Problem-1

a) The use of these membership functions to present the linguistic hedges seems appropriate as they capture the idea of the the linguistic modifiers "very" and "presumably". We can call them as descriptors too. They indicate the fuzziness or uncertainity of the universe.

→ The state of "fast speed" is expressed by a membership function $\mu_F(V)$ of the fuzzy set "F" with elements as V

→ $\mu_F(V)\ \forall\ V \in F$

→ Considering "very fast speed" ~~is~~, it is represented by membership function as $\mu_F(V-V_0)\ \forall\ V \in F$ where $V_0 > 0$.

When $V_0 > 0$, the membership function will ~~shift~~ be shifting towards the right of the graph indicating a "very fast" region. The values of this membership function will more than the original function showing that it is acceptable/appropriate.

When $V_0 < 0$, the membership function ~~is~~ will be shifting towards the left of the graph indicating lower speeds than the original making it ~~is~~ ~~a~~ inappropriate.

→ Considering "presumably fast speed", it is expressed by membership function of $\mu_F^2(V)\ \forall\ V \in F$. The value decreases compared to the original showing contraction for the value [0,1]. The speed decreases saying this one is inappropriate. ~~Al~~/The

b) Given,

Fuzzy Set = $\left\{\frac{0.1}{10}, \frac{0.3}{20}, \frac{0.6}{30}, \frac{0.8}{40}, \frac{1.0}{50}, \frac{0.7}{60}, \frac{0.5}{70}, \frac{0.3}{80}, \frac{0.1}{90}\right\}$

$V = \{0, 10, 20, 30 \ldots 199\ 200\}$ rev/s and $V_0 = 50$ rev/s

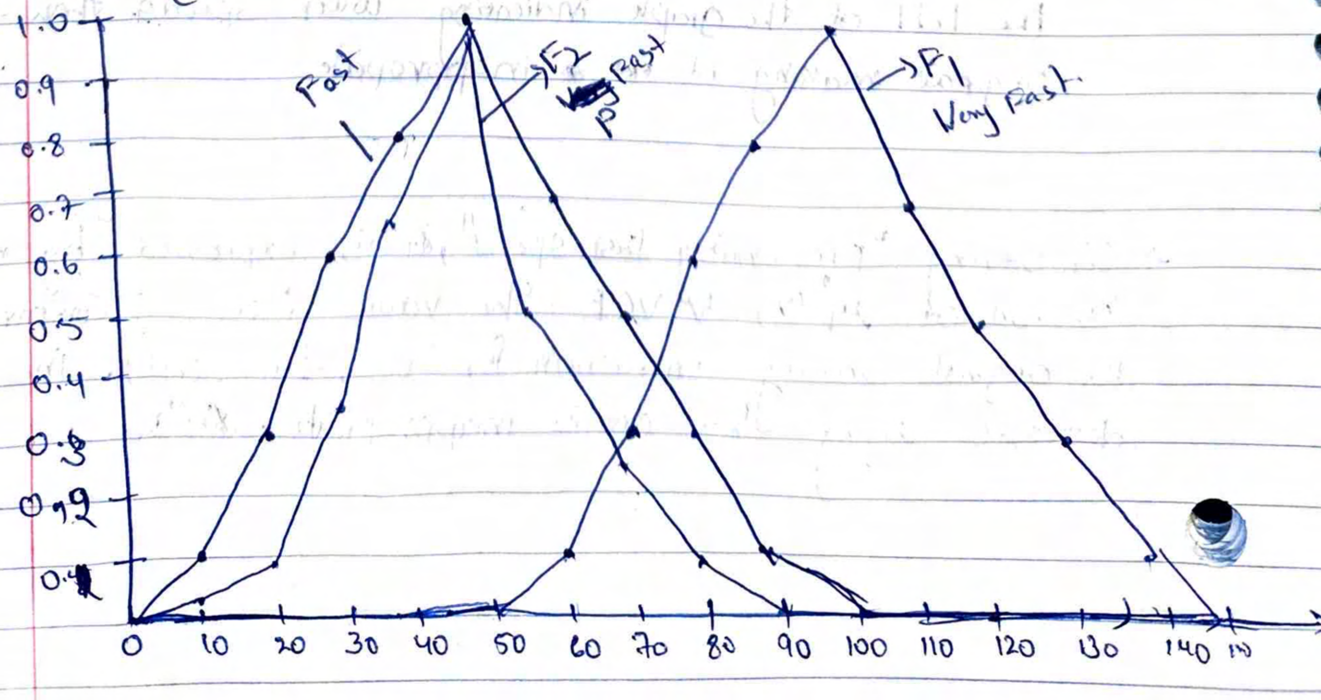
→ Very fast speed → $\mu_F(V - V_0)\ \forall V \in V$ $V_0 = 50$

$$F_1 = \left\{\frac{0.1}{60} + \frac{0.3}{70} + \frac{0.6}{80} + \frac{0.8}{90} + \frac{1.0}{100} + \frac{0.7}{110} + \frac{0.5}{120} + \frac{0.3}{130} + \frac{0.1}{140}\right\}$$

For values, 10, 20, 30, 40, 50, 150, 160, 170, 180, 190, 200 → after $V - V_0$ we don't have the membership values so we considered them as '0'

→ presumably fast speed → $\mu_F^2(V)\ \forall V \in V$

$$F_2 = \left\{\frac{0.01}{10} + \frac{0.09}{20} + \frac{0.32}{30} + \frac{0.64}{40} + \frac{1}{50} + \frac{0.49}{60} + \frac{0.25}{70} + \frac{0.09}{80} + \frac{0.01}{90}\right\}$$



Problem-2

Given,

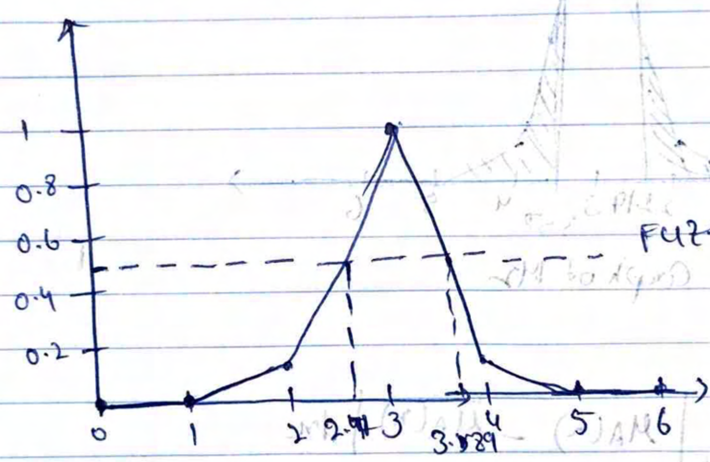
membership function $\mu_A(x) = e^{-\lambda(x-a)^n}$ for $\lambda = 2, n = 2$ and $a = 3$ for the Support Set $S = [0, 6]$

Let's plot the original one:

$$\mu_A(x) = e^{-2(x-3)^2}$$

The

$$\mu_A(x) = \left\{ \frac{0}{1} + \frac{0.135}{2} + \frac{1}{3} + \frac{0.135}{4} + \frac{0}{5} + \frac{0}{6} \right\}$$



Fuzziest part of the system.

a) $M_1 = \int_S f(x)\,dx$ where $f(x) = \mu_A(x)$ $\mu_A(x) \le 0.5$

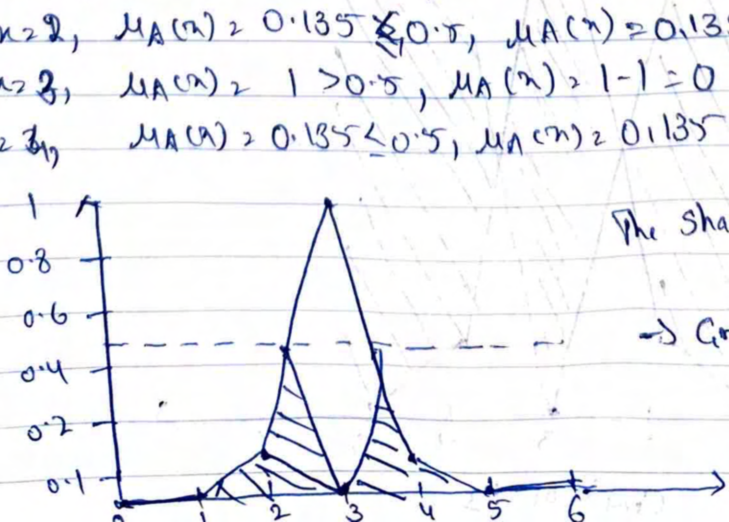
$= 1 - \mu_A(x)$ $\mu_A(x) > 0.5$

$x = 0$, $\mu_A(x) = 0 \le 0.5$, $\mu_A(x) = 0$

$x = 2$, $\mu_A(x) = 0.135 \le 0.5$, $\mu_A(x) = 0.135$

$x = 3$, $\mu_A(x) = 1 > 0.5$, $\mu_A(x) = 1 - 1 = 0$

$x = 4$, $\mu_A(x) = 0.135 \le 0.5$, $\mu_A(x) = 0.135$



The shaded area represent M_1

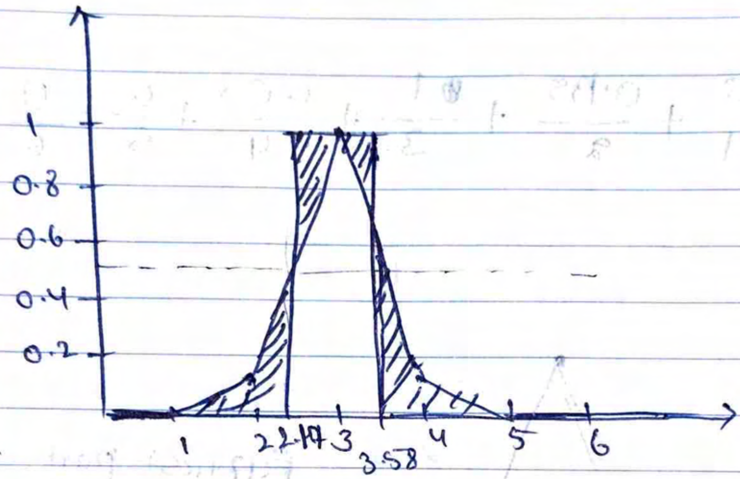
→ Graph for M1

b) $M_2 = \int_S |\mu_A(x) - \mu_{A^{1/2}}(x)|\, dx$

where $\mu_{A^{1/2}}$ is α-cut of $\mu_A(x)$ for $\alpha = 1/2$

So, $\mu_{1/2} = 1$ when $\mu_A(x) \geq 0.5$

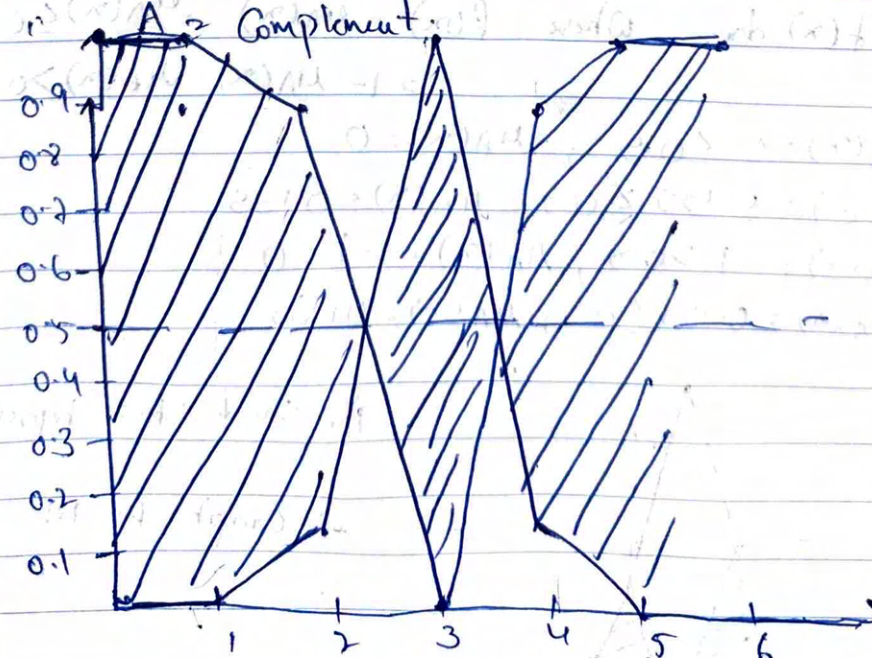
$\mu_{1/2} = 0$ when $\mu_A(x) < 0.5$



Graph of M_2

c) $M_3 = \int_S |\mu_A(x) - \bar{\mu}_A(x)|\, dx$

$\bar{A}$ = Complement



Graph of M_3

(i) Establishing relationship b/w M_1, M_2 and M_3

$$M_1 = \int_S f(x)$$

$$\Rightarrow \int_{2.4}^{2.41} \mu A(x)\,dx + \int_{2.41}^{3.589} (1-\mu A(x))\,dx + \int_{3.589}^{6} \mu_A(x)\,dx$$

$$M_1 \Rightarrow \int_0 \mu A(x)\,dx + \int [3.589 - 2.41] + \int_{2.41}^{3.589} \mu_A(x)\,dx + \int_{3.589}^{6} \mu_A(x)\,dx$$

~~separation~~

$$M_1 \Rightarrow \int_0^{2.41} \mu A(x)\,dx + \int_{3.589}^{6} \mu A(x)\,dx - \int_{2.41}^{3.589} \mu A(x)\,dx + 1.179 \rightarrow (1)$$

$$M_2 = \int_S |\mu A(x) - \mu A_{1/2}(x)|\,dx \quad \text{where } \mu A_{1/2}(x) = 1 \quad x \geq 0.5$$
$$= 0 \quad x \leq 0.5$$

$$\Rightarrow \int_0^{2.41} \mu A(x)\,dx + \int_{2.41}^{3.589} (1-\mu A(x))\,dx + \int_{3.589}^{6} \mu A(x)\,dx$$

This represents
α-cut here

$\Rightarrow$ This is same as the eqn (1)

$\therefore \quad M_1 = M_2$

$$M_3 = \int_S |\mu_A(x) \cdot \overline{\mu_A(x)}|\,dx \Rightarrow \int_S |\mu_A(x) - [1-\mu_A(x)]|\,dx$$

$$= \int_S |2\mu_A(x) - 1|\,dx$$

$$\Rightarrow 2\left(\int_{2.41}^{3.589} \mu A(x)\,dx - \int_{0}^{2.41} \mu A(x)\,dx - \int_{3.589}^{6} \mu A(x)\,dx\right) + 2.41 + 2.4 - 1.179$$

Usg eqn ①

$$= 2[1.179 - M_1] + 4.82 - 1.179$$

$$\Rightarrow M_3 = 6 - 2M_1$$

$\therefore \; M_1 = M_2 = \frac{1}{2}[S - M_3]$ S = length of support set.

Evaluation M_1, M_2 and M_3 for $\mu A(x)$

$$M_1 = \int_0^{2.41} e^{-2(x-3)^2}dx + \int_{2.41}^{3.589} \left(1 - e^{-2(x-3)^2}\right) dx + \int_{3.589}^{6} e^{-2(x-3)^2}$$

$$= \left[\frac{e^{-2(x-3)^2}}{-4[x-3]}\right]_0^{2.41} + \left[\frac{x - e^{-2(x-3)^2}}{-4[x-3]}\right]_{2.41}^{3.589} + \left[\frac{e^{-2(x-3)^2}}{-4[x-3]}\right]_{3.589}^{6}$$

$$\Rightarrow [0.212 + 1.179 + 0.4233 + 0.212] = M_1 = M_2$$

$$M_3 = 6 - 2M_1 = 1.95$$

(ii) The fuzziness measures M_1, M_2 and M_3 in fuzzy logic's provide valuable information about the degree of fuzziness or uncertainity with a membership function of a fuzzy set.

→ By analyzing and comparing these fuzziness measures, we can gain insights into overall fuzziness of the membership function. The combination of M_1, M_2 and M_3 helps us to assess the uncertainity, spread and centering of the membership function, allowing us to draw better conclusions.

(iii) $\lambda = 1, a = 3, n = 2$, Support Set $= [0, 6]$

$$\mu_B(x) = e^{-(x-3)^2}$$

$x = 0 \to \mu A(0) = 0$

$x = 1 \to \mu A(1) = 0.018$

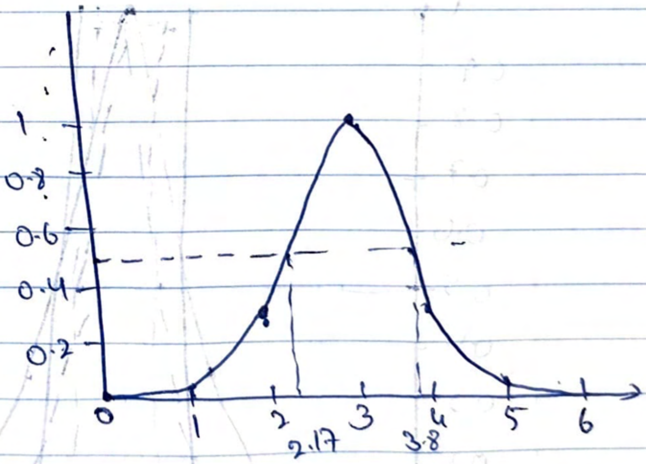
$x = 2 \to \mu A(2) = 0.367$

$x = 3 \to \mu A(3) = 1$

$x = 4 \to \mu A(4) = 0.367$

$x = 5 \to \mu A(5) = 0.018$

$x = 6 - \mu_6(6) = 0$



a) $M_1 = \int_3 f(x)\,dx$ $\quad f(x) = \mu A(x) \quad \mu A(x) \leq 0.5$

$= 1 - \mu A(x) \quad \mu A(x) > 0.5$

$x = 0 \quad \mu A(0) = 0$

$x = 1 \quad \mu A(1) = 0.018$

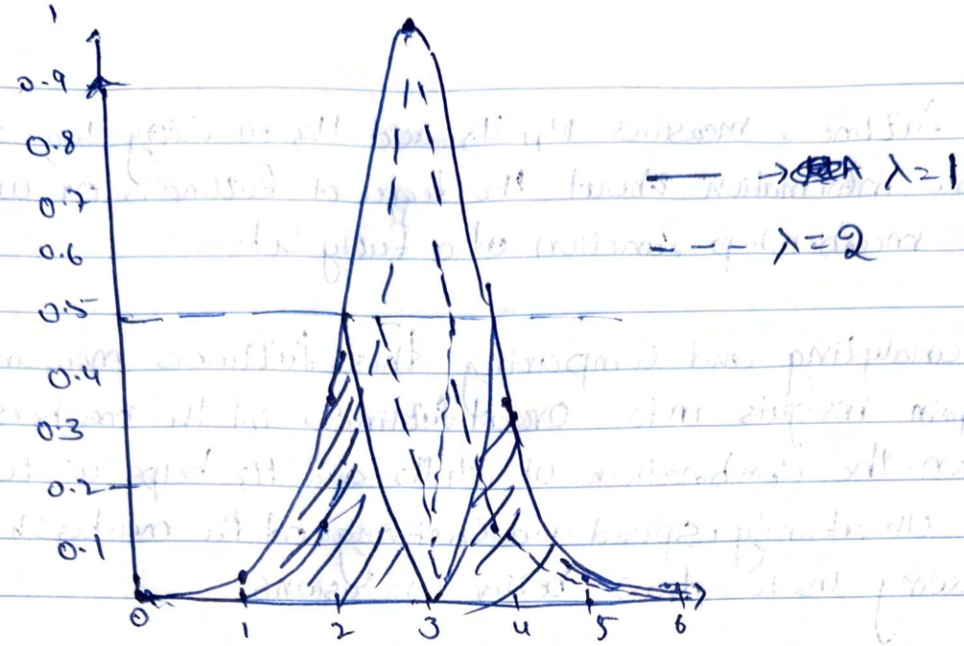
$x = 2 \quad \mu A(2) = 0.367$

$x = 3 \quad \mu A(3) = 0$

$x = 4 \quad \mu A(4) = 0.367$

$x = 5 \quad \mu A(5) = 0.018$

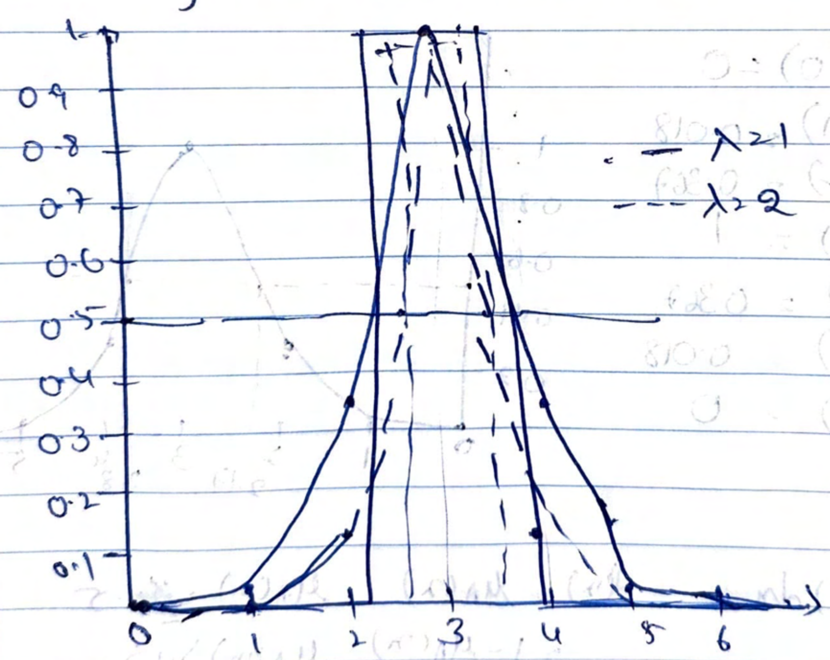
$x = 6 \quad \mu A(6) = 0$



λ=1, Graph for M_1

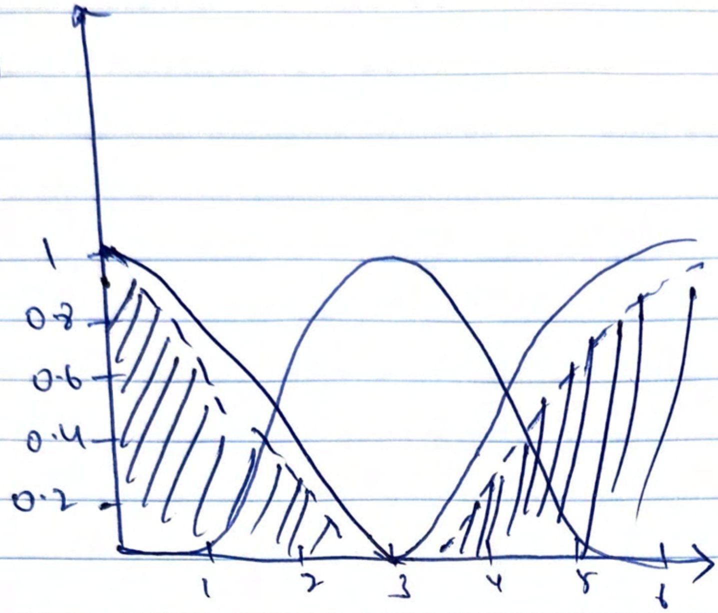
Fuzziness in M_1 is less when $\lambda=1$ than $\lambda=2$

b) $M_2 = \int | \mu_A(x) - \mu_{A1/2}(x) | \, dx$



→ The fuzziness of λ=1 is less for values less than 0.5 and more for values greater than 0.5.

3)



M3

Here, by comparing the measure of fuzziness for both, we can clearly conclude that $\mu_B(x)$ is fuzzier than $\mu_A(x)$.

Question 3

Given,

$\chi_A(x) = 1$ if $x \in A$

$= 0$ otherwise.

let's us consider three sets.

$A = \{1, 2, 3, 4\}$

$B = \{2, 3, 4, 5\}$

$X = \{1, 2, 3, 4, 5, 6\}$

To prove :-

$\bar{\chi}_A = 1 - \chi_A$ where $\chi_A = \{1, 2, 3, 4\}$

(values)	χ_A	$\bar{\chi}_A$	$1 - \chi_A$
1	1	0	0
2	1	0	0
3	1	0	0
4	1	0	0
5	0	1	1
6	0	1	1

(Compare each value with set A if it's present χ_A is 1 }

from the above table $\bar{\chi}_A = \{5, 6\}$ and it's also clear that both $\bar{\chi}_A$ and $1 - \chi_A$ shows same results which implies $\bar{\chi}_A = 1 - \chi_A$

(ii) To prove.

$\chi_{A \cup B} = \max(\chi_A, \chi_B)$

Considering the sets A and B and if the selected value is present in the respective sets the value is 1.

(values)	x_A	x_B	$x_{A\cup B}$	$\max(x_A, x_B)$
1	1	0	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	0	1	1	1
6	0	0	0	0

From the above table $x_{A\cup B} = \{1, 2, 3, 4, 5\}$ and it's clear that both $x_{A\cup B}$ and $\max(x_A, x_B)$ produces same results which implies $x_{A\cup B} = \max(x_A, x_B)$

(iii) To prove

$$x_{A\cap B} = \min(x_A, x_B)$$

Similarly let's consider the sets A & B & if the value is present in the respective set the value is 1.

(values)	x_A	x_B	$x_{A\cap B}$	$\min(x_A, x_B)$
1	1	0	0	0
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	0	1	0	0
6	0	0	0	0

From the above table it is clear that $x_{A\cap B} = \{2, 3, 4\}$ and it is clear that both $x_{A\cap B}$ & $\min(x_A, x_B)$ provides same results which indicates $x_{A\cap B} = \min(x_A, x_B)$

(iv) To prove

$$\chi_{A\to B}(x,y) = \min[1, \{1 - \chi_A(x) + \chi_B(y)\}]$$

let's have different sets and universes for this

$X = \{1,2,3,4\}$ $Y = \{3,4,5,6\}$

Set A is defined in X; $A = \{1,2\}$

B is defined in Y: $B = \{4,5\}$

truth table would be

x	y	$\chi_A(x)$	$\chi_B(y)$	$\min[1, \{1-\chi_A(x)+\chi_B(y)\}]$	$\chi_{A\to B}$
1	3	1	0	0	0
1	4	1	1	1	1
1	5	1	1	1	1
1	6	1	0	0	0
2	3	1	0	0	0
2	4	1	1	1	1
2	5	1	1	1	1
2	6	1	0	0	0

In all the cases the result of $\chi_{A\to B}$, is indicating that the implication holds and it is clear from the truth table that $\chi_{A\to B} = \min[1, \{1-\chi_A(x) + \chi_B(y)\}]$

Implications

(i) $\bar{\chi}_A = 1 - \chi_A$

This is the complement of fuzzy set A. It capture the opposite of the original fuzzy set membership

(ii) $\chi_{A \cup B} = \max(\chi_A, \chi_B)$

This equation represents the union between two fuzzy sets. The membership value is determined by maximum of the corresponding membership values in A & B

(iii) $\chi_{A \cap B} = \min(\chi_A, \chi_B)$

This equation represents the intersection between fuzzy sets A & B and in this the membership value can be determined by taking the minimum of values in A & B.

(iv) $\chi_{A \to B}(x, y) = \min\{1, (1 - \chi_A(x)) + \chi_B(y)\}$

The equation is implication operation in fuzzy logic and essentially provides the if then relationship between sets A & B

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Question 4. To show: $\max(0, x+y-1)$ is T-norm.

Non-decreasing

For all x, y in the interval $[0,1]$ and if $x \le y$ then $\max[0, x+z-1] \le \max[0, y+z-1]$ for all z in $[0,1]$

Since $x \le y$ let consider.

Case 1: $x = 0$
$\max[0, z-1] = 0$ and since $z-1$ is non positive and $y \ge x$ $\max[0, x+z-1] \le \max[0, y+z-1]$

Case 2: $x > 0$
$\max[0, x+z-1] = x+z-1$
$\max[0, y+z-1] = y+z-1$

Since $x > 0$ and $x \le y$
$x+z-1 \le y+z-1$

which implies $\max[0, x+z-1] \le \max[0, y+z-1]$

Commutative property

For all x, y in $[0,1]$ $\max[0, x+y-1] = \max[0, y+x-1]$

Since addition is commutative we have $x+y = y+x$
Hence both sides of the equation are equal
$\Rightarrow \max[0, x+y-1] = \max[0, y+x-1]$

Boundary Condition

For all x in $[0,1]$ $\max[0, \cancel{0}+y-1] = 0$ and
$\max[0, 1+y-1] = y$

when $x=0$
$\max[0, 0+y-1] = \max[0, y-1] = 0$ since y is non-negative and the y in $[0,1]$ and maximum with 0 is 0.

when $x=1$
$\max[0, \cancel{1}+y-1] = \max[0, y] = y$ Same y is non-negative hence maximum with y is y.

Associative Condition

for all x, y and z in $[0,1]$ $t(t(x,y),z) = t(x, t(y,z))$

$t(t(x,y),z) = t(\max[0, x+y-1], z]$
$= \max[0, \max[0, x+y-1] + z - 1]$

Considering $x+y-1 \leq 0$

$= \max[0, \max[0, 0 \text{ or negative value}] + z - 1]$
$= \max[0, z-1]$
$= 0$

considering $x+y-1 \geq 0$
$= \max[0, \max(0, x+y-1) + z - 1]$
$= \max[0, x+y-1+z-1]$
$= \max[0, x+y+z-2)$

$t(x, t(y,z)) = t(x, \max[0, y+z-1])$
$= \max[0, x + \max[0, y+z-1] - 1]$

Considering
$y+z-1 \leq 0$ then $\max[0, y+z-1] = 0$

$\Rightarrow \max[0, x+0-1]$
$= \max[0, x-1]$
$= 0$

considering
$y+z-1 \geq 0$ then $\max[0, y+z-1] = y+z-1$

$= \max[0, x+y+z-1-1]$
$= \max[0, x+y+z-2]$

Since $t(x, t(y,z)) = t(t(x,y), z)$ are both equal hence associate property is satisfied.

Since $\max[0, x+y-1]$ satisfies all the properties it is indeed a t-norm.

corresponding t-conorm (s-norm)

$s(x,y) = x+y - t(x,y)$
$s(x,y) = x+y - \max[0, x+y-1]$