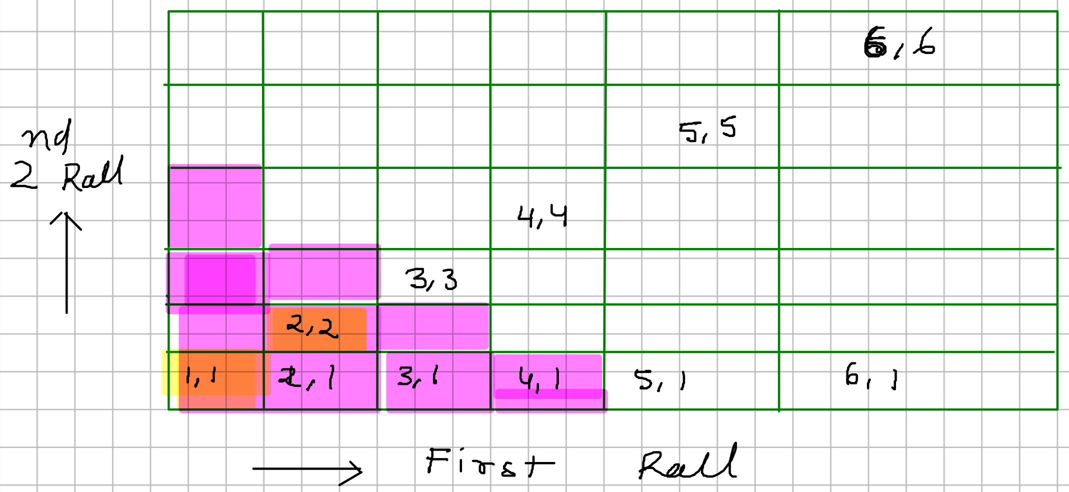
$$P(A_i/B) = \frac{P(A_i)\,P(B/A_i)}{\sum_i P(A_i)\,P(B/A_i)}$$

H.W.

Ex We are tossing a six faced die twice.

(I) what is probability of getting a double.

(II) what is the probability of getting a double when we know that sum of the results of two tosses is less than or equal to 5.

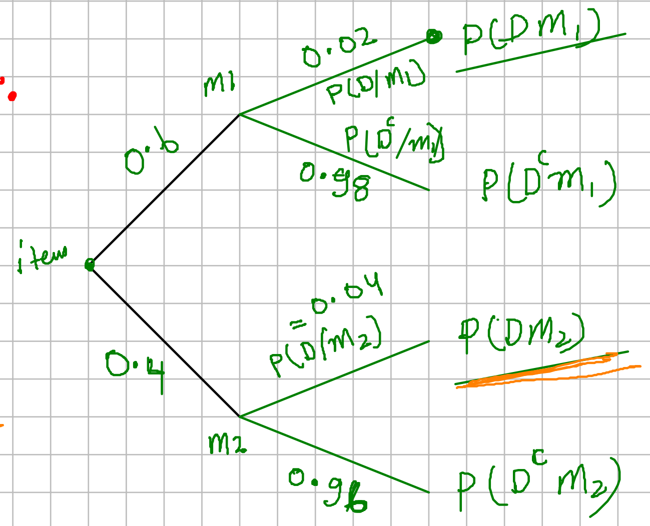


$P(D) = \frac{6}{36} = 1/6$

$P(S \leq 5) = \frac{10}{36}$

$P(D/S \leq 5) = \frac{2}{10}$

Q: A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?



$P(m_2/D) = ?$

$= \frac{P(DM_2)}{P(D)}$

$= \frac{0.016}{0.028}$

$= 16/28$

total probability

$P(D) = P(DM_1) + P(DM_2)$

$= P(m_1)\,P(D/m_1) + P(m_2)\,P(D/m_2)$

$= 0.6 \times 0.02 + 0.4 \times 0.04$

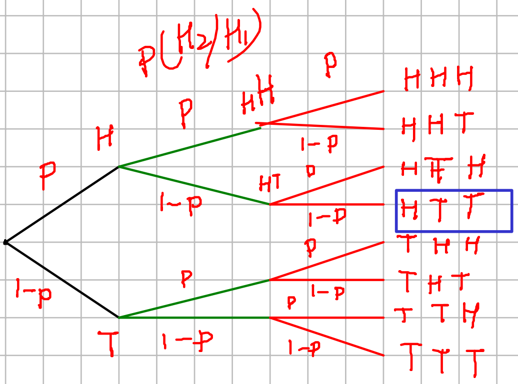
$= 0.012 + 0.016 = \underline{0.028}$

Q An item is found to be defective what is the probability that is is produced by machine-II.

$P(m_2/D) = ?$

⇒ 3 tosses of a baised coin

$p(H) = p \qquad p(T) = 1-p$



$= p(1-p)^2$

$p(THT) = (1-p)\,p\,(1-p)$
$= p\,(1-p)^2$

$p(1 \text{ Head})$
$= p(HTT) + p(THT) + p(TTH)$
$= \underline{3p(1-p)^2}$

$\dfrac{P(1^{st} H \cap 1 \text{ Head})}{P(1 \text{ head})} = P(\text{first toss is Head} / 1 \text{ Head})$

$= \dfrac{p(1-p)^2}{3p(1-p)^2} = 1/3$

Independence of two Events :→

$P(A/B) = P(A)$

$\Rightarrow \dfrac{P(AB)}{P(B)} = P(A)$ assuming $p(B) \neq 0$

✓ $\boxed{P(AB) = p(A)\,p(B)}$ "Definition"

— symmetric w.r.t. A & B

- $P(A/B) = P(A)$
- Applicable even when $P(B) = 0$

Prob Theorem

If A & B are independent, then A and B^c are independent or NOT independent

$$\boxed{A = \underline{AB^c} \cup \underline{AB}}$$

$$P(A) = P(AB^c) + P(AB)$$

Independent

$$P(AB^c) = P(A) - P(AB)$$

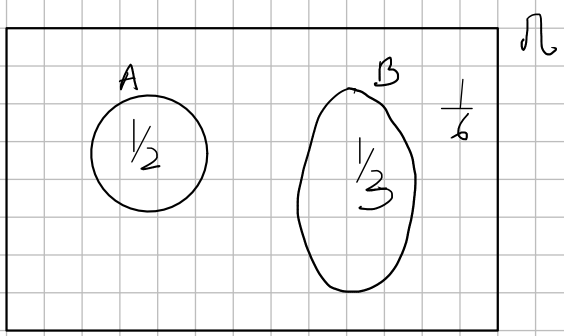
$$= P(A) - P(A)\,P(B)$$

$$= P(A)\,[1 - P(B)]$$

$$P(AB^c) = P(A)\,P(B^c)$$

Independence
disjoint
$>$ twins

$\Rightarrow$



$P(A) = 1/2$

$P(B) = 1/3$

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{0}{1/3} = 0$$

$$P(A/B) \neq P(A)$$

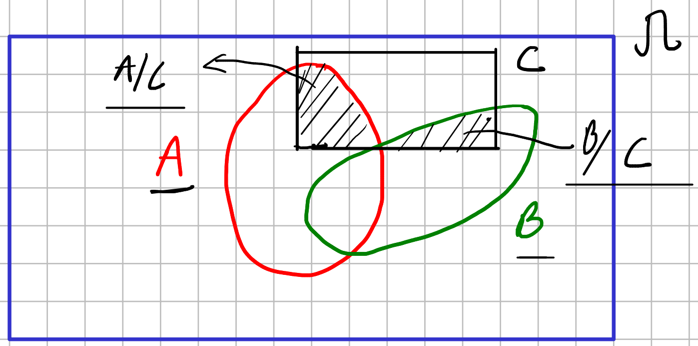
Conditional Independence :→

$$P(A \cap B/C) = P(A/C) \times P(B/C)$$

assume that A and B are independent

→ If we are told that event C has occured

→ Are A & B independent? → may or may not be

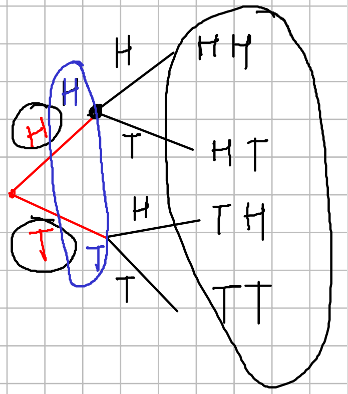


Q We are tossing the coin twice

A : Head on first toss

B : Head on second toss

→ Are A & B independent



1/2

2nd Toss ↑

H T	T T
H H	T H

→ First toss

$A = \{HH\} \cup \{HT\}$

$P(A) = P(\{H,H\}) + P(\{HT\})$

$= \frac{1}{4} + \frac{1}{4} = 1/2$

$P(B) = \frac{1}{4} + \frac{1}{4} = 1/2$

$P(A \cap B) = \frac{1}{4}$

$=$

$P(A)\,P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

A & B are independent

→ C = both tosses are different from one another

→ If C has occurred are, A & B still

Independent?

H T	T T
H H	T H

$$P(A \cap B / C) = P(A/C) \cdot P(B/C)$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$0 \qquad \neq \qquad \underbrace{1/4 \qquad 1/4}_{1/16}$$

A & B are not independent under this new condition

→ D = both tosses are not <u>tail</u>

→ If D has occurred are A & B still Independent

H T	T T
H H	T H

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{2/4}{3/4} = 2/3$$

$$P(A \cap B/D) = P(A/D) \cdot P(B/D)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$1/3 \qquad \neq \qquad 2/3 \qquad 2/3$$

⇒ Not Independent any more

→

Independence of Collection of Events :→

<u>n=3</u>

$A_1 \ , \ A_2 \ , \ A_3$

→ $P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2)\, P(A_3)$ ✓

→ $P(A_1 \cap A_2) = P(A_1) \times P(A_2)$

→ $P(A_2 \cap A_3) = P(A_2) \times P(A_3)$

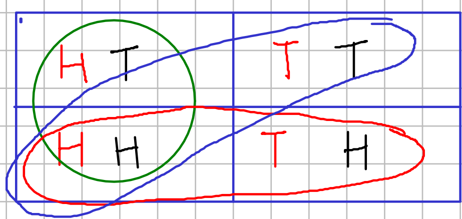
→ $P(A_3 \cap A_1) = P(A_3) \cdot P(A_1)$

→ Flipping the coin twice

A : Head on first toss

B : Head on second toss

C : Both tosses are same



$P(A) = \frac{1}{2}$

$P(B) = 1/2$

$P(C) = 1/2$

$P(A \cap B) = 1/4$

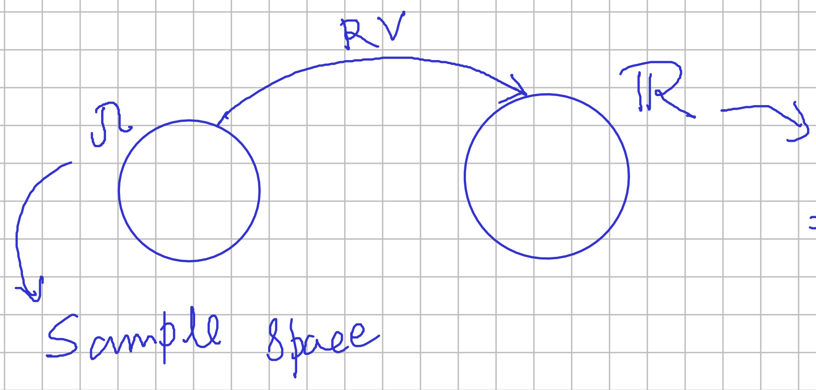
$P(B \cap C) = 1/4$

$P(C \cap A) = 1/4$

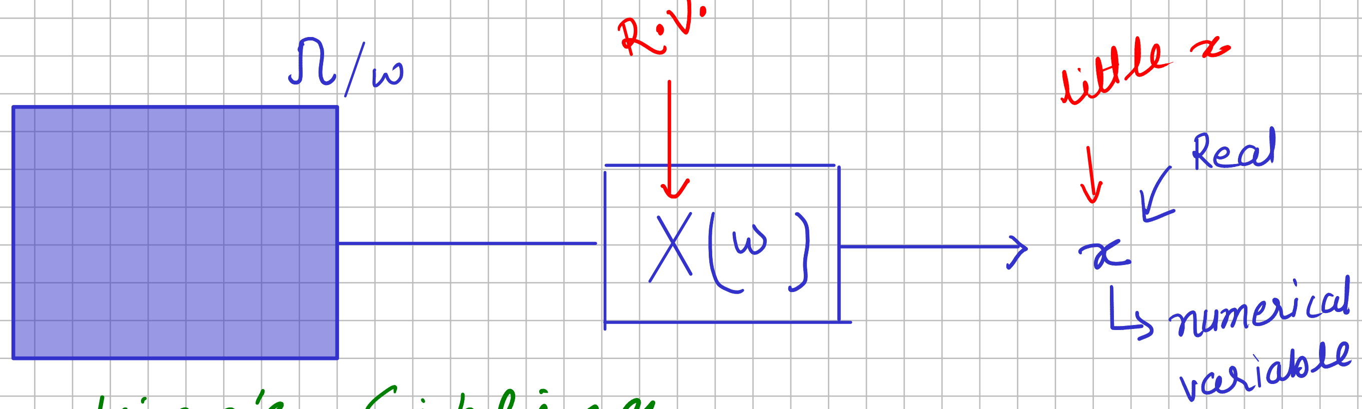
$P(A \cap B \cap C) = \frac{1}{4} \neq P(A) \cdot P(B) \cdot P(C)$

Random variable

RV is not a "VARIABLE"!
It is a "Function".
"Rule"
"Mapping"



→ Reals
= set of Real Numbers



The king's Sibling.

Ex A king comes from a family of two children. What is the probability that his sibling is a Female.

→ Boys has precedence over girl.

→ $P(B) = P(G) = 1/2$

Ω

B G	B B
~~G G~~	G B

$$P(G/K) = 2/3$$

→

Random variable :-> Assigns real number to each outcome.

Ex

Flipping a coin : $\Omega = \{ \overset{\omega}{\downarrow}H, T \}$

	Ram		Shyam
H →	R	$\xrightarrow{2}$	S
T →	R	$\xleftarrow{3}$	S

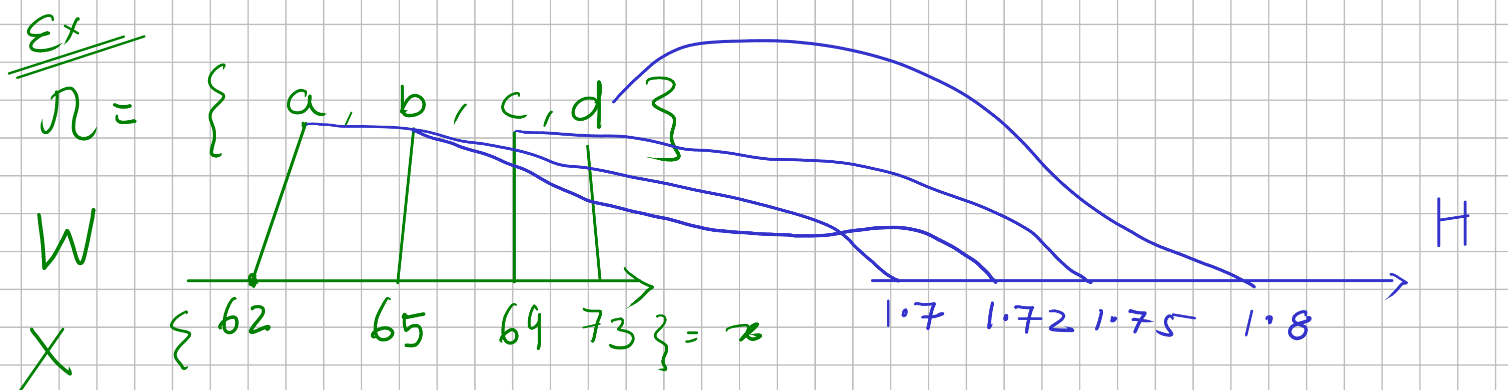
X :- How much Ram will win

$x = +3, -2$

$P_X(X=3) = 1/2$

$P_X(X=-2) = 1/2$

Ex



$B = \frac{W}{H^2}$

- A Random variable (r.v.) associates a value to every possible outcome.
- A Function from sample space to the real numbers.
- R.V. can be continuous or discrete

X : Random variable

x : Numerical value

- Several R.V. can be defined over the same Ω.

PMF (Probability Mass Function) of Discrete R.V.

- "Probability law" or "probability distribution" of Random variable X.

- if we fix some x, then "$X = x$" is an event

$P_X(X=5) = 1/2$ (for example)

$$p_X(x) = P(X=x) = P\{\omega \in \Omega \mid X(\omega) = x\}$$

Ex we are tossing a coin twice

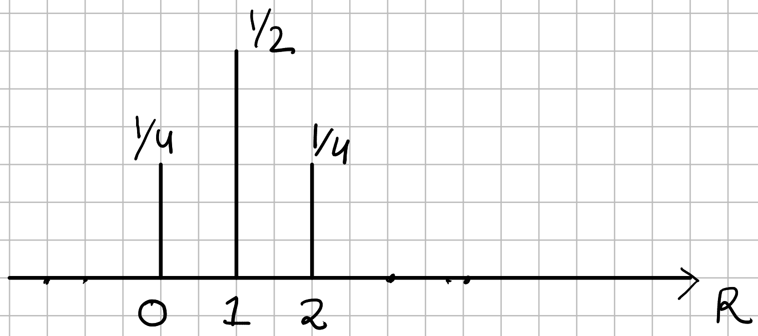
Ω

T H	T T
H H	H T

X = number of Tail in the outcome

$X = \{0, 1, 2\}$

$$
\begin{cases}
p_X(0) = P(X=0) = P(\{HH\}) = 1/4 \\
p_X(1) = P(X=1) = P(\{HT, TH\}) = 2/4 \\
p_X(2) = P(X=2) = P(\{TT\}) = 1/4
\end{cases}
$$



properties

- $p_X(x) \geq 0$
- $\sum_x p_X(x) = 1$

Ex. Two roll of a tetrahedral die. Random variable X is defined as the sum of the numbers that one gets after rolling the die. find the PMF.

Ω

1 4	2 4	3 4	4 4
1 3	2 3	3 3	4 3
1 2	2 2	3 2	4 2
1 1	2 1	3 1	4 1

X

5	6	7	8
4	5	6	7
3	4	5	6
2	3	4	5

$X = \{2, 3, 4, 5, 6, 7\}$

$p_X(2) = 1/16$ $\quad p$

$p_X(3) = 2/16$

$p_X(4) = 3/16$

$p_X(5) = 4/16$

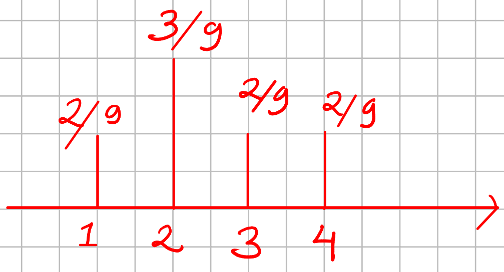
$p_X(6) = 3/16$

$p_X(7) = 2/16$

$p_X(8) = 1/16$

Expectation of RV [Mean / Average] :→

p_X



$$X = \begin{cases} 1 \rightarrow 2/9 \\ 2 \rightarrow 3/9 \\ 3 \rightarrow 2/9 \\ 4 \rightarrow 2/9 \end{cases}$$

$$900 \begin{cases} \rightarrow ₹\,1 \rightarrow 200 \\ \rightarrow ₹\,2 \rightarrow 300 \\ \rightarrow ₹\,3 \rightarrow 200 \\ \rightarrow ₹\,4 \rightarrow 200 \end{cases}$$

$$\text{Average winning} = \frac{200 \times 1 + 300 \times 2 + 200 \times 3 + 200 \times 4}{900}$$

$$= 1 \times \frac{2}{9} + 2 \times \frac{3}{9} + 3 \times 2/9 + 4 \times \frac{2}{9}$$

$$= \frac{2+6+6+8}{9} = 22/9$$

$$E(X) = \sum_x x \, p_X(x)$$

$$\boxed{Y = g(X)}$$

$$E[Y] = \sum_y y \, p_Y(y) = \sum_x g(x) \, p_x(x)$$

$$E[g(x)] = \sum_x g(x) \, p_x(x)$$

in general $\underline{E[g(x)] \neq g[E(x)]}$

$$\alpha X + \beta$$

$$g(x) = \alpha X$$

$$E[g(x)] = \sum_x g(x) \, p_X(x)$$

$$= \sum_x \alpha x \, p_x(x)$$

$$E[\alpha x] = \alpha \, E(x)$$

$$E[\beta] = \sum_x \beta \, p_{\cancel{x}}(\cancel{x}) = \beta \sum_x p_X(x)$$

$$= \beta$$

$$\boxed{E[\alpha X + \beta] = \alpha E[X] + \beta}$$

$$E[X-\mu_x] = E[X]-\mu_x$$ ✓

$$= \mu_x - \mu_x = 0$$

$E[\,|X-\mu_x|\,] \Rightarrow$ difficult to calculate

$$E[(X-\mu_x)^2] = var(X)$$

$$E[X^2+\mu_x^2-2\mu_x X]$$

$$= E[X^2]+\mu_x^2 E[1] - 2\mu_x E[X]$$

$$= E[X^2]+\mu_x^2 - 2\mu_x^2$$

$$= E[X^2]-\mu_x^2$$

$$var(X) = E[(X-\mu_x)^2] = E[X^2]-(E[X])^2$$

$-ve \rightarrow$ can not

$0 \rightarrow$ can be if $p_X(x) = const$

Standard Deviation

$$\sigma_x = \sqrt{var(X)}$$

$$E[(X-\mu_x)^2] = E[X^2] - (E[X])^2$$

$$E[X^2] = E[X^2] + E[(X-\mu_x)^2]$$

Total power = d.c. power + a.c. power

$$\text{Relative Error} = \frac{\sigma_x}{\mu_x}$$

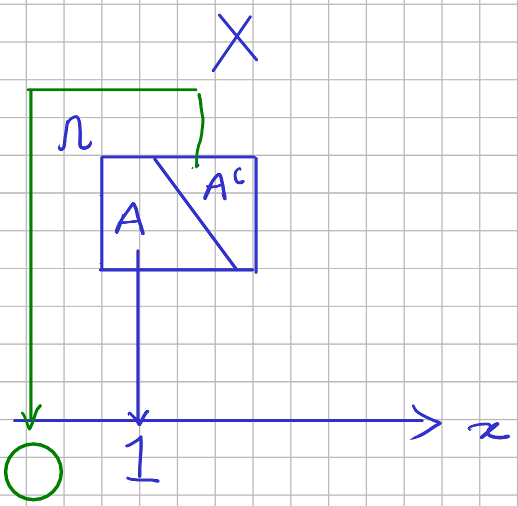
n^{th} order
Central moment (around mean(μ_x))

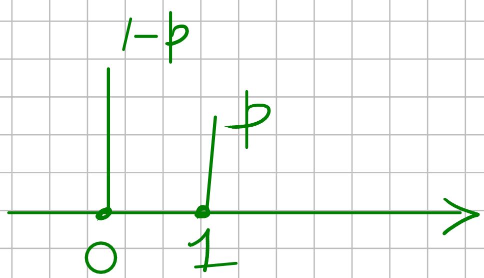
$$E[(X-\mu_x)^n] = \sum_x (x-\mu_x)^n \, p_X(x)$$

moment around origin

$$E[X^n] = \sum_x x^n \, p_X(x)$$

Bernoulli R.V. $\quad p \in [0, 1]$





$I_A = 1$ iff A occurs

$$E[X] = 0 \times (1-p) + p \cdot 1 = p$$

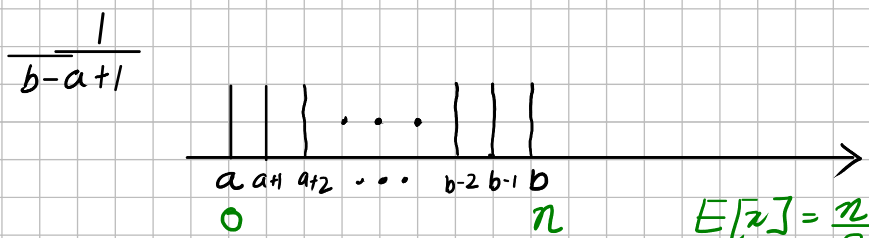
$$E[(X-\mu_x)^2] = E[X^2] - (E[X])^2$$

$$= p - p^2 = p(1-p)$$

$$E[X^2] = 0^2 \times (1-p) + 1^2 \cdot p = p$$

Uniform R.V. $[a, b]$

Experiment: that you pick a number between a & b at Random.

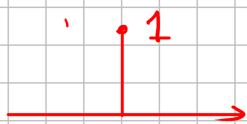


$E[x] = \frac{n}{2}$

Special case

$b = a$

constant



$$\Rightarrow \mu_x = E[X] = \sum_x x \, p_X(x)$$

$$= [a + a+1 + a+2 + \dots + b] \frac{1}{b-a+1}$$

$$= \frac{b-a+1}{2} [a+b] \frac{1}{b-a+1} = \frac{a+b}{2}$$

Binomial R.V.

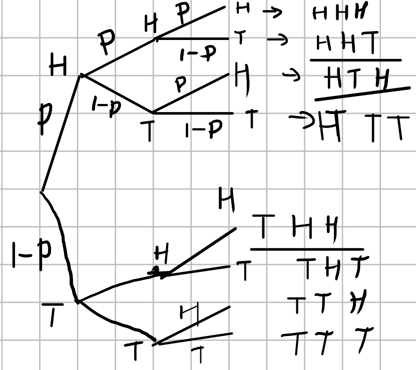
parameters $\left[+ve \text{ integer } n, \; p \in [0,1] \right]$

Ex: n independent tosses of a coin $p(H) = p$

Sample: set of sequence of H & T of length n

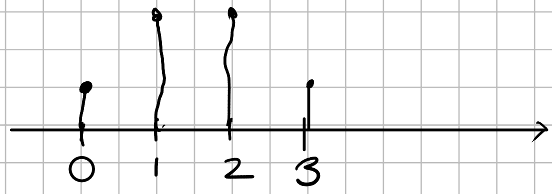
X : numbers of heads observed

$n = 3$



$$P(X=2) = 3\,p^2(1-p)$$

$$= {}^3C_2\, p^2 (1-p)^1$$



$$P(X=k) = {}^nC_k\, p^k (1-p)^{n-k}$$

Geometric Random Variable :→ p $\quad 0 < p \leq 1$

Experiment : Infinite independent tosses of a coin.

Sample space : set of sequence of H & T

Random Variable : number of tosses until the first head

$$\underset{1}{T}\ \underset{2}{T}\ \underset{3}{T}\ \underset{4}{T}\ \underset{5}{T} \cdots \underset{k-1}{T}\quad \underset{k}{H}$$

$$P_X(X=k) = (1-p)^{k-1}\, p = q^{k-1}\, p$$

P(no heads) $\qquad x = \{TTTT\cdots\infty$

$$(1-p)^{\infty} = 0$$

$$= 1 - p - qp - q^2p - q^3p \cdots \infty$$

$$= 1 - p(1 + q + q^2 \cdots \infty)$$

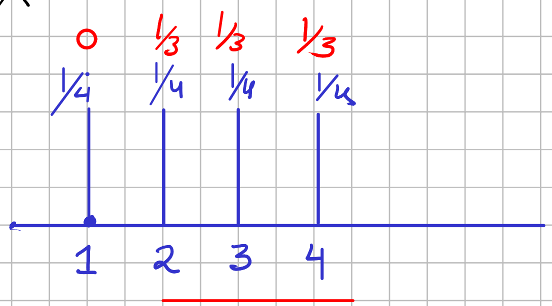
$$= 1 - \cancel{p}\,\frac{1}{\cancel{1-q}} = 1 - 1 = 0$$

Conditional PMF

one R.V. X , $A \Rightarrow$

Let $A = \boxed{X \geq 2}$

$$p_X(x) = P(X = x)$$



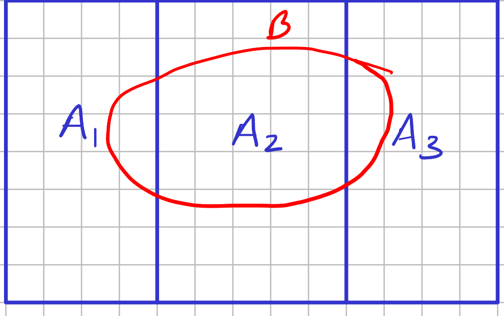
$E[X] = 2.5$

$E[X/A] = 3$

$$p_{X/A}(x) = P(X = x/A)$$

$$E[X/A] = \sum_x x \, p_{X/A}(x)$$

Total Expectation Theorem



Ω : $A_1\, A_2\, A_3$ are partions of Ω

$\sum_i p(A_i) = 1$

$A_i \cap A_j = \phi \quad \forall\, i \neq j$

X ; r.v. ; some event $\mathcal{B}$ to real number

Recall : $p(B) = p(A_1 B) + p(A_2 B) + p(A_3 B)$

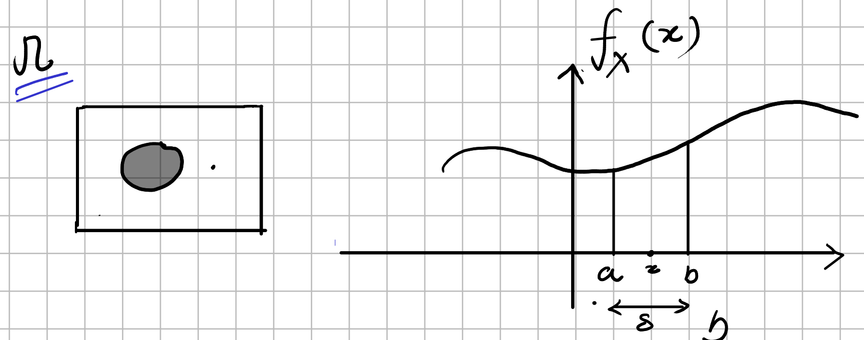
$= p(A_1)\, p(B/A_1) + p(A_2)\, p(B/A_2) + p(A_3)\, p(B/A_3)$

$$p_X(x) = p(A_1)\, p_{X/A_1}(x) + p(A_2)\, p_{X/A_2}(x) + p(A_3)\, p_{X/A_3}(x)$$

$$p_X(x) = \sum_i p(A_i)\, p_{X/A_i}(x)$$

$$E[X] = \sum_i p(A_i)\, E[X/A_i]$$

Continuous Random variable



X

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$P(a < X \le b) = \int_a^b f_X(x)\, dx$$

$\delta \to 0 \qquad P(x) = f_X(x)\, \delta \qquad (a \approx b = x)$

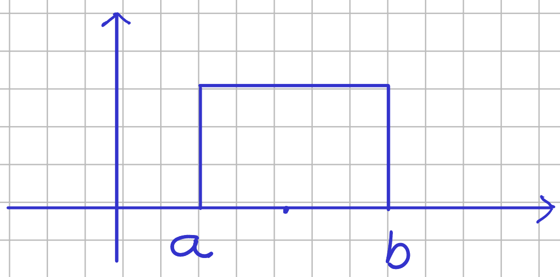
$$\mu_x = E[X] = \sum_{\forall x} x \, p_X(x) = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$$\sigma_X^2 = E\left[(x-\mu_x)^2\right] = \int_{-\infty}^{\infty} (x-\mu_x)^2 f_X(x) \, dx$$

$$\boxed{\sigma_X^2 = E[X^2] - \{E[X]\}^2}$$

$$\boxed{\sigma_X^2 = E[X^2] - \mu_x^2}$$

X



$$f_X(x) = \frac{1}{b-a} \quad a \le x \le b$$
$$= 0 \quad ; \text{ otherwise}$$

$$\mu_x = \int_a^b x \frac{1}{b-a} \, dx = \frac{1}{2(b-a)} \left[b^2 - a^2\right] = \frac{b+a}{2}$$

$$\sigma_x^2 = E[X^2] - \mu_x^2$$

$$E[X^2] = \int_a^b k x^2 \, dx = \left.\frac{k x^3}{3}\right|_a^b = \frac{k}{3}(b^3 - a^3)$$

$$= \frac{1}{3\cancel{(b-a)}} \cancel{(b-a)} (b^2 + a^2 + ab)$$

$$= \frac{b^2 + a^2 + ab}{3} - \left(\frac{b+a}{2}\right)^2$$

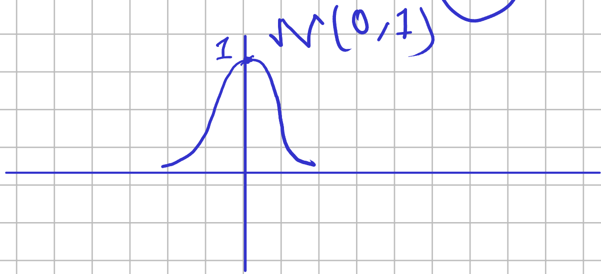
$$= \frac{4b^2 + 4a^2 + 4ab - 3b^2 - 3a^2 - 6ab}{12}$$

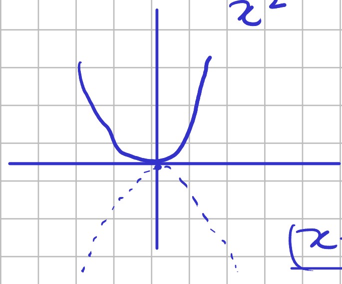
$$= a^2 + b^2 - 2ab/12$$

$$\sigma_x^2 = \frac{(b-a)^2}{12}$$

Gaussian Distribution : $\mathcal{N}(0, 1)$

$$\frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2}\right)}$$





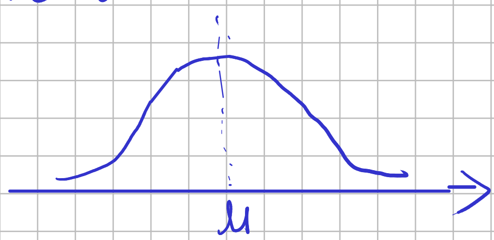
$$\frac{(x-\mu)^2}{2}$$

$\mathcal{N}(\mu, \sigma)$

$$\boxed{\hookrightarrow f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

$\sigma \rightarrow$ std. Deviation

$\mu \rightarrow$ mean



$\boxed{X \rightarrow \mu_x}$

$\sigma_x^2 =$

$Y = aX + b$ ← shifting

a ↳ scaling

$\mu_y = ?$

$$\boxed{\sigma_y^2 = ? = a^2 \sigma_x^2}$$

$$\mu_y = E[Y] = E[aX + b] = a\,E[X] + b$$
$$= a\,\mu_x + b$$

$$\sigma_y^2 = E[(Y - \mu_y)^2] =$$

$$\text{var}(Y) = E[Y^2] - (E[Y])^2$$

$$= E\left[a^2x^2 + b^2 + 2abx\right] - (a\mu_x + b)^2$$

$$= a^2 E[x^2] + b^2 + 2ab\mu_x - (a\mu_x + b)^2$$

$$= \underline{a^2 E[x^2]} + \cancel{b^2} + \cancel{2ab\mu_x} - \underline{a^2\mu_x^2} - \cancel{b^2} - \cancel{2ab\mu_x}$$

$$= a^2\left[E[x^2] - (E[x])^2\right]$$

$$\boxed{\text{var}(Y) = a^2\sigma_x^2}$$