Name: Naga Jayanth Chennupati

Student ID: 21002083

Email: njchennu@uwaterloo.ca

① According to assignment rule and consequence rule, we can infer the program to be three constraints (as the read three parts) as follows:

$$\dfrac{\dfrac{\vdash n \geq 0 \Rightarrow I[0/r, 0/i, 1/p]}{\vdash \{n \geq 0\}\ r:=0;\ i:=0;\ p:=1\ \{I\}} \quad \dfrac{\dfrac{\dfrac{\vdash I \wedge i \neq n \Rightarrow I[(r+p)/r,\ 2p/p,\ (i+1)/i]}{\vdash \{I \wedge i \neq n\}\ r:=r-p;\ p:=2p;\ r:=r+p;\ i=i+1\ \{I[r+p/r,\ 2p/p,\ i+1/i]\}}}{\vdash \{I\}\ \text{while } i \neq n \text{ do } (r:=r-p;\ p:=2p;\ i:=i+1)\{I \wedge x = n\}}}{\vdash \{I\}\ \text{while } i \neq n \text{ do } (r:=r-p;\ p:=2p;\ r:=r+p;\ i:=r+1)\{r = 2^n - 1\}}}{\vdash \{n \geq 0\}\ r:=0;\ i:=0;\ p:=1;\ \text{while } i \neq n \text{ do } (r:=r-p;\ p:=2p;\ r:=r+p;\ i:=r+1)\ \{r = 2^n - 1\}}$$

where I is invariant for while loop

if I is valid to all three constraints, then this program is proved to be correct. Three constraints are as:

1. $\vdash n \geq 0 \Rightarrow I[0/r, 0/i, 1/p]$
2. $\vdash I \wedge i \neq n \Rightarrow I[(r+p)/r, 2p/p, (i+1)/i]$
3. $\vdash I \wedge i = n \Rightarrow r = 2^n - 1$

Let $I = (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n)$, then:

1. $\vdash n \geq 0 \Rightarrow p = 1 \wedge r = 0 \wedge n \geq 0$
2. $\vdash (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i \neq n \Rightarrow p = 2^{(i+1)} - 1$

   $\wedge (i+1) \leq n \wedge (i+1) \neq n$
3. $\vdash (p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i \neq n \wedge i = n \Rightarrow 2^n - 1$

All constraints are valid!

This program is correct.