CS557: Cryptography

Cryptographic Hash Function II

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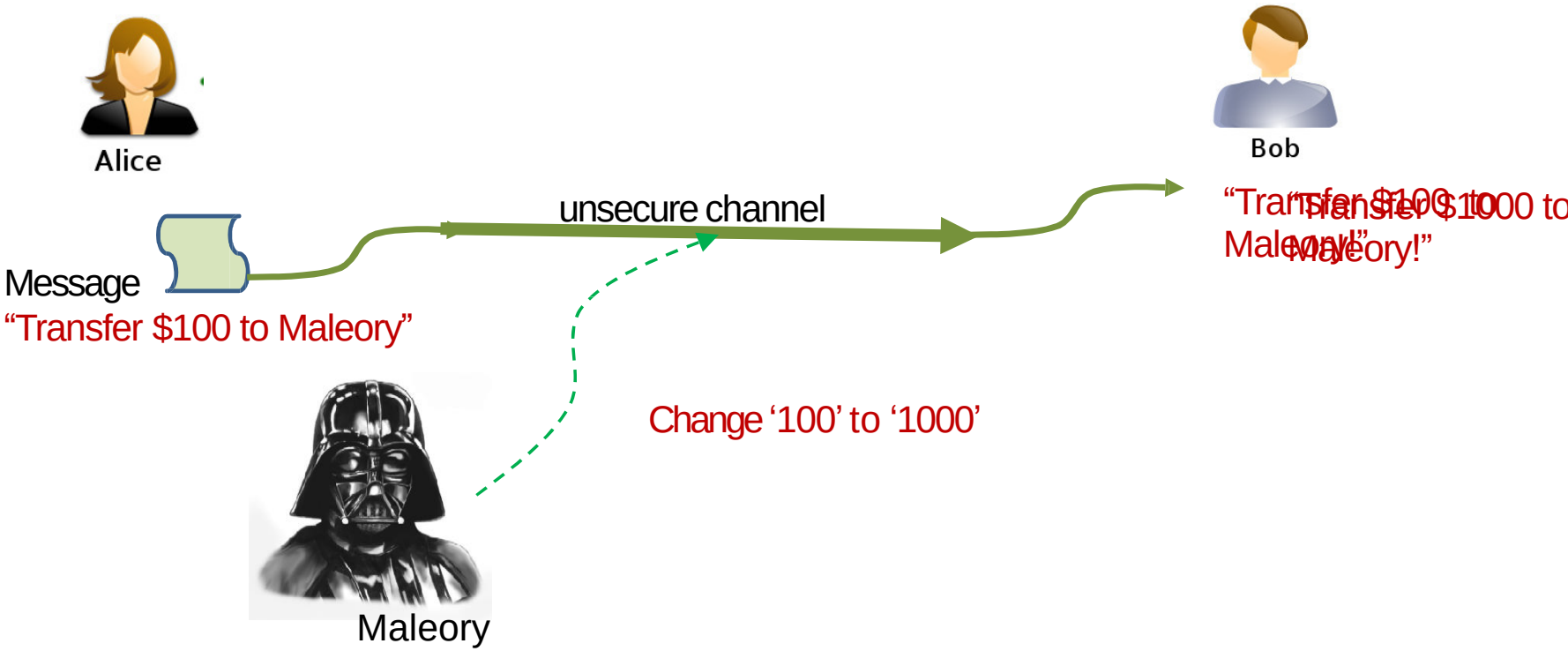
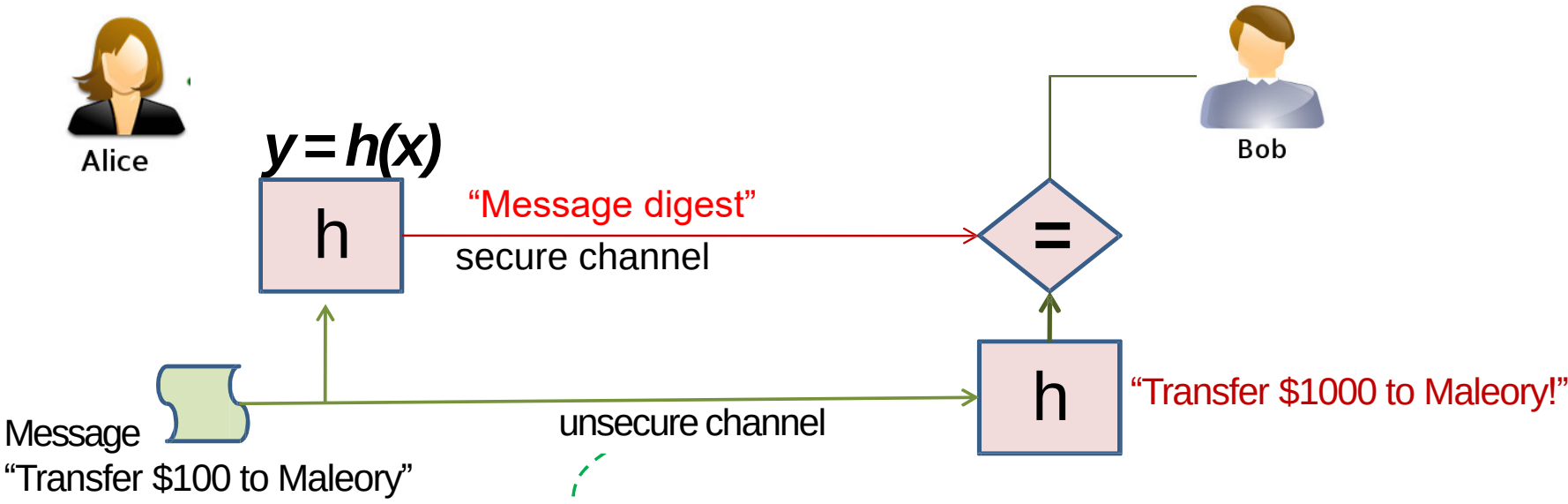
Issues with Integrity

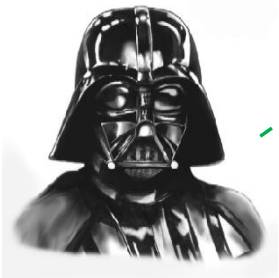
Note…. We are not concerned with confidentiality Now



How can Bob ensure that Alice's message has not been modified ?

Hash (Manipulation Detection code





To suceed

Maleory needs to modify x to x', such that

$h(x) = h(x')$

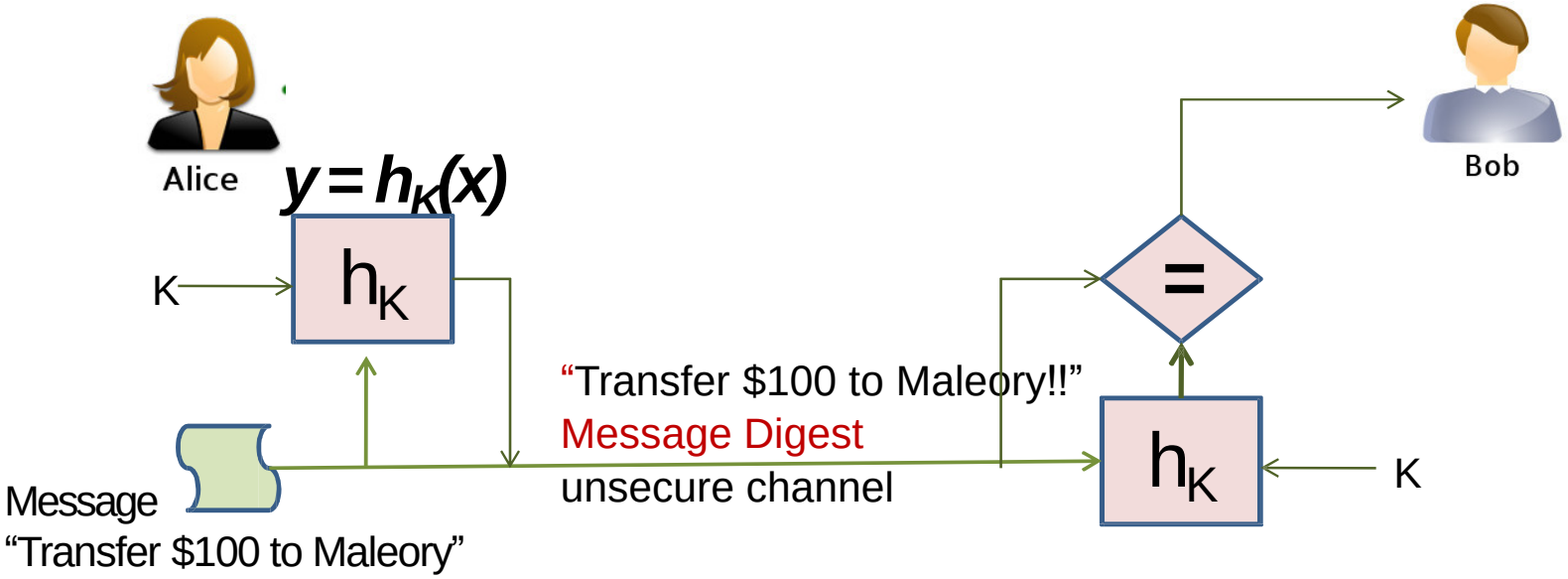
Goal of Hash functions is to resist such collisions

Attacks against MDC

OWHF: given y find x s.t. h(x)=y; or given (x,h(x)) find $x' \neq x$ s.t. h(x')=h(x)

CRHF: find any two inputs $x' \neq x$ s.t. h(x')=h(x) (birthday attack)

Message Authentication Codes (MAC)



MACs can allow the message and the digest to be sent over an insecure channel

However, it requires Alice and Bob to share a common key

Attacks against MAC

without <u>knowing k</u> compute $(x, h_k(x))$ given $(x_i, h_k(x_i))$ with $x_i \neq x$

Applications of Hash functions in Security

- Digital signatures
- Random number generation
- Key updates and derivations
- One way functions
- MAC
- Detect malware in code
- User authentication (storing passwords)

Hash Function

- Hash Function produces a fingerprint of some file/message/data

  $h = H(M)$

Requirements for Hash Functions

Compression:

Efficiency:

Oneway:

Weak collision resistance

Strong collision resistance:

Simple but Insecure Hash Function

- bit-by-bit XOR of every block
  - $C_i = b_{i1}$ *xor* b_{i2} *xor . . . xor* b_{im}
  - reasonably effective as data integrity check
  - in most normal text files, the high-order bit of each octet is always zero
- H(x,y) = ax + by mod m
  - Not Secure

Bruteforce Attacks on Hash Functions

- Attacking one-wayness
  - Goal: given $h:X \rightarrow Y$, $y \in Y$, find x such that $h(x)=y$
  - Algorithm:
    - pick a random value x in X,
    - if h(x)=y, return x; otherwise iterate
    - after failing q iterations, return fail
  - The average-case success probability is

$$\varepsilon = 1 - \left(1 - 1/|Y|\right)^q \approx \frac{q}{|Y|}$$

  - Let $|Y|=2^m$, to get ε to be close to 0.5, $q \approx 2^{m-1}$

Bruteforce Attacks on Hash Functions

- Attacking collision resistance
  - Goal: given h, find x, x′ such that h(x)=h(x′)
  - Algorithm: pick a random set X_0 of q values in X
    for each $x \in X_0$, computes $y_x=h(x)$
    if $y_x=y_{x'}$ for some $x' \neq x$ then return (x,x′)
    else fail
  - The average success probability is
  - Analogous to that of Birthday paradox

Birthday Paradox

- The probability that in a group of people two will share the same birthday

Event A :atleast two people in the group have the same birthday

Event A': no two people in the group have the same birthday

$$\Pr[A] = 1 - \Pr[A']$$

$$\Pr[A] = 1 - \prod_{i=1}^{Q-1}\left(1 - \frac{i}{365}\right)$$

- If 23 people are there in a room, the probability that two people would share the same birthdays is 1/2

Hash Functions

Effort Required for length = n-bit

One Way / Pre-image	2^n
Weak Collision/ 2^{nd} Pre-image Resistance	2^n
(Strong) Collision Resistance	$2^{n/2}$

- Finding collisions is easier than solving pre- image or second preimage

- Thanks