

Efficient Algorithms for Learning Sensor Scheduling Strategies in Intrusion Detection Games with Uncertain Payoffs

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Abstract

We study the problem of sensor scheduling for an intrusion detection task. We model this as a 2-player zero-sum game over a graph, where the defender (Player 1) seeks to identify the optimal sensor scheduling strategy to minimize the probability of missed detection at minimal cost, while the intruder (Player 2) aims to identify the optimal path selection strategy to maximize missed detection probability at minimal cost. The defender’s strategy space grows exponentially with the number of sensors, making direct computation of the Nash Equilibrium (NE) strategies computationally expensive. To tackle this, we propose a distributed variant of the Weighted Majority algorithm that exploits the structure of the game’s payoff matrix, enabling efficient computation of the NE strategies with provable convergence guarantees. Next, we consider a more challenging scenario where the defender lacks knowledge of the true sensor models and, consequently, the game’s payoff matrix. For this setting, we develop online learning algorithms that leverage bandit feedback from sensors to estimate the NE strategies. By building on existing results from perturbation theory and online learning in matrix games, we derive high-probability order optimal regret bounds for our algorithms. Finally, through simulations, we demonstrate the empirical performance of our proposed algorithms in both known and unknown payoff scenarios.¹

Keywords: Online Learning, Matrix Games, Sensor Scheduling, Finite Sample Convergence

1. Introduction

In the modern landscape of security and surveillance, efficient deployment and scheduling of sensors is critical in ensuring comprehensive monitoring and intrusion detection (Guvensan and Yavuz (2011); Murray et al. (2007)). In managing directional sensor resources, the challenge involves not only their placement, but also their dynamic scheduling to maximize detection efficiency while minimizing resource usage and operational costs (Osais et al. (2009)). Traditionally, sensor scheduling has been approached through various optimization techniques, yet these methods often fall short in adversarial settings where an intelligent intruder can actively attempt to evade detection. In such scenarios, game theory offers a robust framework for modeling and solving the intricate interactions between the defending system and the intruding adversary (Pirani et al. (2021); An et al. (2017)). Additionally, the defending system may not have accurate knowledge of sensor performance, as it can be influenced by environmental factors, hardware limitations, or adversarial interference. These uncertainties lead to discrepancies between the simulated and real-world game dynamics, requiring the defending system to learn the true game online, and adaptively refine strategies using feedback from sensors to ensure effective detection and maintain performance in the face of evolving conditions.

1. An extended version of this paper that includes all the omitted proofs can be found here.

1.1. Related Work

Security Games: Many works in the literature have modeled security resource allocation tasks using the Stackelberg game framework (Sinha et al. (2018)). In Paruchuri et al. (2008), the authors model a security patrolling task to protect airports from attackers as a Stackelberg game, where the defender commits to a mixed strategy over an exponentially sized set of pure strategies for allocating patrolling resources (e.g., guards). The attacker observes the defender’s mixed-strategy and best-responds with a pure attack strategy. The authors of Korzhyk et al. (2010) study the complexity of computing Stackelberg equilibrium strategies for large games. However, we consider the setting where both the players commit to mixed-strategies. Additionally, for the security game studied in this paper, the Stackelberg and Nash equilibrium are equivalent, as neither of the agents have a first-mover advantage (Korzhyk et al. (2011)).

Solving Large Games: A line of work focuses on developing efficient techniques for approximately solving games with large strategy spaces (Ganzfried and Sandholm (2015); Lipton et al. (2003); Sandholm (2015)). The authors of Li and Wellman (2020) introduce an iterative structure-learning approach to search for approximate solutions of many-player games. In Li et al. (2023), the authors study team-adversary games with combinatorial action-spaces. They propose a counterfactual regret minimization framework (also studied in Brown et al. (2019)) for learning individual strategies for each player using regret matching. Along the lines of these works, we propose a distributed variant of the well known Weighted Majority algorithm (proposed in Freund and Schapire (1997)) which leverages the structure in the game’s payoff function, to efficiently and iteratively estimate the Nash equilibrium for a given instance of the game, and establish performance guarantees for convergence.

Online Learning in Games: In Fiez et al. (2021), the authors study online learning in periodic zero-sum games and establish convergence properties for the equilibrium strategies. The authors of Bailey and Piliouras (2019) study online learning in two-person two-strategy games and provide convergence guarantees for gradient descent dynamics. However, for a general class of zero-sum games, the dynamics of the equilibrium strategies generated by the learning process can be very intricate and instances may fail to converge (Andrade et al. (2021); Cheung and Piliouras (2019)). In contrast to the above works, we propose an online learning algorithm that converges to the Nash equilibrium of the true game. Two closest papers to our work on learning in matrix games with bandit feedback are (i) O’Donoghue et al. (2021) - where the authors propose a UCB style algorithm for no-regret learning in matrix games; and (ii) Chen et al. (2024) - where the authors present an adaptation of online mirror descent algorithm with last-iterate convergence rates. In this paper, we build on these works and develop online learning algorithms that exploit the structure of the game matrix, and establish tighter regret bounds, which guarantee faster convergence to NE strategies.

1.2. Contributions

We begin by modelling the sensor scheduling task as a zero-sum matrix game. First, we propose a distributed variant of the Weighted Majority algorithm that leverages the structure of the game’s payoff matrix to efficiently compute Nash Equilibrium (NE) strategies, even in exponentially large strategy spaces, with provable convergence guarantees. Second, recognizing that the defender may lack accurate knowledge of the true sensor models and hence the game’s payoff matrix, we develop online learning algorithms that rely on bandit feedback from the sensors to adaptively estimate the game matrix and refine the defender’s sensor scheduling strategies. By building on results from perturbation theory and online learning in matrix games, we derive high-probability regret bounds

for these algorithms. Finally, we demonstrate the empirical performance of our methods through numerical simulations, for both known and unknown payoff scenarios.

2. Problem Formulation

We model the game environment as a graph $G = (V, E)$, where the set of nodes V denotes the regions of an environment and the set of edges E denotes the possible transitions for the intruder between the regions. Let $\mathcal{S} = \{s_1, \dots, s_p\}$, with $|\mathcal{S}| = p$, denote the set of sensors deployed on a subset of nodes in the graph. Each sensor has d possible orientations, $\Theta = \{\theta_1, \dots, \theta_d\}$, $|\Theta| = d$. Let $c_k^q \in \mathbb{R}_{\geq 0}$ be the cost of orienting the sensor $s_q \in \mathcal{S}$ in the direction $\theta_k \in \Theta$. Each sensor covers a subset of nodes in the graph depending on its orientation, which may not be restricted to its neighboring nodes. Let $I = \{i_1, i_2, \dots, i_m\}$, $m = |I|$, denote the set of joint strategies of the defender, i.e., a set of all possible sensor orientations, and let $\{c_1, c_2, \dots, c_m\}$ be the costs associated with each joint strategy. The cost c_i of a joint strategy $i \in I$ is the sum of the individual costs associated with each sensor orientation in that strategy, i.e., $c_i = \sum_{q=1}^p c_{k_{q,i}}^q$, where $k_{q,i}$ is the orientation of sensor s_q in the joint strategy i . We note that the number of joint strategies is $m = |\Theta|^{|\mathcal{S}|}$, which is exponential in the number of sensors. We assume that the intruder starts at a source node S and aims to enter a protected facility at a terminal node T in the graph. Let $J = \{j_1, j_2, \dots, j_n\}$, $n = |J|$, denote the set of pure strategies of the intruder, i.e., the set of paths in the graph that the intruder can take to reach the node T from node S and let $\{r_1, r_2, \dots, r_n\}$, $r_j \in \mathbb{R}_{\geq 0}$, be the associated costs of the respective paths. Without loss of generality, we do not consider cycles, as it will only hurt the intruder, and thus n is finite. Let X and Y denote the set of mixed-strategies for the defender and intruder, respectively. We have the following:

$$X = \left\{ x \in \mathbb{R}^m; \sum_{i=1}^m x_i = 1, x \geq 0 \right\}; Y = \left\{ y \in \mathbb{R}^n; \sum_{i=1}^n y_i = 1, y \geq 0 \right\}. \quad (1)$$

We first consider the case of homogeneous sensors, i.e., all sensors are identical and have the same probability of detection p_{detect} ². Specifically, if the intruder traverses through a node which is covered by a sensor, it will be detected by that sensor with a probability p_{detect} , and will go undetected with probability $(1 - p_{detect})$. We make the following assumption on the sensor model.

Assumption 1: The sensors are imperfect with $p_{detect} \in [p_{min}, p_{max}]$, where $0 < p_{min} < p_{max} < 1$.

The goal of the defender is to minimize the probability of not detecting the intruder. If a certain node is covered by multiple sensors and the intruder has traversed that node, then the overall probability of missed detection at that node is the product of the probability of missed detection of all the sensors covering that node. For a sensor orientation strategy $i \in I$ and a path $j \in J$, let V_{ij} be the number of nodes that are covered by the sensor orientation strategy $i \in I$ and contained in the intruder's path $j \in J$, with repetition. The overall probability of missed detection for a pure strategy pair (i, j) is given by:

$$p_{miss}(i, j) = (1 - p_{detect})^{V_{ij}}. \quad (2)$$

We consider the log-likelihood representation for the probability of missed detection, given by

$$\log(p_{miss}(i, j)) = V_{ij} \log(1 - p_{detect}). \quad (3)$$

2. We later discuss the setting with heterogeneous sensors in Section 4.1.

Let $A, B \in \mathbb{R}^{m \times n}$ denote the payoff matrices for the defender and intruder, respectively. We will refer to the defender as the x -player and the intruder as the y -player. The x -player plays $x \in X$, the y -player plays $y \in Y$, and they receive payoffs $x^\top Ay$ and $x^\top By$ respectively. The defender's payoff value for a pure strategy pair (i, j) is given by $A_{ij} = \log(p_{miss}(i, j)) + c_i$. The defender aims to find the mixed strategy x^* that minimizes the expected value of its payoff function, given the mixed strategy y of the intruder. In other words, the defender aims to solve the following optimization problem.

$$x^* = \min_{x \in X} x^\top Ay. \quad (4)$$

The intruder's payoff value for a pure strategy pair (i, j) is given by $B_{ij} = -\log(p_{miss}(i, j)) + r_j$. Given the mixed strategy x of the defender, the intruder aims to solve the following optimization problem.

$$y^* = \min_{y \in Y} x^\top By. \quad (5)$$

We will denote this bimatrix game by the following tuple: $\mathcal{G} = \{A, B\}$. Note that \mathcal{G} is a non-zero sum game, since $A + B \neq 0$ in general. We will now transform this bimatrix game into an equivalent zero-sum game and establish that these two games have the same Nash equilibrium. A bimatrix game $\mathcal{G} := \{A, B\}$ is zero-sum if $A + B = 0$, and thus can be specified by a single matrix A (since $B = -A$). The solution to a zero-sum game, i.e., the Nash equilibrium strategies, can be obtained by solving the nested min-max optimization problem below:

$$\min_{x \in X} \max_{y \in Y} x^\top Ay. \quad (6)$$

Definition 1 (Nash Equilibrium) A pair of mixed-strategies (x^*, y^*) is a Nash Equilibrium (NE) of the zero-sum game A if $x^{*\top} Ay^* \leq x^\top Ay^*$ and $x^{*\top} Ay^* \geq x^{*\top} Ay$, $\forall x \in X, \forall y \in Y$.

Consider the following equivalent game $\mathcal{G}' := \{A', B'\}$, where matrices A' and B' are such that

$$A'_{ij} = A_{ij} - r_j; \quad B'_{ij} = B_{ij} - c_i. \quad (7)$$

From the result presented in Section 2.1 of [Kannan and Theobald \(2010\)](#), we have the following result characterizing the equilibrium property of \mathcal{G}' . All detailed proofs supporting our theoretical results are deferred to Appendix B of the extended version.

Proposition 2 The game $\mathcal{G}' := \{A', B'\}$ is zero-sum and has the same set of Nash Equilibria as \mathcal{G} .

With a slight abuse of notation, we will denote the modified game (which is zero-sum) by \mathcal{G} , which can be completely specified by the payoff matrix A , where $A_{ij} = \log(p_{miss}(i, j)) + c_i - r_j$. We consider the setting where the defender has the knowledge of c_i 's and r_j 's for all $i \in I$ and $j \in J$. However, the defender may not know the sensor detection probabilities p_{detect} accurately. As a result, the true game payoff matrix A is may not be known accurately to the defender.

Remark 3 We assume that the entries of A are bounded in $[0, 1]$. If this is not the case, we normalize A using its maximum and minimum values, $A_{min} = \min_{i,j} A_{ij}$ and $A_{max} = \max_{i,j} A_{ij}$, respectively. This normalization preserves the Nash equilibrium strategies, as scaling and shifting the payoffs affect only the game's value, not the equilibrium strategies themselves.

3. Distributed Weighted Majority Algorithm

In this section, we propose a fast and scalable algorithm which leverages the structure of the game matrix A for efficiently solving for the Nash equilibrium strategies. For a moderate number of strategies, the min-max optimization problem in (6) can be solved to compute the Nash Equilibrium using a Linear Programming (LP) formulation. However, for the sensor scheduling problem considered in this paper, the defender has an exponentially sized set of scheduling strategies. For example, if $d = |\Theta| = 4$ (i.e., each sensor has 4 possible orientations) and $|\mathcal{S}| = 10$ (i.e., there are 10 sensors), then $|I| = 4^{10} = 1,048,576$. Solving for the exact Nash Equilibrium can become computationally intractable in such large-strategy space games. To this end, we present the Distributed Weighted Majority (DWM) algorithm (Algorithm 1), a variant of the *Weighted Majority (WM)* algorithm originally developed by Freund and Schapire (1997). The DWM algorithm leverages the structure of the game to iteratively estimate the Nash equilibrium strategies using a multiplicative weight update rule and enjoys theoretical guarantees for convergence. We present the algorithm for a general case of heterogeneous sensors, where each sensor $s_q \in \mathcal{S}$ has a probability of detection $p_{detect,q}$.

For sensors $s_q \in \mathcal{S}$, we define the set of sub-game payoff matrices $\mathcal{A} = \{A^1, A^2, \dots, A^p\}$, $A^q \in \mathbb{R}^{d \times n}$, where

$$A_{kj}^q = V_{kj}^q \times \log(1 - p_{detect,q}) + c_k^q, \quad (8)$$

where V_{kj}^q contains all the nodes that are covered by the sensor s_q oriented in the direction θ_k and contained in the intruder path j . At each time step t and for each sensor $s_q \in \mathcal{S}$, we maintain a set of weights $\sigma_t^q \in \mathbb{R}^d$, where $d = |\Theta|$. The joint sensor scheduling strategy $i \in I$ can be denoted as $i = [k_{1,i}, k_{2,i}, \dots, k_{p,i}]$, where $k_{q,i} \in \{\theta_1, \dots, \theta_d\}$ is the orientation of the sensor $s_q \in \mathcal{S}$ in the joint strategy $i \in I$. We update the weights for each joint strategy $i \in I$ as follows:

$$w_t(i) = \prod_{q=1}^p \sigma_t^q(k_{q,i}), \quad (9)$$

where $\sigma_t^q(k_{q,i})$ is the weight assigned to the orientation θ_k for the sensor s_q in the joint strategy $i \in I$. At time step t , given the intruder's mixed strategy $y_t \in Y$, we define the loss per sensor ($s_q \in \mathcal{S}$) for each individual strategy $\theta_k \in \Theta$ as $\ell_t^q(\theta_k)$, given by

$$\ell_t^q(\theta_k) = \sum_{j \in J} A_{kj}^q y_t(j) - \frac{1}{p} \sum_{j \in J} r_j y_t(j). \quad (10)$$

Algorithm 1, provides mixed strategies for the game \mathcal{G} with the following theoretical guarantees.

Theorem 4 *Let $\{x_1, \dots, x_T\}$ and $\{y_1, \dots, y_T\}$ be the mixed strategies returned by Algorithm 1 after T iterations with $\beta = \left(1 + \sqrt{\frac{2|\mathcal{S}||\ln|\Theta|}{L}}\right)^{-1}$, where $\tilde{L} \geq \min_{i \in I} \sum_{t=1}^T \sum_{q=1}^p \ell_t^q(\theta_k)$. We let $\bar{x} = \frac{1}{T} \sum_{t=1}^T x_t$ and $\bar{y} = \frac{1}{T} \sum_{t=1}^T y_t$. Then, \bar{x} and \bar{y} are valid mixed-strategies for the two players, respectively. The pair (\bar{x}, \bar{y}) approximates the game's Nash Equilibrium value within ϵ_T , given by*

$$\epsilon_T = \frac{\sqrt{2\tilde{L}|\mathcal{S}||\ln|\Theta|}}{T} + \frac{|\mathcal{S}||\ln|\Theta|}{T}. \quad (12)$$

Algorithm 1 Distributed Weighted Majority (DWM) Algorithm

Data: Intrusion Detection Game \mathcal{G} , Parameter $\beta \in (0, 1)$, Number of Iterations T

Result: Mixed-strategies: $\{x_1, \dots, x_T\}, \{y_1, \dots, y_T\}$

Initialize: $\sigma_0^q(\theta_k) = 1/|\Theta|, k = 1, \dots, |\Theta|; w_0(i) = 1/|I|, i = 1, \dots, m$

for $t = 1, \dots, T$ **do**

Compute defender's mixed-strategy $x_t(i) = \frac{w_t(i)}{\sum_{i' \in I} w_t(i')}, \forall i \in I$

Estimate intruder's mixed-strategy $y_t = \max_{y \in Y} x_t^\top A y$

Compute per sensor loss $\ell_t^q(\cdot)$ as in Equation (10)

Update weights for each sensor

$$\sigma_{t+1}^q(k_{q,i}) = \sigma_t^q(k_{q,i}) \beta^{\ell_t^q(\theta_k)}; \quad k = 1, \dots, |\Theta| \quad (11)$$

Update joint strategy weights: $w_{t+1}(i) = \prod_{q=1}^p \sigma_{t+1}^q(k_{q,i}), \forall i \in I$

end

Remark 5 *The DWM algorithm exploits the additive structure of the game's payoff function, achieving a substantial computational advantage by performing only $\mathcal{O}(|S||\Theta|)$ multiplicative updates in parallel (i.e., $\mathcal{O}(|\Theta|)$ updates for each sensor $s \in S$ in parallel), to estimate the NE strategy for the defender. In contrast, directly applying the WM algorithm requires $\mathcal{O}(|\Theta|^{|S|})$ updates, which cannot be parallelized, making DWM far more efficient for large-scale instances.*

4. Online Learning and Adaptation for Unknown Sensor Models

In this section, we study the sensor scheduling problem where the defender lacks knowledge of the true sensor models and, consequently, the game's payoff matrix. We present online learning algorithms with performance guarantees for both homogenous and heterogeneous sensor settings, for refining the defender's strategies based on bandit feedback from the sensors.

Since the defender does not have accurate knowledge of the game's payoff, it maintains an estimate of the game matrix, which is updated iteratively using feedback from sensor observations. At each time step, the defender selects a mixed strategy for scheduling sensors, based on its current estimate of the game matrix. At each time step t , the defender and intruder play pure strategies (i_t, j_t) , sampled from their mixed-strategy distributions (x_t, y_t) respectively. The defender receives feedback from the sensors monitoring the intruder's path, indicating whether or not the intruder was detected. We will first consider the setting with homogenous sensors, where the game matrix A can be parametrized by a single (unknown) Bernoulli distribution with parameter p_{detect} . We define $\mathcal{F}_t = ((i_1, j_1), \dots, (i_t, j_t))$ to be the sequence of observations available to the player at round t , and as a shorthand we shall use the notation $\mathbb{E}^t(\cdot) = \mathbb{E}(\cdot \mid \mathcal{F}_t)$. Two aspects of this problem distinguish it from other setups considered in the literature (Cheung and Piliouras (2019); Andrade et al. (2021); O'Donoghue et al. (2021)). Firstly, the defender receives the (pure-strategy) action played by the intruder (j_t) as feedback (as opposed to the mixed strategy y_t), and secondly, the intruder does not have accurate knowledge of the game payoff function/matrix. In particular, our setting is different from the one considered in O'Donoghue et al. (2021), where the players receive a noisy feedback of the payoff, and the noise process is assumed to be 1-sub-Gaussian with a zero mean. At each

time step t , the defender updates its estimate of \hat{p}_{detect}^t of the true parameter p_{detect} and estimates the game matrix \hat{A}_t (see Algorithm 2). Another key distinction from O'Donoghue et al. (2021) is that our problem setting under homogenous sensors does not need complete exploration of the strategy spaces of the players. As a result, we establish tighter regret bounds that do not depend on the strategy spaces of the players, i.e., $(\tilde{O}(\sqrt{T}))$ instead of $\tilde{O}(\sqrt{|I||J|T})$.

Algorithm 2 Online Learning and Adaptation of Nash Equilibrium Strategies

Feedback samples: $\mathcal{H} = \emptyset$

for $t = 1, \dots, T$ **do**

 Compute sample average \hat{p}_{detect}^t using samples in \mathcal{H}

 Clipping estimator: $\hat{p}_{detect}^t = \min(\hat{p}_{detect}^t, p_{max})$

 Estimate the game matrix: $\hat{A}_t(i, j) = V_{ij} \log(1 - \hat{p}_{detect}^t) + c_i - r_j$

 Solve the estimated game \hat{A}_t :

$$(x_t^*, y_t^*) = \arg \min_{x \in X} \arg \max_{y \in Y} x^\top \hat{A}_t y \quad (13)$$

 Play pure strategy $i_t \sim x_t^*$ and observe intruder's pure strategy $j_t \sim y_t$

 Collect V_{i_t, j_t} samples from sensor feedback into \mathcal{H}_t : $\mathcal{H} \leftarrow \mathcal{H} \cup \mathcal{H}_t$

end

In Algorithm 2, we use a clipped estimator for the Bernoulli parameter \hat{p}_{detect}^t , which keeps the estimate within a feasible range by bounding it from above at p_{max} ; this ensures stability in small sample sizes or noisy data without affecting long-term convergence to the true parameter p_{detect} , since as t grows, the sample mean naturally converges to p_{detect} , making clipping increasingly unnecessary and preserving consistency. Additionally, we note that the estimated game (Equation (20)) in Algorithm 2 can be efficiently solved for the NE strategies using Algorithm 1.

We will perform analysis from the perspective of a single player, i.e., the defender, who does not control the actions of the opponent (i.e., the intruder). Let (x^\dagger, y^\dagger) denote the Nash equilibrium strategies of the true game A , and $V_A^* = x^{\dagger T} A y^\dagger$ be the value of the true game. In order to analyze the performance of the online learning algorithm, we consider the individual regret for the defender, i.e., the difference in expected cumulative payoffs relative to the value of the game \mathcal{G} , given by:

$$\mathcal{R}_T = \sum_{t=1}^T \left(x_t^\top A y_t - V_A^* \right). \quad (14)$$

We have the following result, characterizing the bound on the cumulative regret defined in (14).

Theorem 6 *Under Assumption 1, for a specified $\alpha \in (0, 1)$, we have the following: With probability at least $1 - \alpha$, Algorithm 2 has the following guarantees for the cumulative regret defined in (14):*

1. $\mathcal{R}_T \leq \frac{5\sqrt{2}V_{max}}{(1 - p_{max})} \sqrt{T \log \frac{2T}{\alpha}} = \tilde{O}\left(\sqrt{T \log \frac{1}{\alpha}}\right), y_t = y^\dagger;$
2. $\mathcal{R}_T \leq \frac{2\sqrt{2}V_{max}}{(1 - p_{max})} \sqrt{T \log \frac{2T}{\alpha}} = \tilde{O}\left(\sqrt{T \log \frac{1}{\alpha}}\right), y_t \neq y^\dagger,$

where $V_{\max} = \max_{i,j} V_{ij}$ and p_{\max} is the maximum probability of detection of the sensors.

Remark 7 In [Cai et al. \(2024\)](#), the authors propose an online learning algorithm which is an adaptation of EXP3-IX, where they assume the existence of mixed-strategy Nash equilibrium with full support. Particularly, at each iteration t of their algorithm, they restrict the space of NE strategies to $\Omega_t = \{x \in X : x_a \geq \frac{1}{|I|t^2}, \forall a \in I\}$. However, many zero-sum games are not guaranteed to have a mixed-strategy Nash equilibrium with full support, since players typically can have some dominated strategies. In our algorithm, we do not restrict the space of NE strategies for the players. Additionally, our results presented in Theorem 6 provide tighter regret guarantees $\tilde{O}(T^{-\frac{1}{2}})$ compared to $\tilde{O}(T^{-\frac{1}{8}})$ presented in [Cai et al. \(2024\)](#).

4.1. Intrusion Detection Game with Heterogeneous Sensors

In this section, we consider the setting with heterogeneous sensors, i.e., each sensor has a different probability of detection. In this setting, the game's payoff matrix A is parameterized by a set of $|\mathcal{S}|$ Bernoulli distributions, with parameters $\{p_{\text{detect},1}, \dots, p_{\text{detect},p}\}$, $p = |\mathcal{S}|$, each corresponding to a sensor $s_l \in \mathcal{S}$, $l = 1, \dots, p$. The game's payoff matrix in this setting can be expressed as follows:

$$A_{ij} = \sum_{l=1}^p V_{ij,l} \log(1 - p_{\text{detect},l}) + c_i - r_j, \quad (15)$$

where $V_{ij,l}$ is the number of nodes covered by the sensor l in the joint strategy $i \in I$ and contained in the intruder's path $j \in J$, and c_i, r_j are sensor and path costs as specified before, respectively. Define $\max_{ij} V_{ij,l} = V_{\max,l}$. Assumption 1 in the heterogeneous sensor setting implies the following: $0 < p_{\min,l} \leq p_{\text{detect},l} \leq p_{\max,l} < 1$. We note that each entry of A may not depend on all $|\mathcal{S}|$ Bernoulli parameters, as $V_{ij,l}$ can be zero for a subset of sensors in \mathcal{S} . This is due to the fact that every intruder path $j \in J$ will intersect nodes covered in the graph by only a subset of the sensors. Thus, unlike the homogenous sensors setting, we will need exploration of strategies of both the players to ensure better estimates of these Bernoulli parameters.

We adapt the Upper Confidence Bound (UCB) algorithm for learning in matrix games proposed in [O'Donoghue et al. \(2021\)](#), by constructing the UCB estimate for the game matrix A based on the confidence bounds of the Bernoulli parameters. We assume that c_i 's and r_j 's are known.

Lemma 8 For the game matrix A defined in (15), the following holds with probability at least $1 - \delta$,

$$A_{ij} \leq \bar{A}_{ij}^t + \sum_{l=1}^p \frac{V_{\max,l}}{1 - p_{\max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}} = \tilde{A}_{ij}^t, \quad (16)$$

where \bar{A}_{ij}^t is A_{ij} (as in (15)) evaluated with sample averages $\bar{p}_{\text{detect},l}^t$, $n_{ij,l}^t$ is the number of times the intruder chooses the path j which contains nodes covered by the sensor l in the joint orientation strategy i up to (but not including round t), and we use the notation $(1 \vee \cdot) = \max(1, \cdot)$.

We now have the following result that characterizes the bound on the regret (Equation (14)) when Algorithm 1 in [O'Donoghue et al. \(2021\)](#) is applied to the instance of the intrusion detection game with the UCB estimate \tilde{A}_{ij}^t as defined in Equation (16).

Theorem 9 *Let Assumption 1 hold for all sensors. For $T \geq |\mathcal{S}||\Theta||J| \geq 2$ and $\delta = 1/(2T^2|\mathcal{S}||\Theta||J|)$. When Algorithm 1 in O’Donoghue et al. (2021) is applied to the instance of the intrusion detection game with the UCB estimate \hat{A}_{ij}^t (as in (16)), the regret (as in (14)) is bounded as:*

$$\mathcal{R}_T \leq 1 + |\mathcal{S}|\sqrt{2|\Theta||J|T \log(4T^2|\mathcal{S}||\Theta||J|)} = \tilde{O}(\sqrt{|\mathcal{S}|^2|\Theta||J|T}). \quad (17)$$

Remark 10 *Applying Algorithm 1 of O’Donoghue et al. (2021) directly would result in a regret bound that depends exponentially on the number of sensors, i.e., $\tilde{O}(\sqrt{|\Theta|^{|S|}|J|T})$. We leverage the specific structure of the game to achieve significant improvements in the regret bound in Theorem 9, i.e., $\tilde{O}(\sqrt{|\mathcal{S}|^2|\Theta||J|T})$, which will lead to faster convergence to the NE strategy for the defender. We present the adapted UCB algorithm (Algorithm 3) in Appendix A of the extended version.*

5. Experiments

In this section, we evaluate the performance of the proposed algorithms through simulations.

5.1. Empirical Evaluation of Online Learning Algorithm

We consider the intrusion detection game under homogenous sensors, and evaluate the performance of Algorithm 2. We set the true Bernoulli parameter to $p_{detect} = 0.8$ and generate several instances by uniform random sampling of $V_{ij} \in [10, 20], \forall (i, j) \in I \times J$, with $I = \{1, \dots, 10\}$ and $J = \{1, \dots, 20\}$. We solve for the NE strategies (x^\dagger, y^\dagger) using Algorithm 1 with $\epsilon_T = 10^{-6}$ and estimate the optimal game value $V_A^* = x^{\dagger\top} A y^\dagger$. We run Algorithm 2 under both settings: (i) $y_t = y^\dagger$; and (ii) y_t being a random point on the probability simplex Y . We plot the cumulative regrets \mathcal{R}_T and \mathcal{R}_T^\dagger by setting $T = 1000$ in Figure 1(a). We observe the sublinear growth of the regret, demonstrating that the learning algorithm is effectively adapting and improving its performance over iterations.

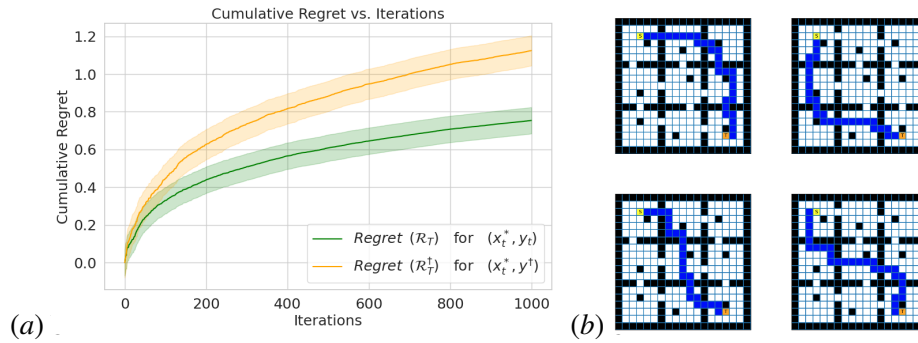


Figure 1: (a) Cumulative regret plots for Algorithm 2; (b) Sample paths (pure strategies) of the intruder in the 9-room grid world

5.2. Empirical Evaluation of Distributed Weighted Majority Algorithm

In this section, we present numerical evaluations to show the computational efficiency of DWM algorithm. We consider the nine-room grid world environment as shown in Figure 2. This is a 19×19 cell environment, which can be represented by a graph. Sensors are installed on the cells (resp. nodes in the graph) marked in red (and numbered), and all cells marked in black are obstacle

cells. We set $|\Theta| = 4$, i.e., each sensor has 4 possible orientations: { East, North, West, South}. We vary the number of sensors $|\mathcal{S}| \in \{2, 3, 4, 5, 6, 7, 8\}$ to generate multiple instances of the game. The defender’s strategy space, given by $4^{|\mathcal{S}|}$, scales exponentially with the number of sensors in \mathcal{S} . The intruder’s strategy space is set to $|J| = 50$, i.e., there are 50 possible paths for the intruder (e.g., see Figure 1(b)) from node S (marked in yellow) to node T (marked in orange) in the grid. All simulations are run on a 2.6 GHz 6-Core Intel Core i7 machine. We set $\epsilon_T = 0.001$, i.e., we wish to compute a ϵ -Nash Equilibrium for the game with $\epsilon = 0.001$. For each instance of the game, we compute the number of iterations T based on $|I|$ and ϵ_T using Equation (21) by setting $\tilde{L} = T$. We use to solve Linear Programs and compute the Nash equilibrium strategies.

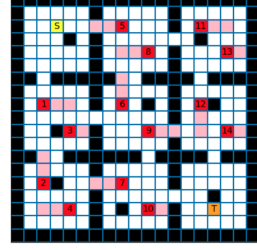


Figure 2: Sample sensor scheduling strategy (pure strategy) of the defender

We solve for the NE strategies using three approaches: (i) a standard bi-level Linear Program (LP), (ii) WM algorithm (as in Freund and Schapire (1997)) and (iii) DWM algorithm (Algorithm 1) for T iterations, and record the running times in Table 1. We observe that the WM and DWM algorithms solve the game to within a 0.001 approximation of the game value much faster than the LP. We observe that as the strategy space increases, the DWM algorithm significantly outperforms the WM algorithm. Due to limitations on the problem sizes in Gurobi Solver (academic license), we cannot solve LPs for problem instances with strategy spaces more than 2000.

$ \mathcal{S} $	Strategy Spaces	Linear Program	Weighted Majority	Distributed Weighted Majority
2	16×50	3.22	2.73	2.68
3	64×50	7.90	3.04	3.65
4	256×50	24.88	6.09	4.28
5	1024×50	75.72	12.15	6.13
6	4096×50	-	35.51	7.88
7	$16,384 \times 50$	-	107.65	9.92
8	$65,536 \times 50$	-	365.43	11.83

Table 1: Comparison of running times (in seconds) of Distributed Weighted Majority Algorithm (Algorithm 1) with standard *Weighted Majority* algorithm and Linear Program

6. Conclusions

In this paper, we study sensor scheduling for intrusion detection by formulating the problem as a zero-sum game over a graph. We proposed a distributed algorithm, which leverages the structure of the game’s payoff, for efficiently computing equilibrium strategies for instances with exponentially large strategy spaces. Next, we studied the game under unknown sensor models, and proposed online learning algorithms with high-probability regret guarantees for learning equilibrium strategies under bandit feedback. Empirical evaluations demonstrate the practical effectiveness of our approach, showing near-optimal and scalable performance over several instances of the game. In future work, we aim to extend the framework to evolving game environments, where strategies dynamically adapt to changes in graph structure and stochasticity in the game dynamics.

Acknowledgments

This material is based upon work supported by the Office of Naval Research (ONR) and Saab, Inc. under the Threat and Situational Understanding of Networked Online Machine Intelligence (TSUNOMI) program (grant no. N00014-23-C-1016). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the ONR and/or Saab, Inc.

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Appendix A. Upper Confidence Bound Algorithm for Matrix Games

In this section, we present the adaptation of the UCB algorithm in [O'Donoghue et al. \(2021\)](#) for the intrusion detection game studied in this paper. The UCB algorithm and regret analysis in [O'Donoghue et al. \(2021\)](#) is presented with respect to the max-player, however, in our game setting, the defender is the min-player. From Remark 3, we have the following:

$$(x^*, y^*) = \arg \min_{x \in X} \arg \max_{y \in Y} x^\top A y = \arg \max_{x \in X} \arg \min_{y \in Y} x^\top (-A) y \quad (18)$$

$$V_A^* = -V_{-A}^*. \quad (19)$$

Therefore, without loss of generality, we will consider the game matrix to be $-A$ and note that the UCB estimate for analyzing the regret will remain the same, as the following relationship holds for the regret $\mathcal{R}(A, \text{UCB}, T)$ defined in [O'Donoghue et al. \(2021\)](#): $\mathcal{R}(A, \text{UCB}, T) = -\mathcal{R}_T$, where \mathcal{R}_T is the regret measure defined in our paper in (14).

Algorithm 3 UCB Algorithm for Online Learning and Adaptation

Feedback samples: $\mathcal{H} = \emptyset$

for $t = 1, 2, \dots, T$ **do**

Compute sample average $\bar{p}_{detect,l}^t$ using samples in \mathcal{H}

Clipping estimator: $\bar{p}_{detect,l}^t = \min(\bar{p}_{detect,l}^t, p_{max,l})$; $l = 1, \dots, p$

Compute $\tilde{A}_{ij}^t = \bar{A}_{ij}^t + \sum_{l=1}^p \frac{V_{max,l}}{1 - p_{max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}}$

Solve the estimated game \tilde{A}^t :

$$(x_t^*, y_t^*) = \arg \max_{x \in X} \arg \min_{y \in Y} x^\top \tilde{A}^t y \quad (20)$$

Play pure strategy $i_t \sim x_t^*$ and observe intruder's pure strategy $j_t \sim y_t$

Collect samples from sensor feedback into \mathcal{H}_t : $\mathcal{H} \leftarrow \mathcal{H} \cup \mathcal{H}_t$

end

Appendix B. Theoretical Proofs

B.1. Proposition 2

Proposition *The game $\mathcal{G}' := \{A', B'\}$ is zero-sum and has the same Nash Equilibrium as \mathcal{G} .*

Proof We have $A'_{ij} = A_{ij} - r_j$ and $B'_{ij} = B_{ij} - c_i$. By substitution, we have $A'_{ij} = \log(p_{miss}(i, j)) + c_i - r_j$ and $B'_{ij} = -\log(p_{miss}(i, j)) + r_j - c_i$. It follows that $A'_{ij} = -B'_{ij}$ and thus $A'_{ij} + B'_{ij} = 0$. Thus, the game $\mathcal{G}' = \{A', B'\}$ is a zero-sum game. The equivalence of Nash Equilibrium strategies for \mathcal{G} and \mathcal{G}' follows directly from the results presented in Section 2.1 in [Kannan and Theobald \(2010\)](#). ■

B.2. Theorem 4

Theorem Let $\{x_1, \dots, x_T\}$ and $\{y_1, \dots, y_T\}$ be the mixed-strategies returned by Algorithm 1 after T iterations with $\beta = \left(1 + \sqrt{\frac{2|\mathcal{S}|\ln|\Theta|}{\tilde{L}}}\right)^{-1}$, where $\tilde{L} \geq \min_{i \in I} \sum_{t=1}^T \sum_{i_q \in i} \ell_t^q(i_q)$. We let $\bar{x} = \frac{1}{T} \sum_{t=1}^T x_t$ and $\bar{y} = \frac{1}{T} \sum_{t=1}^T y_t$. Then, \bar{x} and \bar{y} are valid mixed-strategies for the two players, respectively. The pair (\bar{x}, \bar{y}) approximates the game's Nash Equilibrium within ϵ_T , given by

$$\epsilon_T = \frac{\sqrt{2\tilde{L}|\mathcal{S}|\ln|\Theta|}}{T} + \frac{|\mathcal{S}|\ln|\Theta|}{T}. \quad (21)$$

Proof We establish this result by showing the equivalence between the multiplicative weight updates made by the DWM algorithm (Algorithm 2) to that of the Weighted Majority algorithm in Freund and Schapire (1997). Consider the per-agent loss function

$$\ell_t^q(\theta_k) = \sum_{j \in J} A_{kj}^q y_t(j) - \frac{1}{p} \sum_{j=1}^n r_j y_j. \quad (22)$$

Now, consider the joint strategy weight update rule given by

$$w_{t+1}(i) = \prod_{q=1}^p \sigma_{t+1}^q(i_q). \quad (23)$$

Substituting for $\sigma_{t+1}^q(i_q)$, we have

$$w_{t+1}(i) = \prod_{q=1}^p \sigma_t^q(i_q) \beta^{\ell_t^q(i_q)}, \quad (24)$$

$$= \left(\prod_{q=1}^p \sigma_t^q(i_q) \right) \beta^{\sum_{q=1}^p \ell_t^q(i_q)} \quad (25)$$

From definition of A_{kj}^q in (8), we have $\sum_{q=1}^p \ell_t^q(i_q) = \sum_{q=1}^p \left(\sum_{j \in J} A_{kj}^q y_t(j) - \frac{1}{p} \sum_{j=1}^n r_j y_j \right) = \sum_{j \in J} A_{ij} y_j$.

From Equations (11) and (9) we have, $w_{t+1}(i) = w_t(i) \beta^{\ell_t(i)}$, where $\ell_t(i) = \sum_j A_{ij} y_j$. The weight update rule of the DWM algorithm (Algorithm 1) is equivalent to the multiplicative weight update over the defender's joint strategy space when the Weighted Majority Algorithm (as in Freund and Schapire (1997)) is applied to compute the mixed strategies $x \in X$ for the defender. The guarantees presented in this theorem directly follow from the Theorem 5 for the Hedge(β) algorithm presented in Freund and Schapire (1997). We refer interested readers to Littlestone and Warmuth (1994) and Nehme (2009) for detailed theoretical analysis of the weighted majority algorithm for zero-sum games. ■

B.3. Theorem 6

Theorem Under Assumption 1, for a specified $\alpha \in (0, 1)$, we have the following: With probability at least $1 - \alpha$, Algorithm 2 has the following guarantees for the cumulative regret defined in (14):

1. $\mathcal{R}_T \leq \frac{5\sqrt{2}V_{max}}{(1 - p_{max})} \sqrt{T \log \frac{2T}{\alpha}} = \tilde{O}(\sqrt{T \log \frac{1}{\alpha}}), y_t = y^\dagger;$
2. $\mathcal{R}_T \leq \frac{2\sqrt{2}V_{max}}{(1 - p_{max})} \sqrt{T \log \frac{2T}{\alpha}} = \tilde{O}(\sqrt{T \log \frac{1}{\alpha}}), y_t \neq y^\dagger,$

where $V_{max} = \max_{i,j} V_{ij}$ and p_{max} is the maximum probability of detection of the sensors.

Proof We begin by establishing a bound on $\|\hat{A}_t - A\|_\infty$. For each round of interaction t , the players play strategies i_t, j_t and the defender receives V_{i_t, j_t} feedbacks from the sensors, i.e., for each node l traversed in the path j_t and covered by the joint sensor strategy i_t , the sensors get a 0/1 feedback whether the intruder was detected or not, denoted by the Bernoulli random variable $Z_{ij}^{t,l}$. For round t , the estimate of Bernoulli parameter \hat{p}_{detect}^t is computed as follows:

$$\hat{p}_{detect}^t = \frac{1}{V_{i_t, j_t}} \sum_{l=1}^{V_{i_t, j_t}} Z_{ij}^{t,l}. \quad (26)$$

The sample average for T rounds of interaction is given by

$$\hat{p}_{detect}^\top = \frac{1}{T} \sum_{t=1}^T \left(\frac{1}{V_{i_t, j_t}} \sum_{l=1}^{V_{i_t, j_t}} Z_{ij}^{t,l} \right). \quad (27)$$

After T rounds, the average number of samples obtained per round is given by

$$\bar{k} = \frac{1}{T} \sum_{t=1}^T V_{i_t, j_t}, \quad (28)$$

and the total number of samples is $\bar{k}T$. Let p_{detect} denote the true Bernoulli parameter, i.e., true probability detection of the sensors. Applying Hoeffding's inequality, we have

$$\mathbb{P} \left(|\hat{p}_{detect}^\top - p_{detect}| \geq \epsilon_T \right) \leq 2 \exp(-2\bar{k}T\epsilon_T^2). \quad (29)$$

Define $\eta = 2 \exp(-2\bar{k}T\epsilon_T^2)$. With probability at least $1 - \eta$, for $T > 1/(2\bar{k}T\epsilon_T^2) \log 2/\eta$, we have $|\hat{p}_{detect}^\top - p_{detect}| \leq \epsilon_T$.

We will now establish a bound for $\|\hat{A}_t - A\|_\infty$. By substituting for A_{ij} , we have,

$$\|\hat{A}_t - A\| = V_{ij} |\log(1 - \hat{p}_{detect}^t) - \log(1 - p_{detect})|. \quad (30)$$

By applying Mean Value Theorem to the function $\log(1 - p)$, we have the following bound

$$|\log(1 - \hat{p}_{detect}^t) - \log(1 - p_{detect})| \leq \frac{|\hat{p}_{detect}^t - p_{detect}|}{1 - \max(\hat{p}_{detect}^t, p_{detect})}. \quad (31)$$

By replacing V_{ij} with $\max_{ij} V_{ij} = V_{max}$ and $\max(\hat{p}_{detect}^t, p_{detect})$ with p_{max} .

$$\|\hat{A}_t - A\|_\infty \leq \frac{V_{max} |\hat{p}_{detect}^t - p_{detect}|}{1 - p_{max}}. \quad (32)$$

Denote $\delta_t = \frac{V_{max} |\hat{p}_{detect}^t - p_{detect}|}{1 - p_{max}}$, which is $\delta_t = V_{max} \epsilon_t / (1 - p_{max})$.

With probability $1 - \eta$, we have $\epsilon_t = \sqrt{\log(2/\eta)/2t}$, and thus

$$\|\hat{A}_t - A\|_\infty \leq \frac{V_{max}}{\sqrt{2t}(1 - p_{max})} \sqrt{\log \frac{2}{\eta}}. \quad (33)$$

We will now analyze the regret \mathcal{R}_T as defined in (14). Consider Case 1, where $y_t = y^\dagger$, i.e., the intruder knows the true game matrix A and plays the NE strategy y^\dagger of the true game. Consider the instantaneous regret \mathcal{R}_t for round t :

$$\mathcal{R}_t = x_t^{*\top} A y^\dagger - V_A^* = x_t^{*\top} A y^\dagger - x^{\dagger T} A y^\dagger \quad (34)$$

By adding and subtracting $x_t^* \hat{A}_t y^\dagger$, we have

$$\mathcal{R}_t = \underbrace{x_t^{*\top} A y^\dagger - x_t^{*\top} \hat{A}_t y^\dagger}_{\text{Term-I}} + \underbrace{x_t^{*\top} \hat{A}_t y^\dagger - x^{\dagger T} A y^\dagger}_{\text{Term-II}} \quad (35)$$

Term-I:

$$x_t^{*\top} A y^\dagger - x_t^{*\top} \hat{A}_t y^\dagger = x_t^{*\top} (A - \hat{A}_t) y^\dagger \leq \|\hat{A}_t - A\|_\infty = \delta_t. \quad (36)$$

To Term-II, we add and subtract $x^{\dagger T} \hat{A}_t y^\dagger$, and obtain the following:

$$\text{Term-II} = \underbrace{x_t^{*\top} \hat{A}_t y^\dagger - x^{\dagger T} \hat{A}_t y^\dagger}_{\text{Term-II-I}} + \underbrace{x^{\dagger T} \hat{A}_t y^\dagger - x^{\dagger T} A y^\dagger}_{\text{Term-II-II}} \quad (37)$$

Now consider Term-II-I. We know that x_t^*, y_t^* is the NE for \hat{A}_t . Since the y-player is maximizing the expected game value, we have $x_t^* \hat{A}_t y_t^* \geq x_t^* \hat{A}_t y, \forall y$ and thus we have $x_t^* \hat{A}_t y^\dagger \leq x_t^* \hat{A}_t y_t^*$. As a result, we have

$$\text{Term-II-I} \leq x_t^* \hat{A}_t y_t^* - x^{\dagger T} \hat{A}_t y^\dagger. \quad (38)$$

The above expression can be viewed as the difference in the game values when the NE strategies of a game A , i.e., (x^\dagger, y^\dagger) , are played in a perturbed game \hat{A}_t . From Theorem 4 in [Lipton et al. \(2006\)](#), we have the following:

$$\text{Term-II-I} \leq 3\|\hat{A}_t - A\|_\infty = 3\delta_t. \quad (39)$$

Term-II-II

$$x^{\dagger T} \hat{A}_t y^\dagger - x^{\dagger T} A y^\dagger = x^{\dagger T} (\hat{A}_t - A) y^\dagger \leq \|\hat{A}_t - A\|_\infty = \delta_t. \quad (40)$$

Combining Equations (36), (39) and (40), we have the following bound on the instantaneous regret \mathcal{R}_t :

$$\mathcal{R}_t \leq \delta_t + 3\delta_t + \delta_t = 5\delta_t. \quad (41)$$

The cumulative regret can be bounded as follows. With probability at least $1 - \alpha$,

$$\mathcal{R}_T = \sum_{t=1}^T \mathcal{R}_t \leq \sum_{t=1}^T 5\delta_t; \quad (42)$$

$$\leq \frac{5V_{max}}{\sqrt{2}(1 - p_{max})} \sqrt{\log \frac{2}{\eta}} \int_{t=0}^{\infty} \frac{1}{\sqrt{t}}; \quad (43)$$

$$\leq \frac{5\sqrt{2}V_{max}}{(1 - p_{max})} \sqrt{T \log \frac{2T}{\alpha}}, \quad (44)$$

where $\alpha = T\eta$. This completes the proof of Part 1.

Now consider the scenario where $y_t \neq y^\dagger$, i.e., when the defender is playing against a suboptimal opponent (intruder), who does not play the NE strategy of the true game. The instantaneous regret is given by

$$\mathcal{R}_t = x_t^{*\top} Ay_t^\dagger - V_A^* = x_t^{*\top} Ay_t^\dagger - x_t^{\dagger T} Ay_t^\dagger. \quad (45)$$

By adding and subtracting $x_t^{*\top} \hat{A}_t y_t$, we have

$$\mathcal{R}_t = \underbrace{x_t^{*\top} Ay_t - x_t^{*\top} \hat{A}_t y_t}_{\text{Term-I}} + \underbrace{x_t^{*\top} \hat{A}_t y_t - x_t^{\dagger T} Ay_t^\dagger}_{\text{Term-II}}. \quad (46)$$

Term-I:

$$x_t^{*\top} Ay_t - x_t^{*\top} \hat{A}_t y_t = x_t^{*\top} (A - \hat{A}_t) y_t \leq \|\hat{A}_t - A\|_\infty = \delta_t. \quad (47)$$

In Term-II, we have $x_t^{*\top} \hat{A}_t y_t \leq x_t^{*\top} \hat{A}_t y_t^*$, since the y-player is maximizing the expected game value.

$$x_t^{*\top} \hat{A}_t y_t - x_t^{\dagger T} Ay_t^\dagger \leq x_t^{*\top} \hat{A}_t y_t^* - x_t^{\dagger T} Ay_t^\dagger \quad (48)$$

From Theorem 5 in [Lipton et al. \(2006\)](#), we have that $x_t^{*\top} \hat{A}_t y_t^* - x_t^{\dagger T} Ay_t^\dagger < \|\hat{A}_t - A\|_\infty$, and thus

$$x_t^{*\top} \hat{A}_t y_t - x_t^{\dagger T} Ay_t^\dagger \leq \|\hat{A}_t - A\|_\infty = \delta_t \quad (49)$$

Combining the terms in Equations (47) and (49), we have

$$\mathcal{R}_t \leq \delta_t + \delta_t = 2\delta_t. \quad (50)$$

Using similar arguments as in Case 1, we have with probability at least $1 - \alpha$,

$$\mathcal{R}_T \leq \frac{2\sqrt{2}V_{max}}{(1 - p_{max})} \sqrt{T \log \frac{2T}{\alpha}} \quad (51)$$

■

B.4. Lemma 8

Lemma For the game matrix A defined in (15), the following holds with probability at least $1 - \delta$,

$$A_{ij} \leq \bar{A}_{ij}^t + \sum_{l=1}^p \frac{V_{max,l}}{1 - p_{max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}} = \tilde{A}_{ij}^t, \quad (52)$$

where \bar{A}_{ij}^t is A_{ij} (as in (15)) evaluated with sample averages $\bar{p}_{detect,l}^t$, $n_{ij,l}^t$ is the number of times the intruder chooses the path j which contains nodes covered by the sensor l in the joint orientation strategy i up to (but not including round t), and we use the notation $(1 \vee \cdot) = \max(1, \cdot)$.

Proof Consider the following expression for A_{ij} :

$$A_{ij} = \sum_{l=1}^p V_{ij,l} \log(1 - p_{detect,l}) + c_i - r_j, \quad (53)$$

We have the following confidence bound estimate for $p_{detect,l}$ which follows from Hoeffding inequality (alternatively a special case of Chernoff's bound): With probability at least $1 - \delta$

$$p_{detect,l} \leq \bar{p}_{detect,l}^t + \sqrt{\frac{\log \frac{2}{\delta}}{2(1 \vee n_{ij,l}^t)}} = \tilde{p}_l^t \quad (54)$$

Using the similar technique as in proof of Theorem 6, we apply Mean Value Theorem to the function $A_{ij}(p_{detect,l})$ with respect to $p_{detect,l}$, $l = 1, \dots, p$ (as in (53)) to obtain the following UCB estimate for A_{ij}

$$A_{ij} \leq \bar{A}_{ij}^t + \sum_{l=1}^p \frac{V_{max,l}}{1 - p_{max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}} = \tilde{A}_{ij}^t, \quad (55)$$

where $V_{max,l}$ and $p_{max,l}$ are as defined in Section 4.1. ■

B.5. Theorem 9

Theorem Let Assumption 1 hold for all sensors. For $T \geq |\mathcal{S}||\Theta||J| \geq 2$ and $\delta = 1/(2T^2|\mathcal{S}||\Theta||J|)$, the regret as defined in Equation (14) for Algorithm 1 in O'Donoghue et al. (2021) applied to the instance of the intrusion detection game with the UCB estimate \tilde{A}_{ij}^t (as in (16)) is bounded as:

$$\mathcal{R}_T \leq 1 + |\mathcal{S}| \sqrt{2|\Theta||J|T \log(4T^2|\mathcal{S}||\Theta||J|)} = \tilde{O}(\sqrt{|\mathcal{S}|^2|\Theta||J|T}). \quad (56)$$

Proof We establish proof by making similar arguments as in Theorem 1 of O'Donoghue et al. (2021). Note that we discuss the equivalence of our regret definition and problem setting with the one presented in O'Donoghue et al. (2021) in Appendix A.

Since the upper confidence matrix (55) overestimates the true matrix, we have $V_{\tilde{A}_t}^* \geq V_A^*$. Let E_t be the event that there exists a pair i, j such that $p_{detect,l} > \tilde{p}_l^t$ for $l = 1, \dots, p$. By definition,

$E_t \in \mathcal{F}_t$. Consider for a moment that E_t does not hold and let x_t^* be the best-response of the x-player at round t . We have the following for the instantaneous regret \mathcal{R}_t :

$$\mathcal{R}_t = \mathbb{E}^t(V_A^* - x_t^\top A y_t) \leq \mathbb{E}^t(V_{\tilde{A}_t}^* - x_t^\top A y_t) \quad (57)$$

$$= \mathbb{E}^t(x_t^* \tilde{A}_t y_t - x_t^\top A y_t) \quad (58)$$

$$\leq \mathbb{E}^t(x_t(\tilde{A}_t - A)y_t) \quad (59)$$

$$= \mathbb{E}^t \left[\sum_{l=1}^p \frac{V_{max,l}}{1 - p_{max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}} \right] \quad (60)$$

The cumulative regret is given by

$$\mathcal{R}_T = \sum_{t=1}^T \mathcal{R}_t \quad (61)$$

$$\leq \mathbb{E} \left[\sum_{t=1}^T \sum_{l=1}^p \frac{V_{max,l}}{1 - p_{max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}} \right] + T\mathbb{P}(\cup_{t=1}^T E_t) \quad (62)$$

We bound the second term as follows: $T\mathbb{P}(\cup_{t=1}^T E_t) \leq 2T^2|\mathcal{S}||\Theta||J|\delta \leq 1$.

The first term is bounded by

$$\mathbb{E} \left[\sum_{t=1}^T \sum_{l=1}^p \frac{V_{max,l}}{1 - p_{max,l}} \sqrt{\frac{\log(\frac{2}{\delta})}{2(1 \vee n_{ij,l}^t)}} \right] \leq \sum_{ij} \mathbb{E} \sum_{l=1}^p \sqrt{2n_{ij,l}^T \log \frac{2}{\delta}} \quad (63)$$

$$\leq |\mathcal{S}| \sqrt{2|\mathcal{S}||\Theta||J| \log \frac{2}{\delta}}. \quad (64)$$

By substituting for δ , we have with probability at least $1 - \delta$,

$$\mathcal{R}_T \leq 1 + |\mathcal{S}| \sqrt{2|\Theta||J|T \log(4T^2|\mathcal{S}||\Theta||J|)} = \tilde{O}(\sqrt{|\mathcal{S}|^2|\Theta||J|T}). \quad (65)$$

■